

BOOKS BY CHARLES B. BREED
AND
GEORGE L. HOSMER

THE PRINCIPLES AND PRACTICE OF SURVEYING

Volume I. Elementary Surveying.

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BY CHARLES B. BREED

SURVEYING

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SURVEYING

By

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PREFACE

There has been a persistent trend in the technical colleges toward broadening the instruction in civil engineering to include the newly developed branches of the profession. This is a natural tendency and is in line with technological progress. To obtain time for these new subjects, instruction has had to be curtailed in some of the older branches of the profession. The time given to Surveying has been particularly subject to abridgment. Hence a demand has been created for short texts on surveying that will adequately cover the fundamentals in the limited time now allotted to the subject. This text has been prepared to meet the requirement.

Many of the shorter courses in surveying are so limited for time, that emphasis must be placed on the principles and on the use of surveying instruments. Consequently, this part of the text has been made quite complete, and many field exercises and problems have been included, which, if carefully worked out, will give the student a sound fundamental knowledge of surveying technique.

The usual field and office practices in land surveys, in leveling, and in topographical surveys are thoroughly covered. The chapter on Lines and Grades is particularly designed to give an insight into the surveying problems that arise in connection with engineering construction. Aerial Surveying is covered to the extent necessary to identify when it should be employed, and how its accuracy may be controlled by means of ground surveys. The text relating to Public Lands Surveys contains the latest practice. The chapter on Cross-sections and Volumes is rather complete. A single land survey has been carried along from chapter to chapter: the original survey, its field-notes, its computations, and, finally, the finished map. The student may follow the entire process as applied to one typical survey.

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CHAPTER

SURVEYS AND THEIR PURPOSES SOURCES OF SURVEYING INFORMATION

1. INTRODUCTION. Surveying is the art of measuring and locating lines, angles, and elevations on the earth's surface. It requires care, skill, attention to details, and good judgment. The principles are few; the applications are many and varied. Each survey presents particular problems, but if the principles are properly mastered, the resourceful surveyor will have no difficulty in making the necessary applications.

It is essential that the surveyor shall understand the limitations of the instruments which he is using and the ever present possibility of errors inherent in the process and mistakes due to carelessness. The sources of error inherent in every survey must be thoroughly understood and guarded against while the survey is being conducted. The exactness, or precision, desired will vary according to the purpose of the survey, but whether the survey is a rough or a precise one, sufficient checks must be applied in the field and in the preparation of plans so that inexcusable errors and mistakes may be eliminated from the final results.

Measurements should never be accepted as correct until they have been checked either by repetition or in some other manner.

2. KINDS OF SURVEYS. The surface of the earth is curved. The mathematical figure that best represents the shape of the earth is a spheroid. Surveys that cover so small a part of the earth's surface that its curvature need not be taken into account are called "*plane surveys*." Surveys in which the earth's curvature must be taken into account are called "*geodetic surveys*."

The common types of surveys are known as cadastral, topographic, engineering, route, underground, aerial, and hydrographic.

Cadastral surveys (land surveys) are surveys which are made for determining and defining land boundaries and areas;

they are also used for locating and identifying property lines.

Topographic surveys are those made for determining the shape of the ground and the location of both natural and artificial features upon it.

Engineering surveys are surveys of the sites where construction of an engineering nature is to be undertaken; they include the laying out of lines, grades and detailed dimensions, which serve as guides for the construction work.

Route surveys may also be properly classed under engineering surveys. They comprise surveys of linear construction such as railroads, highways, canals, transmission lines and include the layout of lines and grades for these projects.

Underground surveys consist of surveys for locating the workings below the earth's surface, as in mines, tunnels, and aqueducts.

Aerial surveys are made from an airplane, and for the purpose of mapping the terrain. The control of such a map is obtained from ground surveys; but the details are obtained from the aerial photographs.

Hydrographic surveys are those made for determining the shape of the bottom of lakes, rivers, harbors, and oceans. They also include the measurement of flow of water in streams and the estimation of water resources.

3. DUTIES OF THE SURVEYOR. *The land surveyor* should be expert in subdividing land, retracing old boundaries, analyzing evidence as to the legality of boundaries, identifying boundaries, and making surveys of properties. He furnishes accurate descriptions of boundaries and areas which are required in preparing land titles. Such titles are usually drawn by an attorney called a conveyancer. The surveyor has not the legal authority to determine property boundaries, but his expert knowledge and advice will often enable him to bring owners of adjoining land to an agreement as to boundaries and thereby possibly avoid adjudication. For this reason he must know and be able to apply the legal principles governing competency and weight of evidence as to boundaries.

A large amount of the country's wealth is either in land itself or in properties which are permanently fixed to the land. It is the responsibility of both the surveyor and the convey-

ancer to protect the public in the purchases and sales of lands.

The topographic surveyor uses the control points established by government or private surveys and fills in the details of the earth's surface and physical features, such as hills, valleys, streams, roads, railroads, buildings, and fences. For small surveys the control is usually provided by a transit and tape survey which, wherever practicable, is tied in to one or more government survey points.

The surveyor engaged in engineering works is responsible for an accurate determination of the terrain upon which such works are to be built. The design of the structure depends in many respects upon the information shown on the surveyor's plans. He has a further responsibility in laying out on the ground the lines and grades of the proposed work. An error in the survey which is not discovered until the construction work has started may result in such costly changes in design and construction that the cost of the survey may be many times exceeded. The surveyor also measures and computes the pay quantities of the different classifications of construction work designated by the contract.

The geodetic surveyor's field of interest includes practical astronomy, terrestrial magnetism, oceanography, gravity, the figure of the earth, and seismology. His service to the ordinary surveyor lies in locating, with high precision, survey stations for horizontal control, and "bench marks" (permanent points of elevation) for vertical control of detailed surveys.

4. SOURCES OF SURVEYING INFORMATION — Local Sources. The engineering or the street departments in counties, cities, and towns have on file the lines and grades of all "accepted" streets and of many "unaccepted" streets. The determined elevations and descriptions of permanent points called bench marks are also on file in such departments.

In metropolitan districts there are usually special commissions from which may be obtained many engineering and surveying data relating to parks, sewage works, water supply, waterfront improvements, and rapid transit facilities.

The registry of deeds will furnish a description of every parcel of land that has been conveyed. These descriptions, as a rule, are not sufficiently definite to permit of plotting a plan or of com-

puting the area of the land, but they often, though not always, are sufficient to identify the boundaries. The surveyor usually retains the details of his fieldwork and does not put them all in the deed description or on the plan. Such details are his private property and he is entitled to compensation for releasing them to another surveyor, who may wish to use them in making a later survey of the same property.

5. STATE SURVEYS. State governments keep records of boundary surveys for the state and its subdivisions (counties and towns). Several states have extended the work of the Federal Government by establishing a plane coordinate system covering the entire area, and by determining the coordinates of many control points which are useful to the surveyor. State highway departments have plans of all state highway locations and definitions of rights-of-way in which these highways lie.

6. FEDERAL SURVEYS. The following Federal agencies have a wide range of information useful to the surveyor:

United States Coast and Geodetic Survey, Dept. of Commerce

United States Geological Survey, Dept. of the Interior

General Land Office, Dept. of the Interior

Forest Service, Dept. of Agriculture

Corps of Engineers, War Department

Agricultural Adjustment Administration, Dept. of Agriculture

Soil Conservation Service, Dept. of Agriculture

International Boundary Commission

United States Lake Survey

Mississippi River Commission

National Parks Service

Post Office Department

Board of Surveys and Maps

U. S. Army Airforce

Hydrographic Office, U. S. Navy

The U. S. Coast and Geodetic Survey has available for public use all data obtained from its various classes of field surveys, as well as its manuals of instruction. The scope of this survey covers:

Airway Maps

Astronomical Work

Base-Lines

Cartography

Coast Pilots

Currents

Geodesy

Gravity

Hydrography

Leveling

Nautical Charts

Oceanography

Seismology

Terrestrial Magnetism

Tides

Topography

Traverse

Triangulation

Of particular interest to local surveyors are the publications listing the triangulation stations (latitude and longitude), coordinates of survey points, bench marks (elevations) for the various states, and the magnetic declination.

The U. S. Geological Survey has available for public use all data obtained by its Geologic, Topographic, and Hydrographic branches. Among its publications are:

The Geologic Atlas of the United States. The atlas is a series of folios which contain descriptions of, and maps showing, the geology of various regions of the country.

Topographic Maps of the United States. These maps are printed on paper about $16\frac{1}{2}$ by 20 inches and are published on the following scales: $\frac{1}{250000}$, $\frac{1}{125000}$, $\frac{1}{62500}$, and $\frac{1}{31250}$.

Water Supply Papers give measurements of stream flow and descriptions of streams and gauging stations.

In addition to the above, many special maps and publications on geology, mineral sources, topography, hydrography, and hydrology are available.

The General Land Office (Public Land Surveys) is in charge of the survey of the public lands of the United States. A manual specifying in detail the regulations and methods of carrying on the public land surveys, maps showing the locations of the surveys that have been made, and data concerning the actual surveying, may be procured from this office.

The Bureau of Standards is engaged in tests and comparisons of weights and measures; in such research as is involved in the establishment and maintenance of various standards and units of measurement; in the development of measuring instruments and methods of determining physical constants and the properties of materials. Its activities cover a wide range of subjects in the fields of physics, chemistry, engineering, and commercial practice. For a small fee surveyors may have their tapes standardized by the Bureau.

The Corps of Engineers (U. S. Army) has made a large number of surveys in connection with navigable waters the jurisdiction of which it controls. At its district offices will be found maps defining pier heads, harbor lines, channel lines,

depths of water, and bridge clearances in the harbors and rivers of its districts.

The U. S. Lake Survey has established bench marks in all harbors of the Great Lakes, the elevations of which are based on the Lake Datum. It also has records of the stages of the Lakes since about 1860.

The Mississippi River Commission of the War Department has made extensive surveys of the Mississippi Basin and has published topographic quadrangle sheets on the scale of about 1 inch = 1 mile. The Commission has available also airplane photographs, triangulation, and bench-mark data. It keeps a record of channel lines and depths, river stages and discharge, and it maintains guides for navigation.

The Board of Surveys and Maps issues general specifications for the precision of various types of surveys, and for topographical symbols (conventional signs) to be used on maps issued by the various agencies. The members of the Board are drawn from the various Federal organizations which are engaged in surveying operations.

The remaining Federal bureaus are engaged in surveying work and in the preparation of maps that are especially adapted to their needs.

CHAPTER II

MEASUREMENT OF DISTANCES

7. **HORIZONTAL DISTANCES.** In surveying, the distance between two points means the **horizontal** distance. If the boundary lines of a parcel of land are along sloping surfaces of the ground, the distances recorded in a deed will be the horizontal distances between the corners, and not the slope distances along the ground. In determining the distance between two points the horizontal distance is measured either directly or inclined measurements are made and subsequently reduced to the corresponding horizontal distance. Distances plotted on engineers' plans are always horizontal.

INSTRUMENTS FOR MEASURING DISTANCES

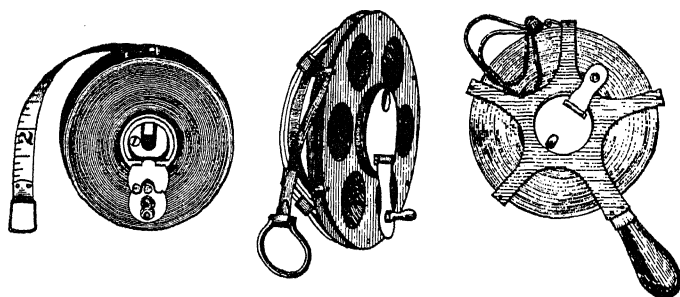
8. **CHAINS.** Old surveys were sometimes made with the *Surveyor's* (or *Gunter's*) *Chain*, which was 66 ft. in length and divided into 100 links, each link 7.92 in. long. Later, the *Engineer's Chain*, which was 100 ft. long, came into use; this had 100 links, each 1 ft. long. Both of these chains are obsolete, but the surveyor may have occasion to use old plans on which the distances are expressed in chains.

The following table shows the simple relation which the Gunter's (or Surveyor's) Chain bears to the mile and acre.

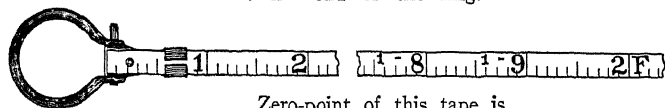
1 Gunter's Chain	= 4 Rods = 66 feet = 100 Links.
1 Mile	= 80 Chains.
1 Acre	= 10 Square Chains = 43,560 square feet.

9. **STEEL TAPES.** For accurate measurements steel tapes are used almost exclusively. The common lengths are 50 ft., 100 ft., and 200 ft., although longer tapes are available. Some steel tapes are graduated throughout their length in feet, tenths, and hundredths; in others the end foot or an additional foot on one end of the tape is graduated either in tenths or in tenths and hundredths. Fig. 1 shows some of the common types of tapes

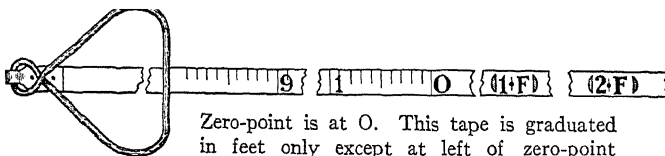
and their graduations. On some tapes the zero-point is at the extreme end of the brass loop; on some it is at the extreme end



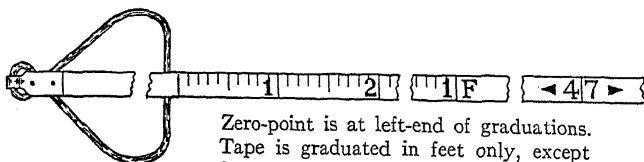
Zero-point of this tape is at the end of the ring.



Zero-point of this tape is the end of the steel ribbon.



Zero-point is at O. This tape is graduated in feet only except at left of zero-point where one foot is graduated in hundredths.



Zero-point is at left-end of graduations. Tape is graduated in feet only, except first foot which is graduated in hundredths.

FIG. 1. COMMON TYPES OF STEEL TAPES.

of the steel ribbon; on others it is marked on the steel ribbon a short distance from the end.

10. **Relation Between Decimals of Foot and Inches.** Surveyors' measurements are made in feet and decimals and the tapes

TAPES

are always so graduated. It is frequently necessary in construction work to convert decimals of a foot into inches and fractions, but the inch is not commonly used in surveying fieldwork. The following table gives relations which enable the surveyor to convert rapidly from one unit to the other.

TABLE 1

DECIMALS OF FOOT IN INCHES			
DECIMAL OF FOOT	INCHES	DECIMAL OF FOOT	INCHES
.01	$\frac{1}{8}$ —	.25	= 3 (exact)
.08	1 —	.50	= 6 (exact)
.17	= 2 +	.75	= 9 (exact)

Decimals of a foot can be easily converted mentally into inches by use of the equivalents in the above table, for example, $0.47 \text{ ft.} = 0.50 - 0.03 = 6'' - \frac{3}{8}'' = 5\frac{5}{8}''$.

11. Metallic tapes, sometimes called woven tapes, are cloth tapes with fine tinsel twisted around each thread to prevent excessive stretching. They are usually graduated into feet, tenths, and half-tenths and are made in lengths of 25 ft., 50 ft., and 100 ft. They are used for short measurements which are not required nearer than 0.1 ft., such as measurements of buildings for plotting purposes or measurements for topography. When precise results are required, a steel tape should be used.

12. Cloth tapes, without woven wires, stretch so much as to be practically useless in surveying.

13. Invar tapes are made of a nickel steel alloy having a very small thermal coefficient of expansion. They are used for surveys requiring a high degree of precision. For a given change in temperature, the change in length of these tapes is about one-thirtieth that of the ordinary steel tape.

14. THE STADIA. Distances can be measured **approximately** and very rapidly by the stadia method. In this method the distance is obtained by sighting the telescope of a transit on a graduated rod held at the other end of the line, and observing the interval on the rod included between two special horizontal cross-hairs (stadia-hairs) set in the telescope. From the observed interval, the distance from the instrument to the rod is readily computed. The stadia method of surveying is fully explained in Chapter X.

MEASURING DISTANCES

15. MEASURING HORIZONTAL DISTANCES. There are two methods in common use for measuring the horizontal distance between two points: (1) making the measurement by holding the tape horizontally; (2) measuring the slope or inclined distance between the points. By the first method the horizontal distance is obtained directly. By the second method the slope distance is later reduced to the horizontal distance; and for this purpose we must obtain either the difference in elevation of the points or the vertical angle between the line connecting these points and the horizontal line.

If the ground is level, the tape is stretched on the ground and is therefore supported horizontally when the measurements are taken. But if the terrain is not horizontal or if obstructions such

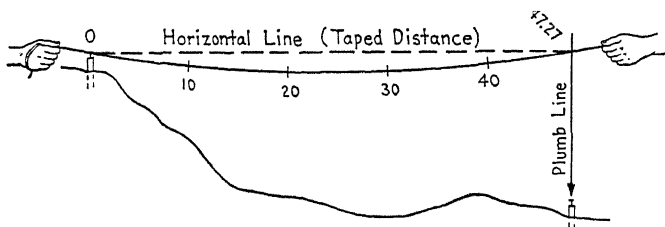
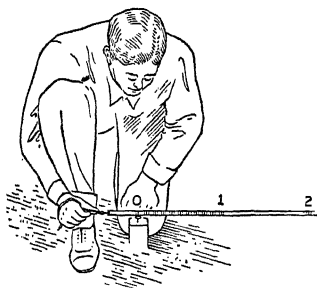


FIG. 2. HORIZONTAL DISTANCE MEASURED DIRECTLY BY PLUMBING.
SUFFICIENT PULL ON TAPE GIVEN TO ELIMINATE EFFECT OF SAG.

as bushes are on line, the tape in method (1) must be held horizontally by supporting it at one or at both ends when the measurement is being made, and the distance must be transferred to the ground by means of a plumb-bob, as in Figs. 2 and 3. Wherever possible one end of the tape should be held on the stake, as in Figs. 2 and 3; plumbing at both ends is to be avoided if possible.

When measuring horizontally the tape must be held as nearly level as is possible by estimation, ordinarily without special instruments to determine that it is horizontal. For ordinary measurements the pull on the tape (tension) is judged by the feeling on the hand; for precise work it is measured with a spring balance. Although some **compensating** error is likely to be introduced in transferring the measurement from the tape to the stake

by means of the plumb-bob, a greater **cumulative** error is likely to occur by failing to hold the two ends of the tape at exactly the same level. **Holding a tape level requires much practice.** Level lines are present in the outside finish of buildings and on the shore of water surfaces; when available these level lines should be used as guides in holding the two ends of the tape level. **The importance of having the two ends of the tape at the same level cannot be overstated.**



A. Holding zero end of tape on a tack in top of stake.



B. Plumbing over tack in stake and reading tape graduations which are on observer's side of tape.

FIG. 3. THESE MEN ARE SHOWN IN PROPER POSITIONS
PERFORMING THE TAPING INDICATED IN FIG. 2.

16. **“Breaking Tape.”** In taping a line by the usual method of holding the tape horizontally, it will be frequently found that on steep ground the length of the line must be made up of a series of horizontal measurements, all less than a full tape-length. To illustrate this method, assume that the horizontal distance AB (Fig. 4) is required. The tape is removed from the reel and laid

on the ground, the zero end being in the direction of B and the 100-ft. end at A . The head-tapeman takes a position about on line, and so selected that the rear-tapeman will not have to plumb higher than his chest to make the tape level. Under the direction of the rear-tapeman, the head-tapeman sets a pin or temporary mark on line. The tape is now held horizontally through the line mark and the necessary tension is gradually applied, the rear-tapeman plumbing the 100-ft. mark on the tape over A . When the rear-tapeman calls "O.K.," the head-tapeman inserts a pin or nail in the ground at some full foot, as point a (say at the 60-ft. mark). It is good practice to repeat this measurement as a check. When the head-tapeman calls "all right," the rear-tapeman drops the tape and advances to point a ,

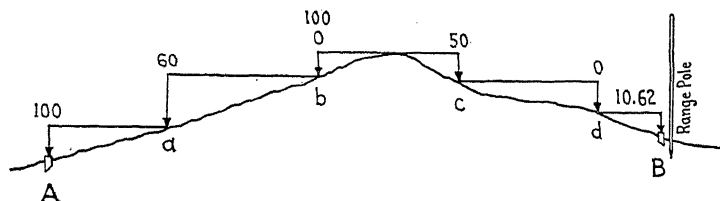


FIG. 4. MEASURING HORIZONTAL DISTANCES BY USE OF PLUMBING.

the head-tapeman taking a position at the zero end of the tape. In a similar manner point b is set, the rear-tapeman plumbing the 60-ft. tape mark over point a and the head-tapeman setting point b at the zero end of the tape. The tape is now pulled ahead until the 100-ft. mark is at b . Similarly, points c and d are set. It will be noted that in measuring bc both tapemen have to plumb, but in setting points c and d only the head-tapeman plumbs. When point d has been set the measured horizontal distance $Ad = 200$ ft.; the distance dB still remains to be measured. The tape is now drawn ahead, the head-tapeman plumbing the zero of the tape over B ; if the tape is graduated throughout its length to hundredths, the rear-tapeman reads 10.62. The total horizontal distance $AB = 210.62$ ft.

If, however, only the end foot is graduated to hundredths, the rear-tapeman holds on a full foot-mark (in this case 11 ft.) at d so that the graduated end foot will come over B . In this case

the rear-tapeman calls "11 ft." The head-tapeman actually reads on the tape 0.38, which he mentally subtracts from 11 ft. The head-tapeman therefore records 10.62 ft.

Measuring a line in the above manner is called "breaking tape." By this method it is an easy matter to keep account of the number of 100-ft. lengths measured. Of course, the line could have been measured by taping each individual distance, as Ab , bc , etc., the sum giving the length AB .

To check the measurement, the line is taped from B to A , using different intermediate points from those used on the forward measurement.

Some surveyors prefer to begin with the zero end; others prefer to begin with the 100-ft. end, as described above. The principles are the same in both methods. A competent surveyor will start with the end of the tape which is more convenient and will make the necessary calculations for the part measurements at the end of the line.

17. Suggestions on Taping. It is easier to measure downhill than uphill. In the former case the rear-tapeman holds the tape steady at the point on the ground while the head-tapeman applies a uniform pull on it, plumbs exactly over the point, and reads the tape. The head-tapeman, who is doing the plumbing, can keep his balance as he applies the pull because he has the whole process so much under his control.

In taping uphill the process is more difficult because in this case the rear-tapeman must hold himself in perfect balance, keep his plumb-bob steady and exactly over the point, while the head-tapeman is applying a steady pull on the tape, which tends to pull the rear-tapeman off balance.

In making tape measurements, both head-tapeman and rear-tapeman should take such positions as will enable them to apply tension to the tape **gradually** without losing their balance and so causing irregular or sudden pull on the tape. The positions which the men should take will depend on the method used in taking the measurement. If, for example, the tape is supported throughout its length, both men should take a position with one knee on the ground, as in Fig. 3-A. If one end of the tape is on the ground and the other end is being plumbed, one man must be

in a standing posture, as in Fig. 3-B. If both ends are plumbed, both men must be standing in perfect balance. In any event, each should retain a firm grip on the tape and should increase tension gradually rather than in jerks. They should stand so that they can look squarely across the tape graduations and at the point to which they are measuring; at the same time they should keep far enough off the line being measured for the transitman to sight past them.

18. SLOPE MEASUREMENTS. When taping "on the slope" the measurement may be made (at the right tension) **directly between the two points** and the distance read from the tape; or the tape may be stretched directly **from a tack in a stake to the horizontal axis of the transit telescope**. The slope distance being known, the horizontal distance may be calculated if either the difference in height of the two ends of the tape or the angle of slope of the tape is known.

19. If the slope distance is measured directly between the points and the difference in their elevations is known, the horizontal distance can be obtained as follows. In the right triangle formed by the slope distance s , the difference in elevation of the ends of the tape h , and the horizontal distance d , $d = \sqrt{s^2 - h^2}$. Because the solution of this expression is rather awkward, a shorter approximate expression is commonly used which gives the difference between slope distance and horizontal distance.

$$s - d = \frac{h^2}{2s}$$

The proof of this relation is demonstrated from Fig. 5 wherein:

$$\begin{aligned}s^2 - d^2 &= h^2 \\(s + d)(s - d) &= h^2\end{aligned}$$

assuming $s = d$ and applying this to first parentheses only,

$$2s(s - d) = h^2 \text{ (approximately)}$$

$$s - d = \frac{h^2}{2s} \text{ (approximately)}$$

$$\text{Similarly, } s - d = \frac{h^2}{2d} \text{ (approximately)}$$

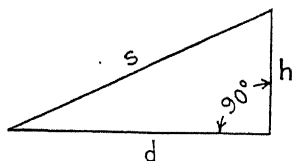


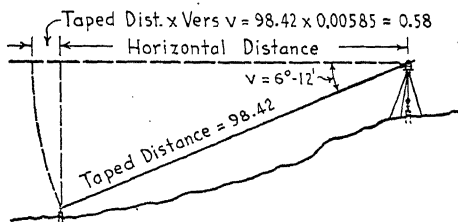
FIG. 5.

The ordinary slide-rule is suitable for this computation.

It is evident that the smaller h is in comparison with the other two sides, the more exact will be the results obtained by this formula. The formula is correct to the nearest 0.01 ft. even when $h = 14$ ft. and $s = 100$ ft., or when $h = 30$ ft. and $s = 300$ ft. Greater precision may be obtained from the following expanded form:

$$s - d = \frac{h^2}{2s} + \frac{h^4}{8s^3}$$

20. If the angle of slope v of the tape is measured with the transit, for instance, the horizontal distance will be $s \cos v$ (Fig. 6). But here again it is easier to compute the difference in length between the slope and the horizontal distance. Instead



$$\text{Hor. Dist.} = \text{Taped Dist.} - \text{Taped Dist.} \times \text{Vers } v = 97.84$$

FIG. 6. SLOPE DISTANCE AND VERTICAL ANGLE MEASURED IN FIELD.
HORIZONTAL DISTANCE COMPUTED BY VERS SINE METHOD.

of computing $d = s \cos v$, compute $s - d = s \text{ vers } v^*$ and obtain the result by slide rule or by simple arithmetic. When the angle v is smaller than 10° , the difference between the inclined and the

* In most texts on Trigonometry $\text{vers } v$ is expressed as being $1 - \cos v$, which is correct; but it is not in this form best suited to the surveyor. He customarily senses each trigonometric function of an angle as the ratio of one side of a right triangle divided by another. It is therefore advisable to think of $\text{vers } v$ as the ratio of the distance M divided by the radius R . In Fig.

7, $\text{vers } v = \frac{M}{R}$, and it is this ratio which should be visualized and kept in mind. It can readily

be shown that, since $M = R - d$, the $\text{vers } v = \frac{R - d}{R} = 1 - \frac{d}{R} = 1 - \cos v$.

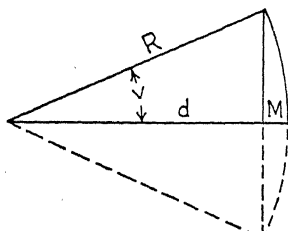


FIG. 7.

horizontal distance is small; for example, if the slope distance is 98.42 and the vertical angle is $6^{\circ} 12'$, the horizontal distance is $98.42 - 98.42 \times \text{vers } 6^{\circ} 12' = 98.42 - (98.42 \times .00585) = 98.42 - 0.58' = 97.84$ ft. (Fig. 6). Generally speaking, a slide rule calculation of $s - d$ is as accurate as a 5-place trigonometric calculation of d would be if the formula $s \cos v$ were used.

21. COMMON MISTAKES IN READING AND RECORDING MEASUREMENTS OF DISTANCES.

1. Failure to observe the correct position of the zero-point of the tape:— (1) End of ring; (2) end of steel band; (3) on steel band.
2. Omitting a whole tape-length.
3. Transposing figures, e.g., 63.56 for 63.65 (mental); or reading tape upside down, e.g., 6 for 9, or 86 for 98.
4. Reading wrong foot-mark, as 63.56 for 62.56.
5. Subtracting incorrectly when using tape graduated on first foot only.

22. PRECISION REQUIRED IN LINEAR MEASUREMENTS.

Surveys require different degrees of precision depending on the purpose of the survey; therefore the linear measurements in some surveys must be made more precisely than in others. By precision is meant the ratio of the error to the distance measured. For example, a precision of one unit in 5000 (also expressed as $\frac{1}{5000}$) means that the error should not exceed one part in 5000 of the distance measured. Experienced surveyors know the precision required for different kinds of surveys and conduct the work with the required result in mind.

Steel tapes of 100 ft. or less in length are so manufactured that they are of standard length when supported throughout on a level surface, at a temperature of 68° F. and under a tension of 10 pounds. For tapes over 100 ft. long the tension is 20 pounds. Although changes in temperature and tension will cause slight variations in length of tape, competent tapemen can usually obtain precisions of $\frac{1}{5000}$ or less, such as are required on suburban and farm surveys, by assuming the tape to be of the length stated by the manufacturer and provided that the temperature does not vary greatly from 68° . For precisions of $\frac{1}{10000}$ or greater, as

required in many city surveys, the true length of the tape should be known under definite conditions, and various corrections, which will be given later, should be applied to the measurements to obtain the prescribed precision.

23. TESTING LENGTH OF TAPE. The U. S. Bureau of Standards, for a small fee, will test a steel tape, giving its length at standard temperature ($68^{\circ}\text{F.} = 20^{\circ}\text{C.}$) when supported its full length and subjected to a 10-pound pull. A certificate, giving the result of such a test, will be furnished provided that the tape is made of a single piece of metal ribbon and that none of the graduations are on pieces of solder, on sleeves attached to the tape, on wire loops, spring balances, tension handles, or on other attachments which are liable to be detached or to become changed in shape. If these requirements are not met, a "*report*" will be furnished, but not a "*certificate*." For an additional fee the Bureau will test the tape when supported at the ends.

It is good practice to have the U. S. Bureau of Standards test, for instance, a **new** 100-ft. tape and a **new** 300-ft. tape and submit a full report, based on tests made with different amounts of tension both when the tapes are supported throughout their length and when they are suspended only at their ends. Thereafter these tapes **should never be taken into the field** but should be kept in the office to serve as available standards with which the tapes used in the daily work can be frequently compared.

It is obvious that in making precise measurements the tape should be used under the conditions for which it was standardized; this seldom can be done. Consequently, the corrections must be computed and applied.

24. SOURCES OF ERROR IN THE MEASUREMENT OF LINES. The principal sources of linear measurement error are

- (1) Tape not stretched straight
- (2) Wind
- (3) Incorrect alignment — Horizontal and Vertical
- (4) Careless plumbing
- (5) Erroneous length of tape
- (6) Variation in temperature
- (7) Incorrect tension
- (8) Sag

It will be observed that some of the above errors are caused by carelessness on the part of the tapemen; others are caused by not accounting for those errors that are inherent in the tape.

In general, most of the sources of error in measurement of distance tend to **increase the length**, i.e., to make the recorded length greater than the true length. For example, sources 1, 2, 3, 4 and probably 8, tend to increase the recorded distances; none of these tend to decrease the recorded distances. Sources 5, 6 and 7 may increase or they may decrease the recorded distances. Consequently, it seems likely that, as stated above, the sources of error in measured distances **tend to cause the recorded distances to be greater than the actual distances**.

25. Tape Not Stretched Straight. The tape should be stretched straight between the two points and not allowed to bend around bushes, trees, or boulders. Failure to observe this condition will cause the measured distance to be **too great**. A foot off line at the middle of a 100-ft. tape will cause an error of .02 ft. in 100 ft. (Art. 19).

26. Wind. Wind will blow the tape off line especially when plumbing, thus causing the measured distance between the points to be **too great**. The error caused by wind is approximately the value given in the preceding article when the 100-ft. tape is blown 1 ft. off line at its middle. The magnitude of the error may be reduced by supporting the tape on line at one or more points along its length.

27. Incorrect Alignment. When a line is longer than a tape-length it must be measured in parts and the intermediate points must be "lined in" either by eye or by the transit. If the distance only is required, it is usually possible to line the intermediate points in so closely by eye that the resulting error in length is negligible. The magnitude of the error may be com-

puted by the approximate formula $s - d = \frac{h^2}{2s}$ (Art. 19). For

example, if one end of the tape is 1 ft. off line in a 100-ft. tape-length the error will be $(1)^2 \div 200 = 0.005$ ft. In 40 ft. a 1-ft. error in alignment will be $(1)^2 \div 80 = 0.012$ ft. Fig. 8 illustrates the amount of error introduced by holding the tape 1 ft.

off line for different distances and shows clearly the **importance of correct alignment when the distances are short**. "Lining in" can ordinarily be done by eye much closer than within one foot, usually within 0.2 or 0.3 ft.

By the same method the error in length caused by failing to hold the tape horizontally may be computed.

It will be observed that **errors in horizontal and vertical alignment are cumulative** (Art. 36).

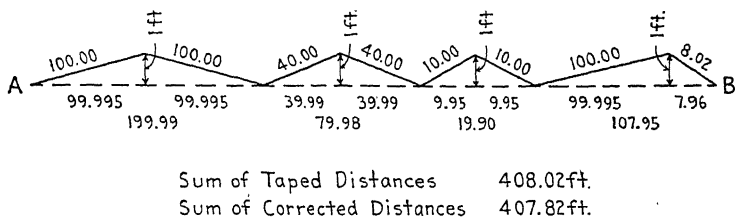


FIG. 8. ERRORS IN MEASUREMENT OF DISTANCES DUE TO CARELESS ALIGNMENT OF INTERMEDIATE POINTS. NOTE THE LARGE ERROR WHEN TAPE-LENGTHS ARE SHORT.

28. Careless Plumbing. In this article, careless plumbing means lack of care in holding the bob over the point; it does not relate to holding one end of the tape too high or too low instead of horizontally; the latter error is explained in Art. 27. It is obvious that careless plumbing will cause such errors that the recorded length of the line may be too small; or it may be too great. Plumbing errors of this kind are therefore **compensating** (Art. 36).

29. Incorrect Length of Tape. Tapes that are either too long or too short will obviously give incorrect measurements of distance. In either case **the error is cumulative**. If the error per tape-length is known, a correction can be computed which, when applied to the observed measured distance, will give the correct distance between the points. The corrections may be + or -, depending on whether the tape is too long or too short; it is obviously **important to apply it in the proper manner**. If the tape is too long it will not go as many times as it should between the two points, and the observed distance therefore will be too short; the correction should be added to obtain the true length.

If the tape is too short it will go more times than it should between the points, and the observed distance will be too long; the correction should be subtracted to obtain the true length.

Suppose, for example, that the distance is known to be exactly 500 ft. between points *A* and *B* and that the line *AB* is to be measured with a 100-ft. tape which is 0.02 ft. short; i.e., its length is actually 99.98 ft. as the standard tape would be if the temperature were at about 30° F. In measuring *AB* with this tape, its full length will obviously be used five times and each length will be recorded as 100 ft., although the tape will fail to reach the point *B* by 0.10 ft., because each time it is laid down it will cover only a distance of 99.98 ft. Consequently, the length of *AB* that this erroneous tape will give is 500.10 ft. To obtain the correct length, $5 \times 0.02 \text{ ft.} = 0.10 \text{ ft.}$ should be subtracted from the recorded length, giving 500.00 ft.

30. Temperature — Effect, Variation, and Application. Steel tapes are of standard length when at a temperature of 68° F. If the length of the tape is known at 68° F. and a measurement of a line is made when the temperature is 30° F., it is evident that the tape will be shorter at 30° F. than at 68° F.; hence a correction must be applied to the observed length of the line to obtain its true length. If the measurement is made when the temperature is 90° F., it is obvious that the tape will be longer than at 68° F. If the temperature varies during the measurement of the line, the error is **cumulative** so long as observed temperatures are consistently below 68° F. or above 68° F. If, during the measurement, some of the observed temperatures are above and some below 68° F., the errors tend to **compensate**.

In computing the change in length of a steel tape caused by a change in temperature, we must know the "*thermal coefficient of expansion*" of the tape. The value adopted for steel tapes is 0.00000645 per 1° F., which is the change per unit length of the tape caused by a change in temperature of 1° F. Thus, the change in the length of a tape 1 ft. long for a change of temperature of 1° F. will be 0.00000645 ft. For a 100-ft. tape, the change will be for 1° F., $100 \times 0.00000645 = 0.000645 \text{ ft.}$ For 15° F. change in temperature, the change in length of a 100-ft. tape will be $15 \times 0.000645 = 0.0097 \text{ ft.}$, or approximately

0.01 ft.; a value which the surveyor **should always keep in mind.**

In determining the temperature only high grade thermometers should be used. Especially constructed thermometers that may be attached to the tape by clips are available. Two of these are commonly used, one placed at, or near, each end. Sometimes the thermometers are placed on the ground. For best results the measurements should be made on cloudy days when the temperature is about constant or changing slowly; on sunny days there may be considerable difference between the temperature of the tape and of the air.

Example: A line measured with a steel tape under a tension of 10 lbs. was found to be 896.72 ft. long when the mean temperature was 36° F. If the tape under a tension of 10 lbs. at 68° F. was 100 ft. long, compute the correct length of the measured line. The change in temperature 68° - 36° = 32°. The correction per tape-length of 100 ft. is 32° (0.00000645) (100) = 0.021 ft. Since there are 8.9672 tape-lengths, the total correction to the measured distance is 8.9672 × 0.021 = 0.19 ft. Therefore, the corrected distance, since the tape is too short, is 896.72 - 0.19 = 896.53 ft.

31. Tension. The standard tension for tapes 100 ft. or less in length when supported throughout their length on a level surface is 10 lbs. It is evident that a different tension applied to the tape, say, 20 lbs., will cause the tape to stretch; hence its length will be changed. Light steel tapes will stretch between 0.01 and 0.02 ft. in 100 ft. if the tension is increased to 20 pounds. Heavier tapes will stretch much less than this. The increase in length caused by an increased tension is sometimes computed by the formula

$$C_p = \frac{L (t - t_0)}{SE}$$

in which C_p is the correction per tape-length in feet

L is the length of the tape in feet

t is the applied tension in pounds

t_0 is the standard tension in pounds

S is the cross-sectional area in square inches

E is the modulus of elasticity of the steel in pounds per square inch.

The increase in length that is caused by an increase in tension may be measured directly for any tape by varying the tension by means of a spring balance and noting the resulting change in length.

In ordinary surveying work, experienced tapemen can estimate the tension sufficiently close to come within the required precision. It is only on precise work that the spring balance is used in applying tension.

32. Sag. A tape is of standard length when it is supported throughout its length and under a definite tension. Suppose that while the same tension (10 lbs.) is being applied, a measurement is made when the tape is supported only at its ends. It is evident that this tape will hang in a "catenary" curve and that the horizontal distance between the end points will be less than the tape reading. The amount of this shortening caused by sag depends on the weight of the tape, the distance between supports, and the tension applied. It may be computed by the formula

$$C_s = \frac{w^2 l^3}{24 t^2}$$

in which C_s is the correction between the supports in feet

w is the weight of the tape in pounds per foot of tape

l is the distance between supports in feet

t is the tension in pounds.

A practical way of determining the effect of the sag for any given tape is, as follows: Select a horizontal piece of ground near a vertical wall of a wooden building where the tape may be stretched out on the level ground, supported throughout its entire length. Drive a nail A firmly into the wall about waist high. The head of the nail should be 2 or 3 in. from the wall and a definite point should be marked on the nail-head. By means of a plumb-bob, set point A' vertically under A , flush with the surface of the ground. Mark A' carefully to indicate the exact point. Next, stretch the tape out on the ground along the wall. Hold the zero end on A' , apply a 10-pound tension with a spring balance at the other end of the tape, and set B' at the same distance from the face of the wall as A' . Indicate by a

mark on B' the exact position of the 100-ft. mark on the tape. By means of a plumb-bob, set a nail B vertically over B' and at the same height as A . Mark the exact point B carefully. With a 10-pound tension on the tape, measure the distance AB along the face of the building, the tape being unsupported in this case. The difference in the readings $A'B'$ and AB is the shortening because of sag. If the pull on the tape is now increased until the 0 and 100-ft. marks on the tape coincide exactly with the elevated points A and B respectively, the additional tension applied is the amount required to eliminate the effect of sag.

It should be kept in mind that the distances $A'B'$ and AB are not necessarily exactly 100 ft. apart, for the reason that the graduated length of the tape may be slightly in error. If the tape has been compared with a standard and its correct length is known, the distance $A'B'$ may be laid out exactly 100 ft., and a temperature correction may be applied, if necessary. (See Art. 30.) It is advisable to lay out AB and $A'B'$ in a shady location where the temperature is likely to change but little while comparisons are being made.

33. MARKING SURVEY POINTS. In survey work, the ends of a straight line, which are frequently angle points, are marked by heavy stakes, called "hubs," which are driven firmly into the ground. These stakes are made from milled lumber or are cut from trees. The exact survey point is indicated by a tack or short nail driven into the top of the stake. When the nature of the ground is such that stakes cannot be used, stout spikes may be driven into the ground; the exact point may then be indicated by a punch hole on the head of the spike. In solid ledge, small drill-holes are frequently used to mark important transit points.

In measuring a line between two stakes, the intermediate points are indicated by taping pins, nails, or stakes; the marking device chosen depends on the purpose of the survey and precision required. For ordinary work pins are frequently used. A pin is made of metal; it is about $\frac{3}{16}$ in. in diameter and 14 in. in length, pointed at one end and with an eye at the other. The pin is stuck in the ground, usually inclined to the vertical, and at right angles to the tape, the measurement being made to the center of the pin where it enters the ground. When a nail is used to

indicate the point, a pin or temporary stake is placed nearby as a "witness mark," so that the nail may be easily found.

In some surveys the intermediate points are set on line with a transit, but in ordinary work where only the length of the line is required, the intermediate points are lined in by eye. For this purpose it is customary to place a range pole* vertically at each end of the line. The rear-tapeman, standing on line, directs the front-tapeman where to place his pin or stake so as to be on line with the range pole up ahead.

34. STATIONING. In some surveys the measured length of each line between stakes is recorded in the notes. In other surveys where distances are to be measured continuously from the initial point of a line without regard to angles in the line, as in route surveys, it is customary to establish the 100-ft. points, called *full stations*. Suppose an angle to occur 870.1 ft. from the point of beginning; this would be called "*station 8 + 70.1*." To set "*station 9*," the 70.1-ft. point of the tape should be held on stake 8 + 70.1 and the stake at station 9 should be placed at the 100-ft. point of the tape. When curves are introduced at angle points, the stationing follows along the curve (Art. 271).

Any point, not necessarily on a continuous traverse, which is used as an instrument point or a triangulation point is also frequently called a "*station*."

35. ACCURACY AND PRECISION. It is common practice when measuring distances and angles to repeat the process one or more times. This is done to avoid mistakes and to determine the measured quantities with greater precision than can be obtained by an individual measurement.

The two words "accuracy" and "precision" are often used carelessly and synonymously. There is, however, a distinction between them which should be clear to the surveyor. **Precision implies refinement of measurements or closeness in agreement between several measurements, whereas accuracy implies correctness or freedom from mistakes and from carelessness.** Pre-

* A range pole, or lining-pole, is usually an octagonal wooden pole about 1" in diameter and 6 to 8 ft. long, with a pointed metal foot. It is usually painted with alternate red and white bands, one foot in width. Metal range poles about half an inch in diameter can be purchased in various lengths.

cision is of no significance unless accuracy is also obtained. A line may be measured in two directions to the nearest hundredth of a foot. The two measurements may be 247.62 and 248.63 ft. The hundredths in these recorded measurements indicate precision, but the fact that there is a foot difference shows clearly that a careless mistake has entered into at least one of the measurements. This is an example of a precise measurement that is inaccurate and therefore useless. Had these results read 247.62 and 247.63 ft. respectively, there would be definite indication that an accurate and a precise measurement had been made. Suppose now that four measurements of the same line are made and recorded as 247.62, 247.63, 247.62, and 247.62 ft., and that there is reason to believe that any one of these four measurements is as good as another. The average of the four is 247.622 ft., but it is improper to record the measurement to thousandths (as 247.622 ft.) because no one of the measurements was made with that precision. The correct mean value of the distance is 247.62.

36. ERRORS OF MEASUREMENT — CUMULATIVE AND COMPENSATING. In general, errors affecting measurements may be divided into two classes. In the first class are the **constant**, or **systematic** errors, which always affect the measurements in the same way and so are **cumulative**. In the second class are the **compensating**, or **accidental** errors, which have a tendency to **compensate** because they may be plus as often as they are minus. The net result of accidental errors, therefore, should be small in measurements of good quality.

One source of the constant error may be the erroneous length of the tape, i.e., the tape may be too long or too short. This error is cumulative and increases in proportion to the length of the line. Another constant error may result from holding one end of the tape too high or too low.

Accidental, or compensating errors, include those caused by plumbing, as, for example, when the plumb-bob may be slightly to one side of the point on one measurement and to the other side when the check measurement is taken.

Care should be used in deciding whether the error is constant or accidental. For example, suppose a tape is standard at 68° and a measurement of a single line is made at an average tempera-

ture of 32° . If no correction is made for the change in temperature from 68° , the error will be cumulative. However, if during the measurement the temperature varies from 30° to 38° but an average of 34° is used to determine the correction, the error caused by the variation in temperature will tend to compensate.

To illustrate the difference in the effect of these two kinds of errors, suppose that a tape is 0.005 ft. longer than the standard 100 ft., but that this fact is not known, and therefore this 0.005 ft. error enters into every 100-ft. length. Suppose also that the **estimated** probable error made in plumbing from tape to the tacks on the stakes is 0.02 ft. per tape length. If a line 4900 ft. long is measured, the tape error caused by erroneous length is $0.005 \times 49 = 0.245$ ft. The plumbing error, which may be either + or -, is found by theory of Least Squares* to be $\pm\sqrt{49} \times .02 = \pm 0.14$ ft. This shows clearly that the larger accidental (compensating) error has less effect than the small constant (cumulative) error. It should be noticed, however, that in a single measurement of 100 ft. the accidental error has much the greater effect.

37. AVOIDING MISTAKES. Mistakes in counting the tape-lengths may be avoided by having more than one person keep the tally of tape-lengths. Mistakes in reading the foot-mark may be avoided by noting not only the preceding foot-mark, but the following foot-mark also, as "63.65 . . . 64 feet," and by holding the tape in such a way that the numbers are **right side up** when being read.

The tapeman should always call off distinctly the distances to the note-keeper, and in such terms that they cannot be mistaken. As an instance of the way in which mistakes of this kind may occur, suppose a tapeman calls, "Sixty-four, three"; it can easily be mistaken for "Sixty-four feet." The note-keeper should repeat the distances aloud, so that the tapeman will know that they are correctly understood. It is useful in doubtful cases for the note-keeper in answering the tapeman to employ different words; this will remove possible ambiguity. For example, if the

* Probable uncompensated error is the product of the probable error of one observation times the square root of the number of opportunities for error. See *Elementary Surveying* by Breed and Hosmer, Vol. I, Art. 23.

tapeman calls, "Thirty-six, five," the note-keeper may answer, "Thirty-six and a half." If the tapeman has meant 36.05, the mistake will be noticed. The tapeman should have called in this instance, "Thirty-six O (pronounced "oh") five" or "Thirty-six point oh five." The following is a set of readings that may easily be misinterpreted unless extreme care is taken in calling them:

60.5 — "Sixty and five" or "Sixty point five."

65.0 — "Sixty-five O (oh)" or "Sixty-five flat."

60.05 — "Sixty,-O (oh) five" or "Sixty point oh five."

All of these may be carelessly called off, "Sixty-five."

In all cases the tapemen should make mental estimates of the distances when measuring, in order to avoid making serious mistakes.

38. Care of Steel Tape. The tape should be kept in the reel when it is not being used, and it should be cleaned at the end of each day. A damp or muddy tape should be cleaned, and wiped first with a dry cloth and then with an oiled one; a tape that is only dusty may be wiped with a dry or slightly oiled cloth. If the tape is taken care of in this manner, it will not get rusty. If, however, through neglect it should get rusty, the spots can be removed with crocus cloth or dry Portland cement, and the tape polished off with an oily cloth. After it has been reeled, it should be placed carefully, not thrown, in the bag.

Steel tapes are not easily broken when used properly. In unreeling and reeling in a tape, the reel (or handle) should be held in such a position that the tape will unwind in a direction **tangent to the reel**. Many tapes may be removed from their handles; when unreeling and removing the tape, care should be used to avoid breaking off the end ring to which the handle is attached.

In placing the tape in the reel, the 100-ft. end should be attached to the reel in such a way that the graduations will face the inside of the coil. If the reel is held in the left hand, the 100-ft. end of the tape should be attached to the reel with the graduations up; the winding handle of the reel should be turned clockwise, using the right hand.

Some surveyors remove the tape from the reel at the beginning

of the day's work and replace it when the day's work is done. Others do not replace the tape, but loop it in one hand in about 4-ft. lengths so that it forms a figure 8, and then give the 8 a twist that turns it into an o as the tape is placed in the kit-bag, or hung on a hook for the night.

Sharp bends and kinks in the tape should be avoided; tapes having kinks in them are easily broken when tension is applied. It is not advisable to allow even rubber-tired vehicles to run over tapes.

FIELD EXERCISES

1. Set 4 stakes on level ground about 300 to 400 ft. apart to represent the 4 corners of a lot of land. First pace these distances, each man using his natural gait, and record the number of paces along each of the 4 sides. Measure the distances with a tape clockwise around the property and record each side separately to the nearest $\frac{1}{100}$ ft.; then measure the distances in the reverse direction. Next, note whether any apparent mistake has been made in the measured distances (the forward and backward measurements of each line should agree within .02 to .04 ft.). Record the average lengths in one column; the paces in the next; and in the third column, record the number of paces per 100 ft. taken on each of the 4 lines. Add the third column and divide by 4 to determine the average number of paces per 100 ft. This will be different for different men. Each student should remember the number of paces he normally takes in 100 ft. It varies from about 34 to 38, depending on the natural gait of the man; for this reason it is always advisable in pacing a line to do so individually, and not in step with anyone else.

2. On nearly level ground, measure a line between tacks set in two stakes about 500 ft. apart, using a 100-ft. steel tape with spring balance attached. Let this tape be stretched out on the ground for 15 minutes or more before the measurement is taken, so that it will acquire a temperature approximately like the air temperature. Note this temperature on a thermometer. Make each tape measurement with a pull of 10 pounds. Measure the line 3 or 4 times. Determine the average length of the line to the nearest 0.01 ft. Correct this length for temperature, bearing in mind that at 68° F. the tape is of standard length and that the coefficient of expansion of steel tapes is 0.0000645. Record the correct distance.

3. Select a line about 500 ft. long on level ground. Measure the line directly on the ground forward and backward, using nails for the intermediate points. Remove the nails and re-measure the line forward and backward, holding the tape about 4 ft. above the ground and plumbing from both ends of the tape. Compare the two measurements.

4. By the method explained in Art. 32, establish a 100-ft. standard for testing tapes, supported throughout their length and also supported at the ends only. Find the tension in the tape when the zero and the 100 marks are exactly 100 ft. apart, the tape being supported at the ends only.

5. Measure a line on a gradual slope requiring plumbing from one end of the tape, first down the slope and then back up the slope, and determine the average value for the length of line. Which result would you consider more reliable and why?

6. Using only the tape, lay out a right angle at point B on the line $AB = 200$ ft. From B , measure toward A a distance of 80 ft. and place a point C on line BA . Point C should be carefully lined in by sighting through a plumb-bob string held over point B and a second plumb-bob string held over point A . From point C , swing an arc of 100 ft. with the tape and mark a number of points on the ground near where a perpendicular from AB at B is estimated to fall. Measure a distance of 60 ft. from B , and find the intersection at D with the arc of 100 ft. from C . The line BD is perpendicular to AB . This exercise can be performed more conveniently by swinging two tapes simultaneously from C and B . The geometric principle involved is that any triangle, the sides of which are multiples of 3, 4 and 5, is a right triangle.

7. Measure a line AB about 600 ft. long. Next, select point C midway between A and B , but 2 ft. off line. Measure AC and CB . Similarly, place a point C' about midway between A and B , but 10 ft. off line. Measure AC' and $C'B$. Reduce the measurements in both cases for error of alignment and compare with the direct measurement.

PROBLEMS

1. A metallic tape which was originally 50 ft. long has stretched uniformly and is found to be 50.12 ft. long. A house 25 ft. \times 35 ft. is to be laid out. Using this tape, what measurements must be made in order that the house shall have the desired dimensions? Using the same tape, what should the diagonals read?

2. In taping down a hill the forward end of the tape was held 2 ft. too low (below a horizontal position) in setting off a half tape-length of 50.00 ft. What error of measurement was introduced? What was the true distance to .01 ft. between the points set?

3. In measuring with a 100-ft. tape the forward end was held 2.5 ft. to the side of the line. What was the error (to nearest .01 ft.) in this one tape-length?

4. A distance measured with a 50-ft. steel tape was recorded as 576.92 ft. The tape was found to be 50.015 ft. long. What was the true length of the line?

5. Two measurements of a line were made with the same tape but on different days. The first length was 624.53 ft. when the temperature was 75° F.; the second length was 624.59 ft. when the temperature was 55° F. The tape is standard at 68° F.; coefficient = 0.0000645. Compute the length of line from each of the two measurements and explain any difference in the reduced readings.

6. A distance is measured on a slope with a 300-ft. tape and found to be 298.94 ft. If the difference of level of the two end points is 16.40 ft., what is the horizontal distance?

7. The following distances were measured around a closed traverse:

LINE	DISTANCE
1-2	319.15
2-3	168.40
3-4	498.80
4-5	394.18
5-1	229.35

The 100-ft. tape used was 99.933 ft. long at 68° F. under a tension of 10 lbs. Correct the above distances if the measurements had been made at an average temperature of 38° F. under a tension of 10 lbs.

8. If one end of 100-ft. tape is consistently held 1.5 ft. too low in measuring a line 1200 ft. long, what error will this cause in the total measurement? (Result to .001 ft.)

9. A steel tape is standardized at 68° F. when supported at the 0 and 100-ft. points and is found to have the following lengths under the following tensions:

TENSION	LENGTH
10 pounds	99.976 feet
15 "	99.997 "
20 "	100.010 "

(a) Under what tension is the tape standard at 68° F.?

(b) Under what tension would the tape be of standard length if the average working temperature were 82° F.?

10. With the transit set up at *B*, a slope distance of 176.92 ft. was measured with a 200-ft. tape from the horizontal axis of the transit to a tack in the top of stake *A*; similarly, a slope measurement of 186.56 ft. was taken to a tack in stake *C* (on the other side of *B* from *A*; *ABC* is a straight line). Vertical angle to point *A* was $-5^{\circ} 45'$, and to point *C*, $+7^{\circ} 23'$. The tape used was 0.015 ft. too long at temperature 68° F. While the work was being done, the tape temperature was 52° to 58° F. The suspended tape was given sufficient pull in both cases to offset the shortening due to sag. (A $-$ vertical angle is an angle of depression, i.e., the point sighted is below the horizontal axis of the transit. A $+$ vertical angle is an angle of elevation, i.e., the point sighted is above the horizontal axis of the transit.)

(a) What is the corrected horizontal distance *ABC*, to 0.01 ft.?

(b) Had the vertical angles been $-5^{\circ} 46'$ and $+7^{\circ} 22'$, how much difference (to the nearest 0.01 ft.) would this have made in the length of *ABC*?

11. A slope distance of 183.64 ft. is measured from the horizontal axis of a transit to a drill-hole in a ledge, vertical angle $+5^{\circ} 17'$.

(a) How many minutes of error must there be in this vertical angle to affect the computed horizontal length by 0.005 ft.?

(b) Had the angle been $-2^{\circ} 04'$, how much error in the angle would cause an error of 0.005 ft. in the horizontal distance?

12. If a line is being measured with a 100-ft. steel tape up a grade which rises about 20 ft. for every 100 ft. horizontal and distances are being measured to the nearest 0.1 ft., how close must the difference of elevations between the ends of the tape be read so that the correction may be made to the nearest 0.1 ft.? If the vertical angle of a slope measurement with a 100-ft. steel tape is $+5^{\circ} 54'$, how close must the vertical angle be read in order that the correction may be made to the nearest 0.01 ft.?

13. A 100-ft. steel tape is 0.006 ft. too long at 68° F. with a 12-lb. pull while supported. A rectangle is measured with this tape (supported). The sides are recorded as 598.58 ft. and 473.51 ft. The temperature of the tape is 78° F. at the start and 84° F. at the end of the job, with a tension of 12 lbs.

(a) If the recorded measurements are used, will the computed area of the field be too great or too small?

(b) What is the error in sq. ft.?

CHAPTER III

MEASUREMENT OF DIRECTION — THE MAGNETIC COMPASS

39. **THE SURVEYOR'S COMPASS.** The surveyor's compass is an instrument for determining the horizontal direction of a line with reference to the direction of a magnetic needle (Fig. 9). The magnetized needle is balanced on a *pivot* so that it swings

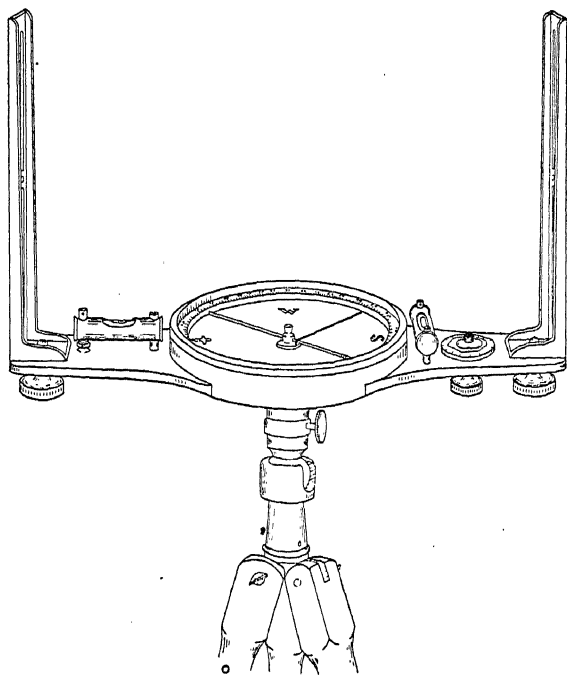


FIG. 9. THE SURVEYOR'S COMPASS.

freely, about its center, in a horizontal plane. The pivot is at the center of a horizontal circle which is divided into quadrants and graduated in degrees and half-degrees. The graduations are numbered from two zero-points 180° apart (marked with the

letters N and S) in both directions to the 90° points (marked E and W). For convenience, letters E and W on the compass-box are interchanged so that the proper quadrant may be read directly from the graduation, as will be explained later. The circle is covered with a glass to protect the needle and the graduations, the part enclosed is the compass-box. The needle is raised from the pivot by means of a lever, controlled by a screw. To prevent dulling the pivot-point, the needle should always be raised when the compass is lifted or carried. Both the circle and the pivot are secured to a brass frame, on which are two vertical sight vanes so placed that the plane through them also passes through the two zero-points of the circle. On the frame, at right angles to each other, are two spirit levels for leveling the compass, i.e., making the graduated circle horizontal. The frame is fixed at right angles to a spindle, the bottom of which is a ball of a ball-and-socket joint attached to a tripod, or Jacob's Staff. By means of the ball-and-socket joint the bubbles are centered, thus making the graduated circle horizontal and the spindle vertical. The frame may be rotated in the spindle and clamped in any desired direction.

The magnetic needle possesses the property of pointing in a fixed direction, namely, the *Magnetic Meridian*. The horizontal angle between the direction of this meridian and the direction of any other line may be determined by means of the graduated circle, and this angle is called the *Magnetic Bearing* of the line, or its *Bearing*. Since the quadrants are numbered from 0° to 90° the angular value of any bearing cannot exceed 90° . Therefore, in giving a bearing it is necessary to state, in addition to the angular value, the quadrant the line is in, such as N $30\frac{1}{4}^\circ$ E, S $62\frac{1}{2}^\circ$ E, S $40\frac{3}{4}^\circ$ W or N 45° W.

Use of the compass is limited to reproducing old property lines and to rough surveying of woodlands and farms.

40. SETTING UP THE COMPASS. When in use the compass is mounted on a tripod and the center of the instrument is set over the point from which the bearings are to be taken. The compass is first placed roughly over the point with sufficient spread on the tripod legs to insure stability; then the center of the instrument is set directly over the point by moving the legs

and by applying varying amounts of pressure to them as they are pressed firmly in the ground. For careful work a plumb-bob is used to center the instrument over the point. In rough surveys the compass may be set over the point by estimation, within a tenth or two of a foot, without the use of a plumb-bob. The compass-box is leveled by moving the head in the ball-and-socket joint until the bubbles are centered.

41. TAKING A BEARING. Set the compass over a point on the line the bearing of which is desired, and level it. Direct the peep sights (or telescope if transit compass is used) to some point along the line, and lower the needle on to the pivot. When the needle comes to rest, determine which end (i.e., the north or south) to read. The *south end* of the needle may be distinguished from the *north end* by a counterpoise attached to the *south end*. The purpose of this counterpoise is to counteract the dip of the needle, north and downward in the northern magnetic hemisphere, so

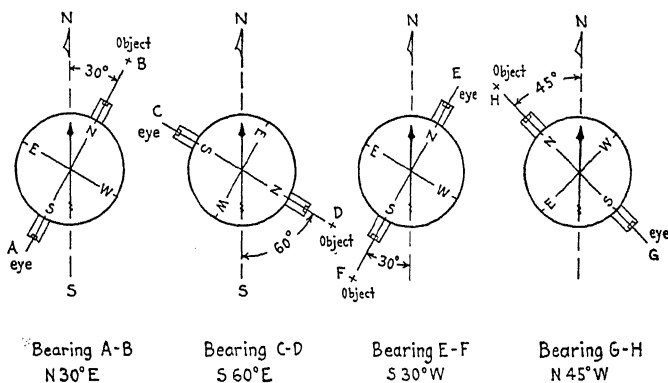


FIG. 10. ILLUSTRATING METHOD OF READING BEARINGS.

that it may swing in a horizontal plane. If the letter N on the compass-box is nearer the object sighted, read the north end of the needle. If the letter S on the compass-box is nearer the object sighted, read the south end of the needle. The bearing is determined by observing the angular value on the graduated scale opposite the end of the needle, and the quadrant is determined by observing between which letters on the compass-box

the end of the needle lies. Bearings are usually read to nearest $\frac{1}{4}^{\circ}$. In stating a bearing the letter N or S always precedes the recorded angular value of the bearing and the letter E or W always follows (Fig. 10). On the compass-box the letter E is marked where one would expect to find the letter W; this interchange of E and W is for convenience in reading the proper quadrant.

In reading the bearing, always keep the eye exactly in line with the needle so as to avoid an apparent displacement of the graduations; such a displacement is called *parallax*.

42. DECLINATION OF THE MAGNETIC NEEDLE. The earth acts as a large spherical magnet with two magnetic poles. The North Magnetic Pole lies just to the northwest of Hudson Bay, and it is toward this pole that the north-seeking end of a freely swinging magnetic needle points. This pole is about 1300 miles from the True Geographic Pole. At present, in the eastern part of the United States, the north end of the needle points west of true north, and in the western part it points east of true north. The angle between the needle (magnetic meridian) and the true or geographic meridian is called the *Magnetic Declination*, or simply the *Declination*, and it is designated as so many degrees East or West. The declination is constantly changing. At present it is $11\frac{1}{2}^{\circ}$ W (i.e., the North end of the magnetic needle points $11\frac{1}{2}^{\circ}$ west of true north) at New York, and 18° E at San Francisco. Between the above localities are points where the declination is zero degrees, i.e., where the magnetic needle points true north. The declination value may be found in tables or from an *Isogonic Chart* published by the U.S. Coast and Geodetic Survey. The Isogonic Chart is a map upon which lines are drawn connecting those points having the same declination. These lines are called *Isogonic Lines*; the line connecting points having a zero declination is called the *Agonic Line*. This line passes through Michigan, Tennessee, and South Carolina.

Fig. 11 is an isogonic chart of the United States for the year 1940. In addition to showing the isogonic lines for every degree of declination, it indicates by dash lines the annual change in declination for different parts of the country. At present the declinations are increasing east of the zero annual change (double-

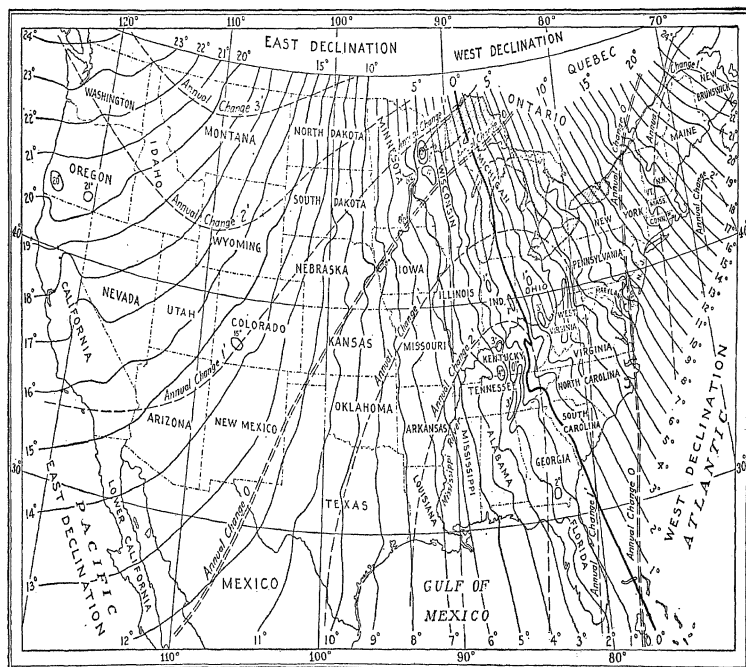


FIG. 11. ISOCONIC CHART OF THE UNITED STATES FOR 1940.
From U. S. Coast and Geodetic Survey.

dash) line running through Pennsylvania and New York, and in the region between the agonic line and the line of zero change running from Mexico to Wisconsin; elsewhere they are decreasing.

The declination of the magnetic needle may be obtained in the field by determining the true bearing of a line by observation on the North Star, and comparing that true bearing with the magnetic bearing. (See Art. 129.)

43. RELATION BETWEEN ANGLES AND BEARINGS. To calculate the angle between two lines the bearings of which are known, it is convenient to make a careful sketch of the lines with respect to N-S and E-W lines. In Fig. 12,

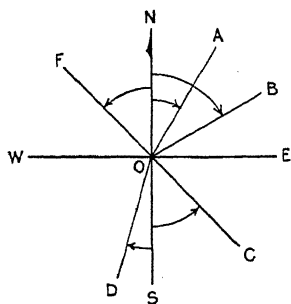


FIG. 12. ANGLES MARKED WITH ARCS ARE BEARINGS.

AOB = difference of bearings.

AOC = 180° — sum of bearings.

AOD = 180° — difference of bearings.

AOF = sum of bearings.

For example, if the bearing of OA is $N\ 20^\circ\ E$, and that of OB is $N\ 70^\circ\ E$, the angle between the lines is $70^\circ - 20^\circ = 50^\circ$. If the bearing of OF is $N\ 50^\circ\ W$ and that of OD is $S\ 10^\circ\ W$, the angle between these lines is $180^\circ - 50^\circ - 10^\circ = 120^\circ$.

Bearings may also be computed from angles. For example, in Fig. 13 the magnetic bearing of AB was observed in the field $N\ 31\frac{1}{2}^\circ\ E$. At B a deflection angle of $8^\circ\ 10'$ was measured to the left and the bearing of BC calculated as $N\ 23^\circ\ 20'\ E$. Had the interior angle ABC been measured instead of the deflection angle, the bearing BC could have been calculated just as readily.

44. Computing the Magnetic Bearing of a Line from an Observed Magnetic Bearing of the Line Taken at Some Other Date. This problem involves the use of the declination on the date when the bearing was taken and on the date when the bearing is required. To illustrate; suppose the observed magnetic bearing of a line in 1850 was $N\ 60^\circ\ E$ and the declination in 1850 was $2^\circ\ W$; required the magnetic bearing of the line in 1940 when the declination was $15^\circ\ W$.

Draw two lines at right angles to each other representing true N & S and E & W lines (Fig. 14). Draw a line ON_1 making the angle NON_1 represent 2° . This line represents magnetic N and S in 1850. Draw the line OA making the angle $N_1OA = 60^\circ$. These lines represent the conditions existing in 1850. Next draw

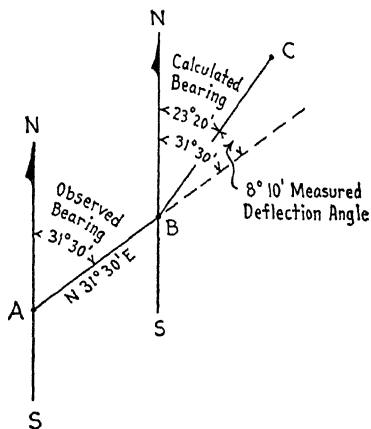


FIG. 13. CALCULATING BEARINGS FROM ANGLES.

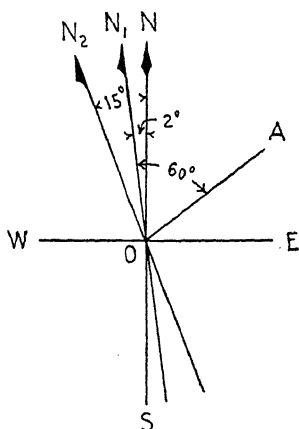


FIG. 14. CALCULATING BEARINGS FOR DIFFERENT YEARS.

the line ON_2 making the angle NON_2 represent 15° . The bearing of OA in 1940 was equal to the angle $N_2OA = 60^\circ + 15^\circ - 2^\circ = 73^\circ$. Hence the bearing of the line in 1940 was N 73° E.

If the bearing of the line in 1850 had been S 60° E, the bearing in 1940 would be $60^\circ + 2^\circ - 15^\circ$, or S 47° E.

If the bearing of OA had been N 85° E in 1850, the angle N_2OA would be $85^\circ + 15^\circ - 2^\circ = 98^\circ$. The bearing of this line in 1940 would be $180^\circ - 98^\circ =$ S 82° E.

45. Rerunning Old Lines. True bearings relate to true north, and magnetic bearings relate to magnetic north. It is therefore necessary to mark all bearings on the plan clearly with a proper note, so that there will be no question as to whether they are true or magnetic. In all cases the year of the survey should appear on the plan. Old deeds usually give magnetic bearings of property lines. It is therefore necessary before the surveyor can rerun these old lines that the difference be determined in the

declination between the date when the deed survey was made and the date of the present survey. All bearings must be corrected by this difference.

For example: Fig. 15 shows the magnetic bearings taken from a deed of a survey made with a compass in 1850 when the declination was 5° W. The declination in 1939 was 15° W. What were the magnetic bearings in 1939?

Between 1850 and 1939 the direction of the needle when pointing to magnetic north has shifted $15^\circ - 5^\circ = 10^\circ$ to the west. Hence the corrected bearings become

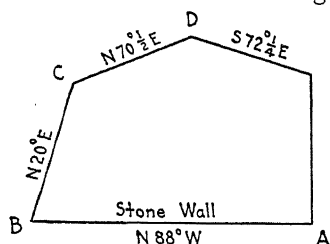


FIG. 15. MAGNETIC BEARINGS IN 1850.

<i>AB</i>	N 88°	W	$- 10^\circ$	= N 78°	W
<i>BC</i>	N 20°	E	$+ 10^\circ$	= N 30°	E
<i>CD</i>	N $70\frac{1}{2}^\circ$	E	$+ 10^\circ$	= N $80\frac{1}{2}^\circ$	E
<i>DE</i>	S $72\frac{1}{4}^\circ$	E	$- 10^\circ$	= S $62\frac{1}{4}^\circ$	E
<i>EA</i>		W	$+ 10^\circ$	= S 11°	W

Another method of rerunning the old lines may be used when one line, such as the stone wall *AB* in Fig. 15, can be identified.

The angles can be computed from the magnetic bearings and the lines rerun beginning with the line *AB*. (See Art. 43.)

Thus the interior angle at

$$A = 88 + 1 = 89^\circ$$

$$B = 180 - (88 + 20) = 72^\circ$$

$$C = (90 - 70\frac{1}{2}) + 90 + 20 = 129\frac{1}{2}^\circ$$

$$D = 70\frac{1}{2} + 72\frac{1}{4} = 142\frac{3}{4}^\circ$$

$$E = 180 - (72\frac{1}{4} + 1) = 106\frac{3}{4}^\circ$$

Another approach may be made by obtaining the present magnetic bearing of *AB*. Comparing this with the old bearing given in the deed gives the change in declination which has taken place, and the other bearings may be adjusted by this amount.

46. LOCAL ATTRACTION OF THE NEEDLE. The direction of the needle as fixed by the earth's magnetic field is altered by any magnetic materials which may happen to be near the compass. The tape, a wire fence, ore-bearing rock, water pipe, or electric power and light lines of direct current will produce errors in the

pointing of the needle. When near powerful electric currents the needle may not even remain still. Consequently **the magnetic needle should be used with caution.** Local attraction may be detected by taking bearings of a line from both its ends. If the bearings have the same angular value it may be assumed that no local attraction exists; if they differ, local attraction does exist and it is imperative to find the correct bearing of the line. This is done by making use of the fact that the angle computed from the bearings taken at the intersection of two lines is the true angle even if the local attraction is deflecting the magnetic needle. The latter principle will only apply when the local attraction at any station remains constant. It will not apply if the cause of the attraction is removed between observations.

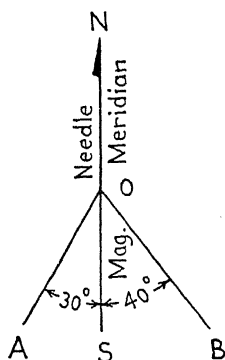


FIG. 16.

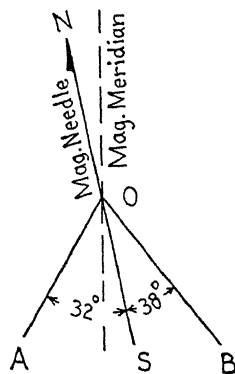


FIG. 17.

In Fig. 16, if NS is the correct magnetic meridian, the correct magnetic bearing of OA is S 30° W and of OB is S 40° E; the angle $AOB = 70^\circ$. In Fig. 17 are the same points, A, O and B. The needle, because of local attraction, has been deflected from the magnetic meridian 2° to the west. The bearing of OA is read as S 32° W and that of OB as S 38° E. The computed value of the angle from these measurements is 70° , as before.

Fig. 18 shows a series of connected lines, the forward and reverse bearings of which have been observed. Starting at A in Fig. 18 the bearing $AO = N 33^\circ E$ was first observed. The in-

strument was then moved to O , and the reverse bearing $OA = S 32^\circ W$ and a forward bearing $OB = S 20^\circ E$ were taken. Similarly, forward and reverse bearings were taken at B and C . As the forward and reverse bearings of OA and OB do not agree, local attraction probably exists at A or O or at both A and O . The problem is to find the correct magnetic bearings of AO and OB .

An inspection of the bearings shows that the forward and reverse bearings of line BC agree, so it is assumed that no local attraction exists at B or C . If there is no local attraction at B , the reverse bearing of BO ($N 18^\circ W$) is correct, and the forward bearing OB should be $S 18^\circ E$ instead of $S 20^\circ E$. The true angle of AOB may be computed from the bearings taken from O , regardless of whether or not there is local attraction at O , as follows: $20^\circ + 32^\circ = 52^\circ$. The correct magnetic bearing of OA therefore is $52^\circ - 18^\circ = S 34^\circ W$, and of AO , $N 34^\circ E$, which differs by one degree from the observed bearing from A of $N 33^\circ E$. This indicates that there is local attraction at A as well as at O .

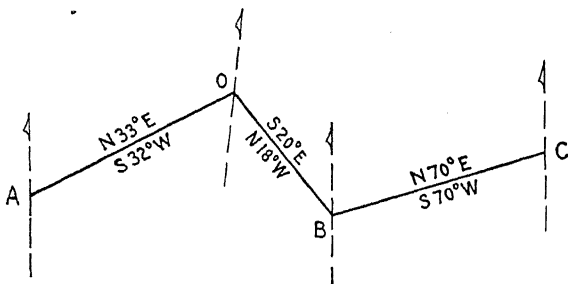


FIG. 18. LOCAL ATTRACTION.

If the forward and reverse bearings fail to agree for any of the lines of the traverse in Fig. 18, the traverse can be extended beyond C to additional selected points and the forward and reverse bearings taken until a line is found for which they agree. The correct bearing of this line will then be established, and the bearings of the desired lines (CB , OB , OA) can be computed from this reference line using the computed angles between traverse lines.

47. COMMON SOURCES OF ERROR IN COMPASS WORK.

1. Iron or steel, or electric current, near compass.
2. Parallax in reading needle.

48. COMMON MISTAKES.

1. Reading wrong end of needle.
2. Not letting needle down on pivot.
3. Reading the wrong side of the 10th degree, viz., reading 41° instead of 39° .

FIELD EXERCISES

1. Set up compass over a point, level it, lower the needle, and read the bearings to several readily identified points, such as distant church spires, trees, property corners, etc. Then compute the angles between these lines of sight.

2. Set 5 stakes in the ground to form the corners of a 5-sided irregular field. Set up the compass at each point and take the bearings of the forward and of the back lines; note the difference between the forward bearing and the reverse bearing of the same line. See if there is evidence of local magnetic attraction which affects the bearings of certain lines and not of others. Compute the interior angles from the average of the forward and reverse bearings (if no local attraction). Add the interior angles computed from these bearings and determine the total error in angles. The sum of the interior angles of a polygon $= (n - 2) 180^\circ$, where n is the number of sides.

3. Mark 5 points $ABCDE$ forming an irregular line (called a traverse). Set one point B near an object that will probably cause local attraction, such as a steel pole, a steel frame building, or an automobile; place the other points so that they are probably remote from local attraction. Observe the forward and reverse bearings of each of these 4 lines, $ABCDE$, and determine from these observations the probable amount of local attraction at point B .

4. Select two inaccessible points. The bearing of the line joining them is desired. By trial and error locate a point on line so that, when looking through the sights in both directions, either point is visible. Observe the magnetic bearing.

5. It is desired to determine by compass the magnetic bearing of an iron fence. Set the compass over one end of the fence A and observe the bearing of the fence AB . Next, observe the bearing of a point C which is some distance away from the fence and which is likely to be free from local attraction. Set over the point C and observe the bearing CA . If the bearings AC and CA are not in agreement, select another point D . Observe the magnetic bearings of CD and DC and note whether they are in agreement. If they are, compute the correct magnetic bearing of the fence; if not, choose another point and observe the direct and reverse bearings. Repeat the procedure until the direct and reverse bearings are found to be in agreement.

PROBLEMS

1. Compute the angles AOB , COD , EOF and GOH from these bearings.

- (a) OA , $N 10^\circ E$
 OB , $N 43\frac{1}{2}^\circ E$
 (b) OC , $N 71^\circ E$
 OD , $S 12^\circ E$

- (c) OE , $S 8\frac{1}{2}^\circ E$
 OF , $N 64^\circ W$
 (d) OG , $S 24^\circ W$
 OH , $S 55\frac{3}{4}^\circ E$

2. Magnetic bearing of line AB is $N 68\frac{1}{4}^\circ E$. Angle ABC is $89^\circ 20'$, C being north of AB . Compute the bearing BC . The deflection angle at C , i.e., angle

between BC produced and line CD , is $25^{\circ} 15'$ to the right. Compute the bearing of CD .

3. (a) In 1860 a line had a magnetic bearing of $N 34^{\circ} W$. The declination of the needle at that place, at that time, was $11^{\circ} 17' W$. In 1935 the declination had changed to $15^{\circ} 55' W$. What was the magnetic bearing of the line in 1935?

(b) In 1885 a line had a magnetic bearing of $S 88^{\circ} 45' W$. The declination was $0^{\circ} 15' E$. In 1930 the declination was $2^{\circ} 10' W$. Find the magnetic bearing of the line in 1930.

4. Magnetic bearings of a closed field are as follows:

STA.	FORWARD BEARING	REVERSE BEARING	STA.	FORWARD BEARING	REVERSE BEARING
A	due E	$S 24^{\circ} W$	D	$N 72\frac{3}{4}^{\circ} W$	$N 42\frac{1}{2}^{\circ} E$
B	$S 12^{\circ} E$	$N 89\frac{3}{4}^{\circ} W$	E	$N 23\frac{3}{4}^{\circ} E$	$S 72\frac{1}{2}^{\circ} E$
C	$S 30\frac{3}{4}^{\circ} W$	$N 15^{\circ} W$			

Errors in magnetic bearings may be caused either by local attraction or by errors in measurement.

(a) Will the former affect the interior angles computed from bearings?

(b) Compute the interior angles of the above traverse from the forward and reverse bearing at each station. Adjust these angles so that the sum will equal the geometric requirements of the figure (See Field Exercise No. 2), by placing the error in the angles adjacent to the shortest side, which is AE . Start from a station that is free from local attraction and compute the corrected bearings of all sides of the figure, using adjusted angles. Show check on these calculations.

(c) At which stations was local attraction present?

5. In running a traverse with a surveyor's compass, the direct bearing of no line agrees with the reverse bearing. State what you would do in order to determine the correct magnetic bearings of the lines of the traverse.

6. Points A and B are on the shore-line of a pond, and point x is on an island in the pond. The shortest distance and its bearing from x to the line AB are desired from the following data:

$$\begin{array}{ll} AB = 652.5 \text{ feet} & \text{Bearing } Bx = S 42^{\circ} W \\ \text{Bearing } AB = S 88\frac{1}{4}^{\circ} E & \quad \quad \quad \text{" } Ax = S 31\frac{1}{4}^{\circ} E \end{array}$$

7. The bearing of one side of a field in the shape of a regular pentagon is $N 52\frac{1}{2}^{\circ} E$. Compute the bearings of the other sides, taken around the field in order.

8. A four-sided piece of land near San Francisco was surveyed in June 1820, and the bearings of the sides recorded as follows: $N 3\frac{1}{4}^{\circ} W$, $N 85\frac{1}{2}^{\circ} E$, $S 20\frac{1}{4}^{\circ} E$ and $S 88^{\circ} W$. The declination in 1820 was $14\frac{1}{4}^{\circ} E$ and in 1940 it was $18\frac{1}{4}^{\circ} E$. Which bearings should be used to retrace the lines in 1940?

9. If the true bearing of a line is $S 41^{\circ} 47' E$, and the observed magnetic bearing of the line is $S 55^{\circ} \frac{1}{4} E$, and the declination is $10^{\circ} W$, what is the error caused by local attraction and its direction referred to the magnetic meridian?

10. If at a place the local attraction is known to be 5° East and the declination is $15^{\circ} W$, what is the true bearing of a line observed at that place with a compass and found to be $N 88^{\circ} W$?

11. Compute the declination from the Isogonic Chart for the northeast corner of Wyoming for the year 1940.

CHAPTER IV

MEASUREMENT OF ANGLES — USE OF THE TRANSIT

49. THE TRANSIT.* The engineer's transit is the "universal instrument" of the surveyor. It is used for measuring horizontal and vertical angles, for setting points in line (horizontally or vertically), for measuring approximate distances by the stadia principle, for leveling operations, and as a compass (when provided with a compass needle).

It consists of a horizontal circular plate (lower plate) graduated around its periphery in degrees and fractions, and is mounted at right angles on a hollow conical vertical axis, or spindle, at its center (Figs. 19, 20, and 21). Another circular plate (upper plate) fits inside of the lower plate and is mounted on a second spindle which rotates inside the spindle of the first plate. Two verniers, for reading the graduated circle at points 180° apart, are attached to this upper plate. The upper plate also carries two uprights (standards) which support the horizontal axis to which a telescope is attached at right angles. On one end of the horizontal axis is a graduated vertical arc, or circle, for measuring vertical angles; this is read by means of a vernier fastened to the standard. Attached to the upper plate are two spirit levels, at right angles to each other, for leveling the circular plates. The outer spindle rotates in a socket which is part of the leveling head. The leveling head contains screws by means of which the plates are leveled, and threads by which the instrument is attached to the tripod.

The movement of both plates, as well as the vertical motion of the telescope about its horizontal axis, is controlled by clamps and slow-motion screws called *tangent screws*.

* European theodolites (transits) are being used to a limited extent in the United States. They are much smaller than the American makes. See *Elementary Surveying* by Breed and Hosmer, Vol. I, Chap. III.

Attached to the telescope is a spirit level for use when the transit is being used as a level. The telescopes of all modern transits may be rotated through 180° (and even through 360° in many transits) about the horizontal axis. When the telescope

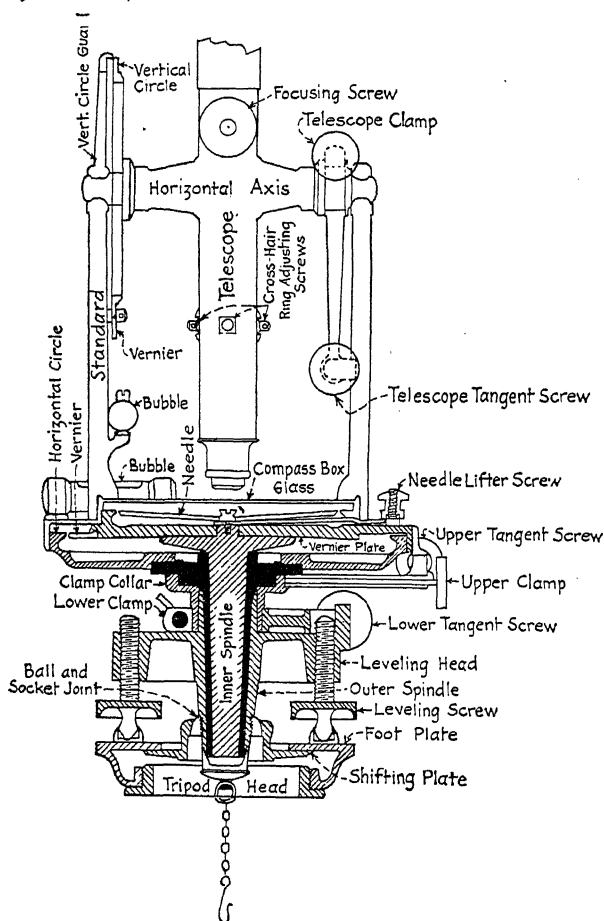


FIG. 19. SECTION OF TRANSIT.

level (Fig. 20) is on the underside of the telescope, it is in the *direct* position; when it is above the telescope, it is in the *inverted* position.

A small hook suspended by a chain is attached to the center of the instrument and hangs through an opening in the top of the tripod. The plumb-bob is attached to this hook when the instrument is being set over a point.

By releasing the pressure on any two adjacent leveling screws

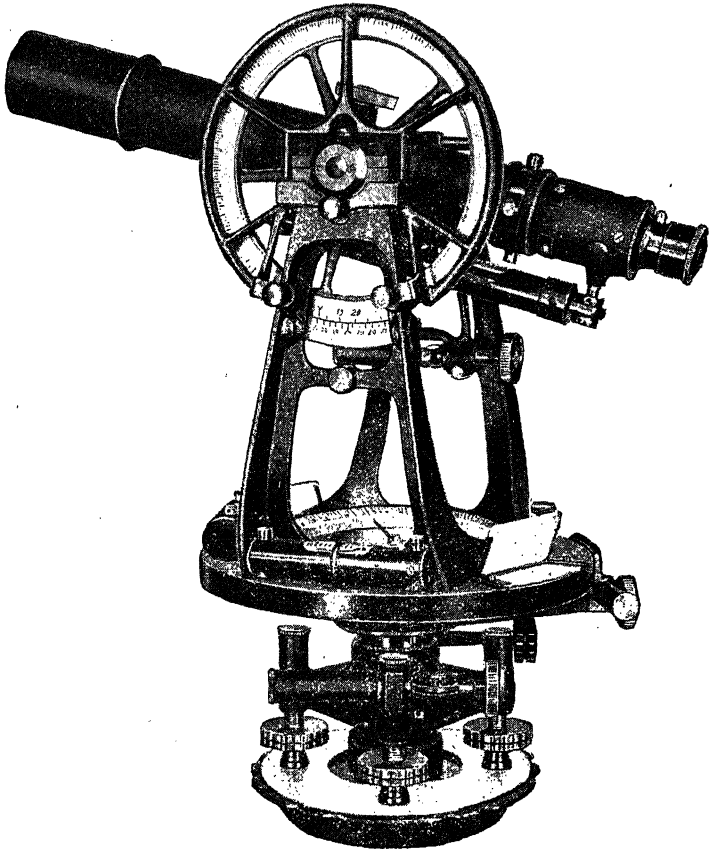


FIG. 20. ENGINEER'S TRANSIT.

the entire head supporting the instrument is slightly lowered, and the head may then be shifted a small amount on the footplate (Fig. 19) without moving the latter. This is of assistance in making a final precise setting over the point.

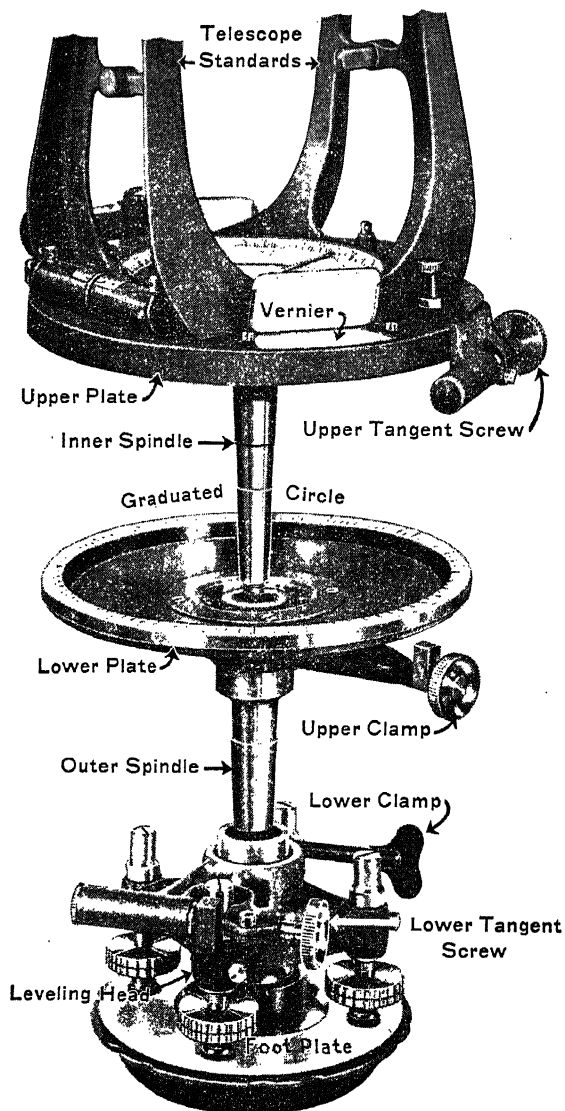


FIG. 21. ENGINEER'S TRANSIT SHOWING HOW ITS PARTS ARE ASSEMBLED.

If the transit is not provided with a level tube and a vertical circle, or arc, then it is called a *Plain Transit*. These transits are used where leveling or stadia work is not required.

50. THE TELESCOPE. The telescope (Fig. 22) is similar to an ordinary telescope, but, in addition to the objective and eyepiece, it contains within the telescope tube an annular ring across which two or more spider web cross-hairs (or fine wires) are stretched at right angles to each other. The cross-hairs furnish a definite means of bisecting objects which are being sighted. The objective is a compound lens. It is mounted in a tube which may be moved by means of a rack and pinion in or out of the telescope tube in the process of focusing.

In some telescopes the objective lens is fixed rigidly to the telescope. Focusing the object is accomplished by means of a lens located between the objective and cross-hairs, and moved by a rack-and-pinion motion. Such telescopes are called interior focusing telescopes.

The cross-hairs consist of two very fine spider threads stretched across a metal ring at right angles to each other, and fastened to the ring by means of shellac. Sometimes fine wires or lines ruled on thin glass are used instead of spider threads. The cross-hair ring (Fig. 23) is held in place by four capstan-headed screws; this permits it to be moved vertically and horizontally in the telescope tube. The holes in the tube through which the screws pass are large enough to allow a slight rotary motion of the ring in adjusting.

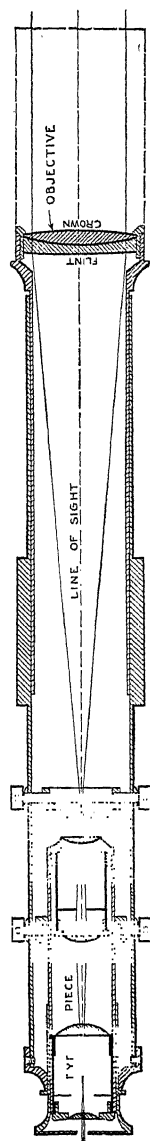


FIG. 22. LONGITUDINAL SECTION OF A TRANSIT TELESCOPE (ERECTING EYEPiece).

Light rays meet at Cross-hairs; in Eyepiece they become inverted and again erected before reaching the eye.

The eyepiece is essentially a microscope and consists of two or of four lenses. When the eyepiece consists of two lenses the objects appear upside down (inverted) when observed through the telescope, and the instrument is called an *inverting* transit. When the eyepiece consists of four lenses the objects appear right side up (erect), and the instrument is called an *erecting* transit. The eyepiece tube may be drawn in or out for focusing the cross-hairs. On some instruments this is done by rotating the eyepiece about

its longitudinal axis; in others the eyepiece is moved by means of a rack-and-pinion.

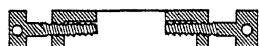
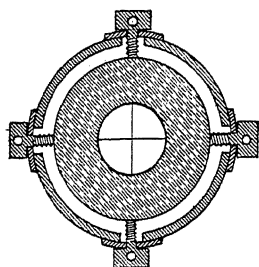


FIG. 23 CROSS-HAIR RING,
OR DIAPHRAGM.

In sighting an object its image is brought to focus in the plane of the cross-hairs; consequently both the image and the cross-hairs are magnified when observed through the eyepiece. To obtain proper results the cross-hairs must be correctly focused, i.e., they must be in the focal plane of the eyepiece. To determine if this condition exists, sight the vertical cross-hair on a point. Move the head

slightly and slowly sideways and observe if the vertical hair appears to move off the point. If it does, a condition known as parallax exists, and this indicates imperfect focusing. To focus the eyepiece, point the telescope at the sky or at some white surface and move the eyepiece until the cross-hairs appear as dark and distinct as possible. Test for parallax. After a few trials a position is found where there is no parallax. When once found, the position of the eyepiece will not have to be changed for that observer. Each observer, however, will have to focus the eyepiece for his personal eye condition.

A line through the optical center of the lens (approximate center of lens) and through the intersection of the cross-hairs is called the "*line of sight*," or the "*line of collimation*." In a properly adjusted instrument it is perpendicular to the horizontal axis of the telescope.

The angular diameter of the circle of view as seen through the

telescope is called the "*field*". Ordinary surveying instruments have a field of one degree or more.

If the lens system is such that the dimensions of an object viewed through the telescope appear to be 20 times those seen by the naked eye, the telescope is said to have a magnifying power of 20 diameters.

51. THE SPIRIT LEVEL. The ordinary carpenter's level, familiar to everyone, has a spirit level which consists of a glass tube partially filled with a non-freezable liquid, and is sealed at both ends. The space above the liquid forms an air bubble which moves as the tube is tilted. In cheap levels the glass tube (or vial) is merely a tube that has been bent by heating it. In those level vials that are used on surveying instruments, the inside upper surface of a straight glass tube of uniform bore is ground by means of an arbor to a longitudinal circular curve of the desired radius (Fig. 24). The radius of this arc depends on the use which is to

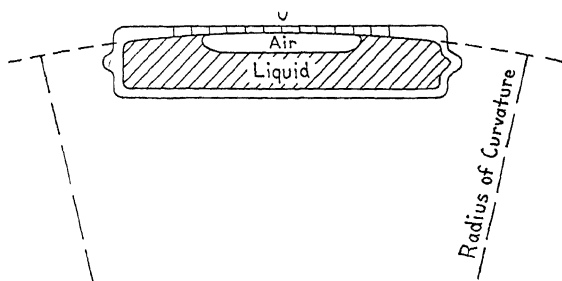


FIG. 24. SPIRIT LEVEL.

be made of the level. In delicate geodetic and astronomical instruments the radius of the curve will be greater than in ordinary surveying instruments. A line that is tangent to this curve at its central point is called *the axis of the bubble tube*. A scale of equal parts reading from the center, or "zero," of the tube in both directions and marked on a metal strip, or etched on the glass vial, is provided for centering the bubble. As the air bubble always moves to the highest point of the tube, this will be the center of the tube when the bubble-tube axis is horizontal. The greater the radius of curvature, the more "sensitive" is the level,

i.e., a small shift in the position of the tube causes a large movement of the air bubble in the tube.

52. PRINCIPLE OF THE VERNIER. The vernier is a device for determining the fractional part of the smallest division of a scale more accurately than it can be estimated by eye. The principle of the vernier is illustrated in Fig. 25, which shows a scale that is to be read to tenths of its smallest division. A dis-

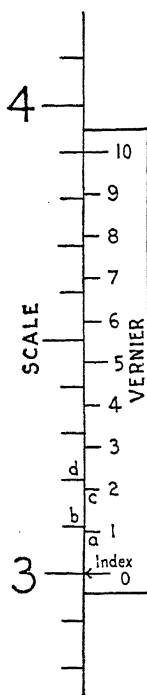


FIG. 25.
READING 3.00.

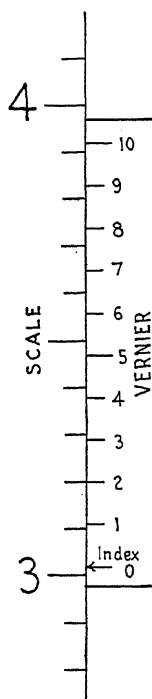


FIG. 26.
READING 3.02.

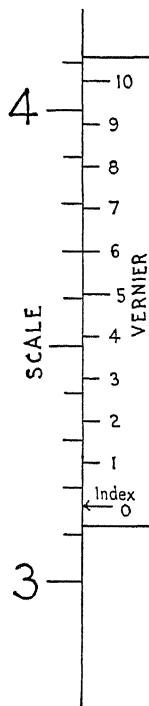


FIG. 27.
READING 3.16.

tance equal to 9 spaces on the scale is made to represent the *full* length of the vernier, and this is then divided into 10 equal parts. Consequently one space on the vernier equals $\frac{9}{10}$ of a space on the scale; hence *ab* equals $\frac{1}{10}$ of a scale space, *cd* equals $\frac{2}{10}$, etc. On the vernier there is an "index" mark, which points to the actual

scale reading. If the vernier in Fig. 25 is now raised until *a* coincides with *b*, the reading becomes 3.01. If the vernier is moved until *c* coincides with *d*, the reading becomes 3.02, as shown in Fig. 26. Referring to Fig. 27 the procedure to follow in reading the scale by use of the vernier is: first, follow up the scale in the direction of increasing numbers and find the nearest scale reading **below** the vernier index, which is 3.1 in Fig. 27; then follow up the vernier in the **same direction** in which the scale was read until the line on the vernier is found which coincides with a line on the scale. In this instance it is 6. The reading, therefore, is 3.16. It will be observed that it is impossible to have more than one coincidence.

53. CIRCLE GRADUATIONS AND VERNIERS OF TRANSITS. The horizontal circles of modern transits are divided into spaces of half-degrees (30 minute), thirds of a degree (20 minute) or sixths of a degree (10 minute). The circle graduations are usually numbered at 10° intervals continuously from 0° to 360°, in both directions from 0°. The inside row of figures increases in a clockwise direction, and the outside row in a counterclockwise direction.

Verniers are provided for reading the angle closer than the smallest circle division, as explained in Art. 52. The graduation of the vernier in every case depends upon the subdivision of the circle. For example, to read to 01' on a circle graduated in half-degrees (30') spaces, the space between each line on the vernier should be $\frac{29}{30}$ of the 30' arc space on the circle; that is, an arc composed of 29 divisions of 30' each (14° 30' on the circle) is subdivided into 30 equal parts to obtain the space between division lines on the vernier. Therefore, one division on the vernier is 01' less in angular measurement than one division on the circle.

Fig. 28 shows a 01' vernier set with its zero (index) opposite the zero (360°) of the circle, ready for measuring an angle. The lines on the vernier on both sides of zero fail to match the lines on the circle by 01'; the second lines on the vernier on both sides of zero fail to match the lines on the circle by 02', and so on. In setting exactly at 0°, this can best be accomplished by noting that the vernier lines on both sides of the zero fail to match the circle divisions by the **same amount**, i.e., they fail to match symmetrically.

In measuring horizontal angles, the outer circle remains fixed in position while the inner circle (with the vernier) moves with the telescope and thus travels along the stationary graduated (outer) circle. If the telescope is turned to the right (clockwise), the zero of the vernier moves clockwise around the circle; if the

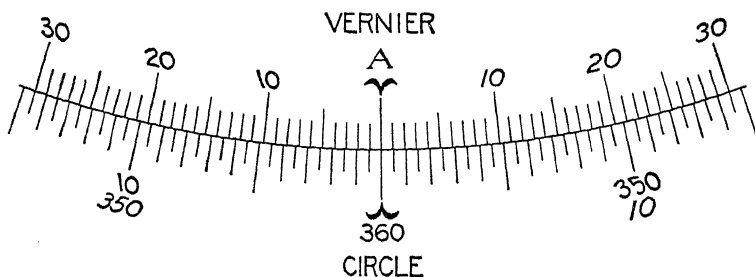


FIG. 28. ONE-MINUTE VERNIER SET AT 0° .

telescope is turned to the left (counterclockwise), the zero (index) of the vernier moves counterclockwise.

The arrow at the zero of the vernier always points to the reading on the circle. In Fig. 29 one can see that the reading on the

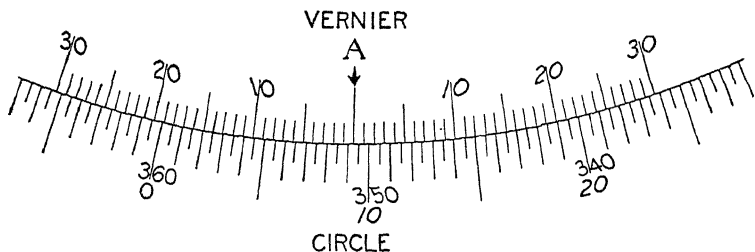


FIG. 29. READING OUTER ROW OF FIGURES $0^\circ 16'$ (angle measured counterclockwise), READING OF INNER ROW, $350^\circ 44'$ (angle measured clockwise).

outer row is approximately $9\frac{1}{4}^\circ$. The exact reading is obtained by reading the degrees and half-degrees on the circle, and then crossing over to the vernier and passing along it in the **same direction** in which the angle has been turned until the graduation on the vernier is found that coincides with a graduation on the circle. The reading on the vernier at this coincidence line is the number

of minutes to be added to the degrees and half-degrees that were read from the circle, i.e., $9^{\circ} 00' + 16' = 9^{\circ} 16'$ in Fig. 29. A reading glass is commonly employed for reading the vernier.

Notice that in both Figs. 28 and 29 there are two verniers, one running in each direction from the common zero-point; this is called a "double vernier." Double verniers are used in order that the circle may be read either clockwise or counterclockwise, depending upon which way the telescope moved in describing the angle being measured.

If the circle is divided into $20'$ divisions and it is desired to read the angle to the nearest $\frac{1}{2}$ minute, or $30''$, the vernier is so

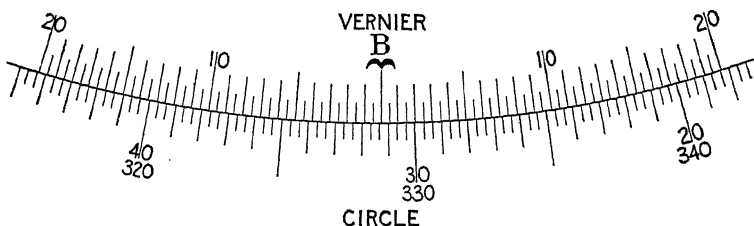


FIG. 30. READING OF INNER ROW, $31^{\circ} 17' 30''$.
READING OF OUTER ROW, $328^{\circ} 42' 30''$.

constructed that 39 of the circle divisions are divided into 40 equal parts. Thus each vernier division is $\frac{39}{40}$ of one of the circle divisions, or $\frac{1}{40}$ less than a circle division; and since each circle division represents $20'$ of angle, $\frac{1}{40} \times 20' = \frac{1}{2}'$, or $30''$. In a similar manner a vernier may be devised for any type of circle graduation. Fig. 30 shows a $30''$ circle set at the readings indicated.

In the example just cited, the value $30''$ is the smallest angular value that can be read directly from this particular vernier. It is called the "least count" and is equal to the smallest division on the circle divided by the number of divisions on the vernier. For example, in Fig. 30, the least count is $20' \div 40 = 30''$.

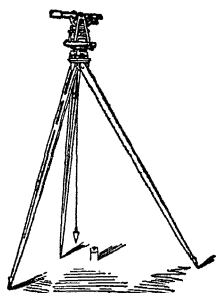
Almost all transits have two verniers, located 180° apart. If the circle is perfectly graduated and the plates perfectly centered, the vernier readings for any part of the circle will differ by exactly 180° . With instruments reading as close as $10''$ it is

often found, owing to a very slight eccentricity in the plates, that the opposite verniers read slightly different in certain parts of the circle. Consequently in precise work, both verniers are read and the mean of the readings is accepted as the correct vernier reading.

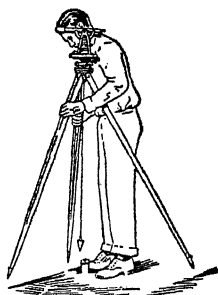
USE OF TRANSIT

54. SETTING UP THE TRANSIT OVER A POINT. The transit is first set up near the point, with the legs spread at such an angle as will insure stability. (See Fig. 31.) The foot plate should be placed approximately horizontal by adjusting the position of one or more of the legs. For the purpose of centering the instrument over the point a plumb-bob is suspended by a string from the hook under, and in the center of, the instrument. The plumb-bob string is tied with a slip knot, or some slipping device is used, so that the height of the plumb-bob may be readily adjusted. Without changing the general set-up the instrument is then moved bodily until the plumb-bob is within about an inch of the point. Sufficient pressure is then applied to the legs to insure firmness in the ground and at the same time to bring the plumb-bob to within $\frac{1}{8}$ in. of the point. While doing this care should be taken to see that the foot plate remains nearly horizontal, thereby requiring little turning of the leveling screws to make the plates level. The final centering of the plumb-bob over the point is made by means of the shifting head of the transit. By loosening two adjacent leveling screws the transit head with the attached plumb-bob is lowered a slight amount, and in this position it may be moved about a quarter of an inch in any direction from its center. When the plumb-bob is over the point, the leveling screws are tightened just enough so that they have a firm bearing. The foregoing steps are illustrated in Fig. 31.

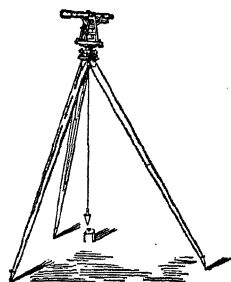
While the above-mentioned six steps may be necessary for the beginner, the first two may be performed by grasping two legs of the tripod as shown in Fig. 32. Place the third leg on the ground at such a point, with respect to the stake, that when the other two legs are allowed to touch the ground the foot



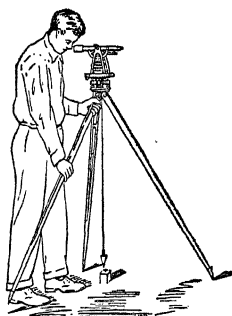
1st. Set instrument on ground near point, with legs spread to give convenient height. Without reference to point, move legs so that plates are nearly level. Tripod leg screws should be firm but not binding.



2nd. Pick up instrument bodily without disturbing relative position of legs and head.



3rd. Place down again with plumb-bob within 1" of point and bring bob to within $\frac{1}{4}$ " to $\frac{1}{2}$ " of point by shifting legs slightly.



4th. Swinging one or more legs in arc of circle will approximately level head without changing materially the position of bob.



5th. Press each leg firmly into ground about same amount, watching position of bob so that it will finally return to within $\frac{1}{8}$ " to $\frac{1}{4}$ " of the tack. Plates should still be nearly level.



6th. Bring bob exactly over tack by loosening two *adjacent* leveling screws and shifting head. Then tighten leveling screws and proceed to level the plates.

FIG. 31. STEPS IN SETTING UP A TRANSIT.

plate will be level and the height of the telescope will be convenient for the transitman and, at the same time, will allow sufficient spread of the legs to keep the instrument stable. It obviously requires practice to perform this step effectively.

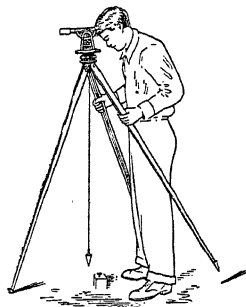


FIG. 32. FIRST STEP IN SETTING UP TRANSIT. (ALTERNATE METHOD.)

When setting up the transit on sloping ground the usual practice is to place one leg of the tripod up, and the other two legs down, the slope.

55. Leveling the Transit. To level the plates, turn the instrument about its vertical axis until the axis of one bubble is parallel to a line through the bases of a pair of opposite leveling screws. The second bubble will then be parallel to the other pair of leveling screws.

Grasp a pair of opposite screws between the thumbs and the forefingers and center the bubble approximately. This is done by **uniformly** turning this pair of leveling screws in such a way as to move the thumbs either toward or away from each other, as shown in Fig. 33, thereby tightening one screw by the **same** amount that the other is loosened. This tilts the leveling head while at the same time sustaining a definite support for it on both screws. The screws should bear firmly on the foot plate at all times, but they should never be allowed to bind. Similarly, the other bubble is centered by means of the other pair of screws. Alternate this process until **both** bubbles are in the center. Then observe the position of the plumb-bob; if it has moved off the point, reset by means of the shifting head and again level. Should one of the screws bind, the other should be turned faster, and if this does not release the tight screw, one of the screws of the other pair should be loosened slightly.

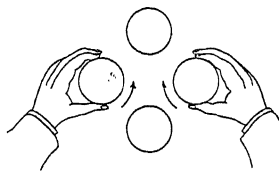


FIG. 33. SHOWING HOW TO MANIPULATE FINGERS WHEN LEVELING.

After both bubbles have been centered in one position, turn the plate about the vertical axis through an angle of 180° . If

the bubbles do not remain central, it indicates that the adjustment of the spirit level is imperfect, but the instrument can still be leveled (without the necessity of adjusting the plate levels) by moving the leveling screws until **each bubble moves half-way back to the center of its tube**. If this is done correctly, the plates will be truly horizontal and the bubble will then stay in the **same place in the tube** (not in the center), as the plates are turned slowly around the vertical axis. The transit is now ready for angle measurements.

56. MEASURING A HORIZONTAL ANGLE. With both clamps loose, set the zero of the vernier by eye close to 0° on the circle; tighten the **upper** clamp (this holds the two plates firmly together), and set the index of the vernier exactly at 0° by means of the upper slow-motion screw, leaving the **lower** clamp loose. Direct the telescope at one of the objects to be sighted and clamp the **lower** clamp when the object is in the field of view and appears near the vertical cross-hair. Set the vertical hair exactly on the object by the **lower** slow-motion screw. The telescope should be carefully focused (Art. 50). Next loosen the **upper** clamp, turn to the second object, sight on it, tighten **upper** clamp, and use slow-motion screw to bring the object behind the vertical cross-hair. The angle is now read by adding to the circle reading the minutes which are read on the vernier. To eliminate errors of adjustment of the cross-hairs and of the horizontal axis, the angle may be measured again in the above manner with the telescope in the reversed (inverted) position, and the mean of the measured angles taken. A better procedure, however, is to measure the angle by repetition, once with the telescope direct and once reversed, as described in Art. 59.

57. Measuring a Deflection Angle. A deflection angle is the angle between the prolongation of a straight line and a line ahead. For example, the deflection angle at B (Fig. 34) between the lines AB and BC is the angle $A'BC$. To measure the angle $A'BC$ the transit is set at B , the index of the vernier set to 0° on the horizontal circle, and a sight taken on point A . The telescope is then plunged, i.e., turned on its horizontal axis, so that it now points in the direction BA' . The upper clamp is loosened and a sight is taken on point C , using the upper clamp

and its tangent screw. The deflection angle $A'BC = 46^{\circ} 13' R$ is then read and recorded, as in Fig. 34. The letter R after the recorded angle means that the telescope was turned to the right to sight point C; had C been to the left of BA' the angle would be marked L. It is important to state whether the angle is to the right or the left. No deflection is complete without this

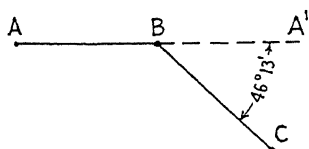


FIG. 34. DEFLECTION ANGLE.

information. It is good practice to repeat this angle mechanically by loosening the lower clamp and pointing the telescope again at A, leaving the telescope in its reversed position. The telescope is again plunged (now in a direct position), the upper clamp is loosened and a pointing is made on C by means of the upper clamp and its tangent screw. One-half of the second circle reading is the correct deflection angle to use. For example, if the first reading was $46^{\circ} 13' R$, the repetition may give $92^{\circ} 25'$. The mean deflection angle is $46^{\circ} 12\frac{1}{2}'$ to the right.

58. DIFFERENT METHODS OF MEASURING A HORIZONTAL ANGLE. Fig. 35 illustrates four different ways in which a horizontal angle may be measured. In the five-sided plot of land represented, the angles can be measured as

- (I) *Interior angles*, such as $206^{\circ} 36'$ at B
- (II) *Exterior angles*, all measured clockwise, such as $153^{\circ} 24'$ at B
- (III) *Deflection angles*, such as $26^{\circ} 36'$ at B
- (IV) *Azimuth angles*, such as $252^{\circ} 16'$ at B

Some surveyors prefer always to measure the **interior** angle of a field. Others prefer always to measure the **angles clockwise** (to the right). In topographical surveys the **azimuth** angle is often measured; it is the angle measured from the south direction, clockwise to the line ahead. Sometimes the magnetic and sometimes the true meridian is used. The way in which the south direction is defined at each angle point is explained in Art. 210.

It will be noticed that in three of the four methods mentioned above, the telescope is not plunged; whereas, when the **deflection** angle is measured, it is plunged, thereby introducing errors if the

line of collimation is not exactly perpendicular to the horizontal axis of the transit. The effect of this error can be entirely eliminated, however, by the method given in Art. 57.

59. MEASURING AN ANGLE BY REPETITION. After an angle has been measured once the inner and outer circles are left clamped together, and by proper manipulation of the instrument a second angle is mechanically added to the first angle. Obviously, half the second reading of the circle should closely check

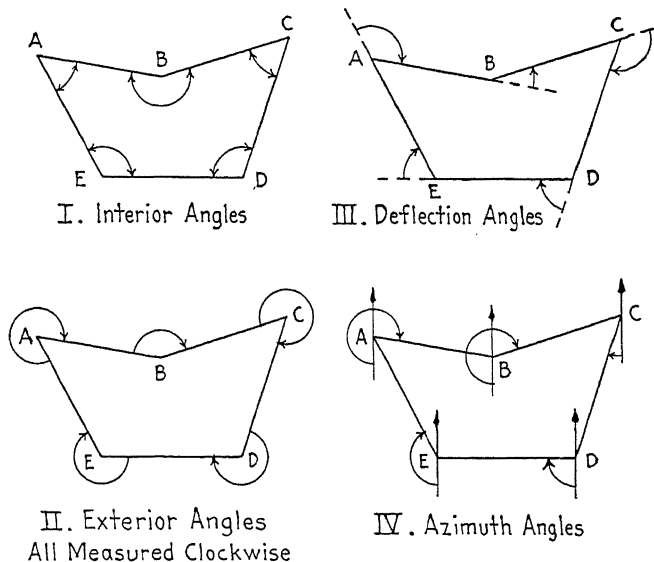


FIG. 35. DIFFERENT METHODS OF MEASURING HORIZONTAL ANGLES.

the reading of the first angle. This procedure is called “measuring an angle *by repetition*.” Thus if a mistake has been made in the first angle, it is at once evident. Another purpose of mechanical repetition of angles is to introduce additional precision. The cross-hair can be pointed on a well-defined object to a precision of about $5''$ of angle, whereas, in a one-minute instrument, the maximum error in angle is not greater than $30''$, which is much less precise than the telescope pointing. The repetition process in detail follows.

The first measurement is made in exactly the same way as

that described for a single angle. Then, **do not touch the upper clamp or upper tangent screw**, but loosen the lower clamp, turn the telescope back to the first object and set exactly on it by means of the lower clamp and tangent screw. The circle now reads, not 0° , but the first (single) angle. Next, loosen the upper clamp, turn the telescope to the second point and set exactly on it by use of the upper clamp and its tangent screw. The index of the vernier now points to the double angle on the horizontal circle. Half the angle now read is the improved value of the required angle. If the process is repeated and a third angle is mechanically added to the last reading, the circle reading is divided by three and a still more exact value of the angle is obtained. Six readings are usually the greatest number taken with the telescope in one position. When using this method it is customary to read half the angles with the telescope direct, and the other half with the telescope inverted, thus eliminating the errors of the line of sight (collimation) and of the standards, provided the plate is truly horizontal throughout the process. (Arts. 80 and 81.)

60. LAYING OFF AN ANGLE BY REPETITION. Lay off the desired angle as close as it can be read with the vernier on the graduated circle and place a temporary point exactly in line with the line of sight in the telescope. Measure this angle that has just been laid off by repetition, as explained in Art. 59; thus determining a more precise value for it by dividing by the number of readings. Suppose the angle intended to be laid off was $25^\circ 12'$, and by repetition it was found to have been actually laid off $25^\circ 12' 10''$. Measure approximately the distance from the instrument to the temporary point and compute the perpendicular distance (called an *offset*) to be laid off from the temporary point so as to reduce the angle by $10''$. Then set the final point by means of this offset distance. This offset usually is so small in comparison with the distance from the transit that, for all practical purposes, it is equal to the arc distance and can be computed as described in Art. 61.

61. RELATION BETWEEN OFFSETS AND SMALL ANGLES. It is always well to remember that the tangent (or sine) of a minute of arc is "three zeros and a three" (0.0003). Consequently the

offset for an angle of a minute at a distance of 100 ft. is 0.03 ft.; for an angle of 10 seconds it is 0.005 ft. The perpendicular offset corresponding to very small angles obviously is practically equal to the arc; the arc of an angle of $01'$ with radius unity is also 0.0003.

Suppose that an angle is measured between points 500 ft. away from the instrument, and that the sighting on one of these points is in error by $\frac{1}{4}$ in., the width of a pencil. How much error in the angle does this make? An error of one minute in angle corresponds to $500 \times 0.0003 = 0.15$ ft.; therefore 0.02 ft. ($\frac{1}{4}$ in.) corresponds to an angle of 8 seconds.

Other useful relations to remember are,

$$\text{Natural tangent of } 1^\circ = \frac{1.75 \text{ ft.}}{100 \text{ ft.}} = \frac{1}{60} \text{ roughly}$$

$$\begin{aligned} \text{Natural tangent of } 00^\circ 00' 01'' &= \frac{0.3 \text{ inch}}{1 \text{ mile}} \\ &= \text{"five z\u00e9ros and a five"} \\ &= 0.000005 \end{aligned}$$

62. PRECISION IN ANGLE MEASUREMENTS — THEIR RELATION TO MEASUREMENTS OF DISTANCES. When an angle is measured and recorded to the nearest $01'$, the minimum error in the angle is zero and the maximum error should be not over $30''$ to $40''$. Suppose the angle measured with a one-minute transit is read $42^\circ 37'$. Its true value can be anywhere from $42^\circ 36' 30''$ to $42^\circ 37' 30''$; there may be a maximum error of $30''$ either way from $42^\circ 37'$. The error in setting at 0° may have been such as to increase this error $05''$ to $10''$, or it may have been such as to decrease it. Strictly speaking then, the reading of the angle may be $35''$ to $40''$ in error, if we assume that both the error of setting at 0° and of reading the angle are the greatest they can be within reason and are both in the same direction. On the other hand, there may be no appreciable error in the angle; 0° on the circle may have been exactly set and the true angle may be exactly a full number of minutes. It is appropriate to assume, however, that, in this discussion, the maximum error in a single

angle read to the nearest minute is about $30''$, and that the average error of a number of angles is probably not over $15''$.

At 300 ft. distance $15''$ of angle corresponds to a maximum offset error of 0.022 ft., or about a quarter of an inch. If, then, the sides of a square 300 ft. long are all measured to the nearest 0.01 ft., the precision in the sides will appear to be greater than the precision in the angles if they are measured to the nearest minute. In practice, however, the fact that the 300.00 ft. distance is recorded to the nearest hundredth is no proof that it is not in error greater than 0.01 ft. If the taping is slightly in error due to temperature changes, to careless plumbing, or to other causes, this distance may readily be in error 0.02 to 0.04 ft. If, on the other hand, the distance is measured and checked, and the tape errors are eliminated so that we are sure that the 300-ft. line is not in error by more than 0.01 ft., the angles should not have a maximum possible error of over $07''$. If such refinement as this had been introduced into the distance measurements, the angles should have been determined to the nearest quarter of a minute by repeating the angles, as explained in Art. 59.

63. REQUIRED PRECISION. The precision required for any survey will depend on the purpose of the survey. It is obvious that a city survey should be made with greater precision than a farm survey. In the survey of the boundaries of a lot of land the observed distances and angles do not form a figure that exactly closes geometrically. The linear amount by which the observed data fail to satisfy the theoretical requirement is called the *error of closure*. This error of closure is made up of errors in distance and errors in angular measurements; not careless mistakes or blunders, but errors inherent in the methods used. The error of closure divided by the perimeter of the lot is called the precision of the survey. It is usually expressed in the form of a fraction with unity in the numerator. If the error of closure is 0.20 ft. and the perimeter is 1000 ft., the precision is $\frac{0.20}{1000}$, or $\frac{1}{5000}$.

To be within a prescribed error of closure the precision in the measured distances and in the measured angles should be consistent. Probably the errors in angles are more nearly those

of the compensating type than those in distances, which have many elements that are likely to produce cumulative errors. In the last analysis, the precision in angle measurements which is required for a given precision in the survey is a matter of experience as well as of mathematics. The following table may be used as a fair guide in this respect:

PRECISION OF SURVEY	READ ANGLES TO	PRECISION OF SURVEY	READ ANGLES TO
$\frac{1}{500}$	05'	$\frac{1}{5000}$	40''
$\frac{1}{1000}$	03'	$\frac{1}{10000}$	20''
$\frac{1}{3000}$	01'	$\frac{1}{20000}$	10''

Precisions of $\frac{1}{500}$ and $\frac{1}{1000}$ would be satisfactory for rough surveys of farms and wood lots; and precisions of $\frac{1}{3000}$ and $\frac{1}{5000}$, for more valuable property in urban districts. In city surveys a precision of $\frac{1}{10000}$ to $\frac{1}{20000}$ is required, depending upon the value of the property being surveyed.

64. SETTING POINTS ON AND EXTENDING A STRAIGHT LINE. In Fig. 36 the line AB is such that A and B are intervisible. If the transit is set over point A and sighted on point B , any point between A and B can be set exactly on the vertical cross-hair. For precise work, it is good practice to repeat this process with the telescope inverted. If the instrument is not in perfect adjustment, slightly different positions will be found for each point; the mean of these positions should be used.

A common surveying operation requires setting points on the prolongation of a straight line. Suppose that in Fig. 36 the transit is at A and a sight is taken on point B , and the points C , D and E are to be set on the prolongation of line BA . These new points can be set by plunging the telescope (revolving it about its horizontal axis) and setting the points in line with the vertical cross-hair. If the instrument is out of adjustment, these points will not be exactly in line with BA produced, but will fall off line, as at C_1 , D_1 and E_1 respectively. The effect of this error in the adjustment of the transit must be eliminated in the following manner: after points C_1 , D_1 and E_1 are set, the instrument is turned about the vertical axis 180° and with the

telescope still inverted is sighted again exactly on point B ; the telescope is again plunged to the direct position and the new points C_2 , D_2 , and E_2 are set. If these do not coincide with C_1 ,

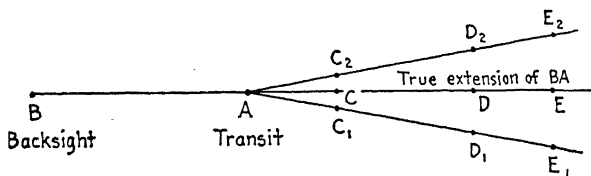


FIG. 36. EXTENDING A STRAIGHT LINE.

D_1 and E_1 , the true points C , D and E lie half-way between C_1 and C_2 , between D_1 and D_2 , and between E_1 and E_2 (Fig. 36).

This method of operating the instrument eliminates the errors which result when the line of sight is not perpendicular to the horizontal axis of the telescope. It also eliminates the error caused by one standard being higher than the other, i.e., when one end of the horizontal axis is higher than the other. In a well-adjusted instrument these errors are not large but it is always well to operate the transit in this manner in order to eliminate the effect of these two possible errors. This method, however, does not eliminate the error caused by the plate bubbles being out of adjustment; but the plate may be made horizontal by the process explained in Art. 55.

65. WORKING ON TO LINE BETWEEN TWO POINTS NOT INTERVISIBLE. In Fig. 37, A and B are points that are not intervisible, but both can be seen from a knoll near C . It is desired to set point C exactly on line AB . This is done by a method of trial called "*working on to line*," or "*jiggling in*"; the procedure is as follows.

Set up the transit at C_1 , estimated by eye to be on the line AB . With the telescope erect sight A , plunge the telescope and observe how near the line of sight hits B , say at B_1 . Estimate the distance BB_1 , or measure it; suppose it is 1.2 ft. Estimate the ratio of AC to AB ; suppose it is 3 to 4, as it appears to be in Fig. 37. Move the transit $\frac{3}{4} \times 1.2 = 0.9$ ft. to point C_2 , and repeat the operation. Do this as many times as is necessary

(always sighting on A with telescope **erect**) until the **inverted** telescope sights **exactly** on B . At this stage the transit is, say, at C_2 . If the line of sight and the standards are in adjustment,

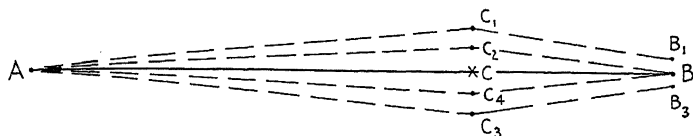


FIG. 37. WORKING ON TO LINE.

the point C_2 will be exactly on the line AB . But if the transit is not in perfect adjustment, C_2 will fall to one side of the correct position, as shown in Fig. 37. Leaving C_2 accurately marked, set up at C_3 , and repeat the process just described, but with the telescope **inverted** when sighting A . Finally, point C_4 will be found from which the plunged telescope (erect telescope) sights exactly on B . Point C lies half-way between C_2 and C_4 . If the transit were in perfect adjustment points C , C_2 and C_4 would coincide. In making these observations the plates must be kept carefully leveled.

66. Working On to Line That Is the Extension of a Line Defined by Two Points. In Fig. 38 points M and N are fixed and it is required to determine point O on MN extended. Neither points M nor N can be occupied with the transit, but both of them can be seen from O . Set the transit at O' , estimated to be on MN extended, and sight on M and measure offset NN' . Move the transit to the left a distance of 5 times NN' if the ratio of MN to MO is approximately 5, as in Fig. 38. Repeat this operation until the line of sight on M passes exactly through N . Then mark point O exactly under the transit.

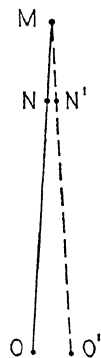


FIG. 38.
WORKING
ON TO
LINE.

This procedure is often required in tunnel work where points M and N are in the roof of the tunnel. (See Arts. 293-4.)

67. Intersecting Two Straight Lines. In finding the point of intersection of the two lines AB and CD (Fig. 39), first set the transit at A and sight B . Drive two stakes in line, at E and at

F , and mark the exact line on each. These stakes should be on opposite sides of line CD . Stretch a string between E and F . Set up the transit at D and sight C . This line will intersect the string at O , the point of intersection of the two lines. Drive a

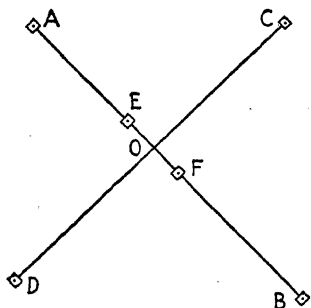


FIG. 39. INTERSECTING TWO STRAIGHT LINES.

stake at O , and mark the point of intersection of the string and the line DC by a tack driven in the top. Stakes E and F should be sufficiently distant from O so that they will not be disturbed when the stake at O is driven.

68. METHODS OF SHOWING SIGHTS. If the point sighted is within a few hundred feet of the transit, a pencil (held in a vertical position) may be used to show a point for the transitman to sight on.

Sighting-rods or range poles are used on long distances.* Where only the top of a lining-pole is visible an error will be introduced if it is not held plumb. A plumb-line is much more suitable for such work, but it may be difficult or impossible to see it on long sights. Under conditions where the plumb-line cannot be readily seen, some surveyors use for a sight an ordinary white card held with one edge against the string, or held so that the center of the card is directly behind the string. If the edge of the card is held against the string, the transitman must be **extremely** careful to sight on the proper edge.†

Whenever the transit is sighted along a line which is to be

* It is desirable that the object used for the foresight should be of such color that the cross-hair can be clearly seen, and of such width that the cross-hair can easily bisect it. A yellow or light colored pencil against a dark background, or a dark pencil against the sky, usually gives good results.

† It is the practice among some surveyors to use a 2-ft. rule for a sight. The rule is opened so that it forms an inverted V (Δ). The plumb-string is jammed into the angle of the Δ by pressing the two arms of the rule together. The rule is then held so that the plumb-string as it hangs from the rule appears to bisect the angle of the Δ .

The man showing the sight for the transitman should always try to stand so that the sun will shine on the object he is holding; on long sights it is difficult (sometimes impossible) to see an object in a shadow.

used frequently or along which the transit is to remain sighted for a long time, the transitman should select, if possible, some well-defined point which he observes to be in the line of sight; this is called a "*foresight*." If no definite point can be found, a nail may be driven through a white card or paper, or a "crow-foot" marked with crayon. By means of this "*foresight*" the transitman can detect if his instrument moves off the line, and can reset the telescope exactly at any time without requiring the aid of another man to show him the line.

69. SIGNALS. The distances are frequently so great in surveying that it is necessary to use hand signals. All signals should be so distinct that there can be no doubt as to their meaning. Care should be taken to stand in such a position that the background will not prevent the signals from being seen distinctly. The palms of the hands should be shown in making the signals; for distant signals a white handkerchief is often used. The following signals are in common use.

"*Right*" or "*Left*" when setting points. — The arm is extended in the direction of the motion desired, the right arm being used for a motion to the right and the left arm for a motion to the left. A slow motion is used to indicate that the point must be moved a long distance; and a quick motion, a short distance.

"*Plumb the Pole*." — The hand is extended vertically above the head and moved slowly in the direction in which it is desired to have the pole plumbed.

"*All Right*." — Both arms are extended horizontally and moved up and then down; or, both hands are shown at same time (without raising and lowering).

"*Give a Foresight*." — The transitman, desiring a foresight, motions to the rodman by holding one arm vertically above his head.

"*Take a Foresight*." — The rodman, desiring the transitman to sight on a point, motions the transitman by holding one arm vertically above his head and then immediately holding his lining-pole, or pencil, vertically on the point.

"*Give Line*." — When the rodman desires to be placed "on line," he holds his lining-pole horizontally with both hands over his head and then brings it down to the ground in a vertical

position. If the point is to be set carefully, as a transit point, the rodman waves the top end of pole in a horizontal circle above his head before bringing it to the vertical position; or he may wave the arm in a similar manner if a pole is not being used.

“*Pick up the Transit.*” — When the chief of the party desires to have the instrument set at another point, he signals to the transitman by extending both arms downward and outward and then raising them quickly.

Where much distant signaling is to be done, flags are attached to the lining-poles. Special signals may easily be devised for different kinds of distant work and for various conditions.

70. MEASURING VERTICAL ANGLES WITH A TRANSIT. In surveying, a vertical angle is the angle in a vertical plane measured from the horizontal plane through the axis of the telescope to the point in question. When the point sighted is above the horizontal plane, the vertical angle is called an angle of elevation, or positive angle; when the point sighted is below the horizontal plane, the angle is called an angle of depression, or negative angle. In the notes the former is preceded by a + sign, and the latter by a - sign.

All engineer's transits are equipped with a vertical arc, or a full vertical circle, and a vernier for measuring vertical angles. In the most common type the arc, or circle, is rigidly attached to the horizontal axis of the telescope, and the vernier is fastened to the standard.

To measure a vertical angle to a point, the transit is carefully set up and leveled. The telescope is directed to the object and when it is observed in the telescope, the vertical motion is clamped. By means of the vertical motion tangent screw the middle horizontal cross-hair is set exactly on the point. The vertical arc reading is the correct vertical angle, provided that the instrument is in adjustment and has been carefully leveled. The conditions that must be fulfilled are the following: the axis of the telescope spirit level and the line of sight must be parallel; when the plate bubbles are centered, the plate must be truly horizontal; and the vertical arc must read 0° when the telescope bubble is in the center of the tube.

If the first condition is not fulfilled it is impossible to obtain

a correct vertical angle by means of the vertical arc only. With a full vertical circle this error may be eliminated by observing the vertical angle first with the telescope direct, then with it reversed, and by taking the average of the two readings.

If the last two conditions are not fulfilled because the transit is slightly out of adjustment, it is possible to determine a correction by means of which the observed vertical angle may be rectified. This correction is called the *index correction*.

To determine the index correction, level the telescope, by means of the vertical motion and the telescope bubble, in the same vertical plane as the point sighted. The amount by which the vertical arc fails to read 0° is the index correction. In applying the index correction to the observed vertical angle, great care should be exercised in applying it in the right direction.

For angles of elevation or depression the index correction is **added arithmetically** when the 0° of the vernier is **outside** the arc used in reading the vertical angle, because the observed angle was too small. It is **subtracted arithmetically** when the 0° of the vernier falls **on** the part of the arc used in reading the vertical angle, because the observed angle was too large.

An interesting old rule used by navigators in applying the index correction is, "When it's on it's off, and when it's off it's on." This means that when the index of the vernier is on the same side of the 0° of the arc as that on which the vertical angle was read, the index correction is off (subtracted from) the observed reading, and vice versa.

71. Vernier Control Bubble. Some instruments are equipped with a vernier control bubble, the purpose of which is to eliminate the index error. A spirit level and vertical arc vernier are mounted on a separate arc entirely independent of the movement of the telescope on its horizontal axis (Fig. 40). The vernier bubble tube may be moved in the plane of the vertical arc by means of a tangent screw. When the vernier bubble is centered, the zero of the vernier scale represents the true horizontal (provided that the vernier bubble is properly adjusted), and the index correction will be zero when a vertical angle is read.

72. USING THE TRANSIT AS A LEVEL. Level the plates, and after revolving through 180° about the vertical axis, bring each

plate bubble **half-way back** to the mid-position. Now bring the telescope bubble to the center of its scale, using the clamp and slow-motion screw of the vertical arc. Turn the telescope

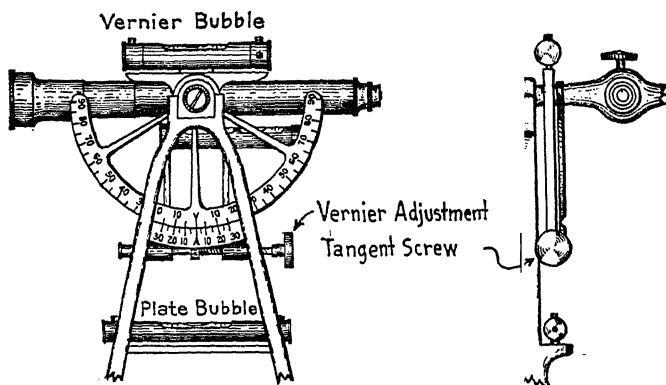


FIG. 40. VERTICAL ARC VERNIER BUBBLE.

toward the rod and read it where the horizontal cross-hair cuts it **at the instant that the telescope bubble is centered**. Care must be exercised when leveling to use the middle cross-hair, not one of the stadia hairs.

Approximate leveling may be done by measuring the vertical angle and the inclined distance from the horizontal axis of the telescope. The vertical height which the point sighted is above (or below) the axis of the transit equals $\text{Slope Distance} \times \sin \text{Vertical Angle}$. The slope distance may be measured by tape or it may be obtained by stadia, as explained in Art. 203. The vertical angle method is not capable of the same degree of precision as is the method of direct leveling, because vertical arcs usually read only to one minute. At 300 ft. an arc of $01'$ subtends about 0.10 ft. In direct leveling, a rod-reading to 0.01 ft. is easily obtained at this distance.

73. COMMON SOURCES OF ERROR IN TRANSIT WORK. Even though the transit is in adjustment, or errors of adjustment have been eliminated by proper manipulation of the transit, the following sources of error may be present:

1. Eccentricity of circle (i.e., imperfectly centered), and errors of graduation.
2. Changes due to temperature and wind.
3. Uneven settling of tripod.
4. Poor focusing (parallax).
5. Inaccurate setting over point.
6. Irregular refraction of atmosphere.

74. COMMON MISTAKES IN TRANSIT WORK.

1. Reading in the wrong direction from the vernier index on a double vernier.
2. Reading the vernier opposite the one which was set.
3. Reading the circle incorrectly, e.g., reading 39° for 41° . If the angle is nearly 90° , reading the wrong side of the 90° point, e.g., 89° for 91° .
4. Using the wrong tangent screw.
5. Neglecting to add $30'$ to a reading; for example, $26^\circ 18'$ for $26^\circ 48'$.
6. Where circle is graduated in both directions from 0° to 360° , reading wrong circle scale near 180° .

75. CARE OF THE TRANSIT. There are certain precautions which every instrumentman should take in handling the transit; the following are the most important. The clamps at the top of the tripod legs should be kept sufficiently tightened so that there will be no apparent looseness. When being transported in a vehicle, the transit should be carried in its box. When being taken out of the box, it should be lifted by the base plate, not by the telescope axis. In the field, it should be carried from place to place on the shoulder, with the tripod legs pointing forward; but in entering buildings the head of the transit should be held forward to avoid striking it against door frames. It should not be jolted at any time and especial care should be taken to avoid jolting it when it is in a horizontal position. It is well to center the leveling screws and the transit head just before picking the transit up to go to the next station. In rainy weather the telescope should not be left pointed upward so that water can collect on the object glass; the moisture may leak down into the barrel of the telescope and cause the cross-hairs to break. The metal

parts of the instrument should be wiped dry before it is put away. Care should be taken to avoid scratching the object glass when cleaning it. A camel's hair brush should be used first, then an old soft and clean piece of linen, moistened with alcohol to dissolve grease. It should not be rubbed, but should be wiped gently and brushed again with the camel's hair brush. A special lens paper is available which may be used. The vertical arc should never be touched with the fingers, as it will tarnish.

Force should never be used in tightening any clamps or screws; a definitely firm, but not severe, tightening is all that ever is necessary. When placing the transit in the box, especial care should be taken to make sure that the telescope does not touch the sides of the box, and to be certain that all clamps are tightened so that the telescope cannot swing against the side of the box while being transported.

ADJUSTMENTS OF TRANSIT

76. TESTS AND ADJUSTMENTS OF THE TRANSIT. Because of careless handling in the field, changes in temperature, and rough handling in transporting, the transit is liable to get out of adjustment.

In general, there are two kinds of adjustments of the transit: shop adjustments, which can be made only in the factory; and field adjustments, which can readily be made by the instrument-man. Only the latter will be explained in this chapter.

When an instrument is in perfect adjustment:

- (1) Axes of the plate bubbles are perpendicular to the vertical axis;
- (2) Vertical hair is perpendicular to the horizontal axis;
- (3) Line of sight is perpendicular to the horizontal axis;
- (4) Standards are exactly at the same height;
- (5) Line of sight and axis of the telescope level are parallel;
- (6) When the telescope is leveled by the telescope level the zeros of the vertical arc and vernier should coincide.

Certain tests of adjustment may be applied to the instrument in the field, and if it is found to be out of adjustment, it may be put back into adjustment by means of adjusting screws, as here-

after described. The transitman can make these adjustments with little or no assistance.

The transit should be set up on firm ground before these tests are made.

In order that the effect of one adjustment may not be reflected in the others, adjustments must be made in the following order:

- (1) Plate bubbles, (2) Vertical cross-hair, (3) Line of sight,
- (4) Standards, (5) Telescope bubble.

77. ADJUSTMENT OF THE PLATE BUBBLES. The purpose of this adjustment is to make the axis of each plate bubble perpen-

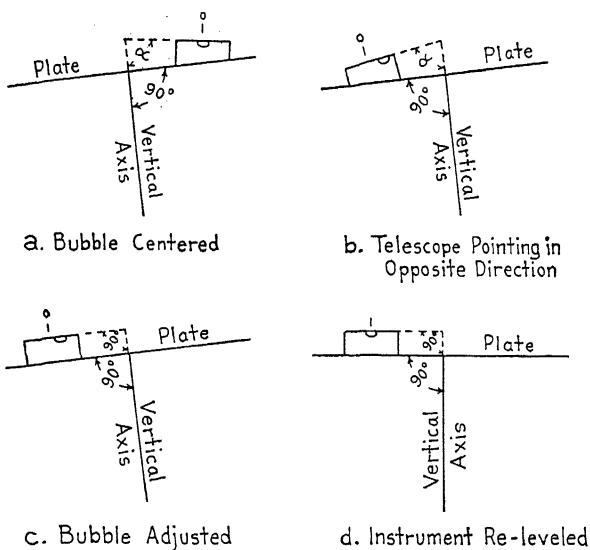


FIG. 41. ADJUSTMENT OF PLATE BUBBLES.

dicular to the vertical axis of the transit. When this has been accomplished, the axes of the plate bubbles will also be exactly parallel to the horizontal plate because the plate is constructed precisely at right angles to the vertical axis. With this adjustment made, the horizontal plate will be truly horizontal when both of the bubbles are centered.

Set up the transit and bring the plate bubbles to their respective centers; Fig. 41a shows one bubble after this first step. Next,

turn the instrument 180° about its vertical axis and observe whether the bubble remains in the center. It will be seen in Fig. 41b that it has moved from the center toward the high end of the tube, which shows clearly that this bubble axis was not perpendicular to the vertical axis of the transit. To adjust, turn the capstan-headed screws on the bubble-tube casing by means of an adjusting pin, thus raising or lowering one end until the bubble moves half-way back to its center, as in Fig. 41c.* Perform these three steps with each bubble separately. Through this third step the leveling screws have not been moved. Consequently the vertical axis is still not plumb and the plate is still not horizontal. If the adjustment has been carefully made the axes of the bubbles are now perpendicular to the vertical axis, or parallel to the horizontal plate. This being true, if the bubbles are now brought to their central positions by means of the leveling screws, this process will tip the vertical axis and the horizontal plate and will make the vertical axis truly vertical and the horizontal plate truly horizontal, as in Fig. 41d.

78. ADJUSTMENT OF THE LINE OF SIGHT. There are two tests; one to determine whether the vertical cross-hair is perpendicular to the horizontal axis of the instrument, and the other to determine whether the line of sight is perpendicular to the horizontal axis.

79. To Make Vertical Hair Truly Vertical. Bisect a well-defined point near the upper end of the vertical cross-hair as viewed in the telescope. Slowly rotate the telescope about its horizontal axis until the point appears near the lower end of the hair. If the point appears to remain on the vertical hair, the hair is in adjustment. If the point appears to move off the hair, rotate the cross-hair diaphragm in its plane by loosening slightly the four capstan-headed screws holding it in place, and then tapping one of the screws with a pencil. (Fig. 23.) Repeat the operation until the point remains on the vertical cross-hair; then tighten the adjusting screws.

* An adjusting pin is furnished by the manufacturer with all transits. It is a small steel pin and is so made that it fits through the holes in capstan-headed screws and nuts, thereby preventing burring the holes. Great care should be exercised in order not to break off the head of the screw.

The process of reversal should be repeated until no further adjustment is required. When finally adjusted, the screws should hold the ring firmly but without straining it.

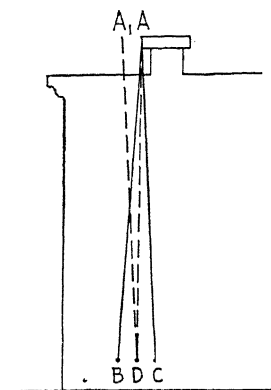


FIG. 43. ADJUSTMENT OF STANDARDS.

81. ADJUSTMENT OF THE STANDARDS. When the instrument is set up and leveled, the horizontal axis should be truly horizontal, and perpendicular to the vertical axis.

To determine whether the standards are in adjustment, set the transit up near a building and sight some definite point *A* on the building as high up as it is possible to observe with the telescope (Fig. 43). Lower the telescope to a *horizontal* position and mark point *B* on the building in the line of sight. Next plunge the telescope (turning it about its horizontal axis) and turn the transit about its vertical axis 180° .

Sight the upper point *A* again, lower the telescope, and set point *C* at same level as *B*. If *C* falls on *B*, the standards are in adjustment. If not, sight point *D* (midway between *B* and *C*), and then raise the telescope until point *A* is on the horizontal wire, but the vertical hair will then be to one side of *A*, as at *A*₁. The adjustment is made by raising or lowering the adjustable end of the horizontal axis, by means of the capstan-headed screw, until the vertical cross-hair coincides with *A*. Repeat the operation and re-adjust if necessary.

82. ADJUSTMENT OF THE TELESCOPE BUBBLE. The line of sight and the axis of telescope bubble must be parallel. This test and adjustment is like the Peg Adjustment, Art. 117., with one exception; in the case of the transit, after the rod-reading for a level line has been determined, the horizontal hair is set on the rod at that point. By means of the capstan-headed nuts, the telescope level is then adjusted by raising or lowering one end of the bubble tube until the bubble is in the center. After this adjustment has been made, the vertical arc and the vernier zero-line should be examined to see whether they coincide when

the telescope bubble is in the center of the tube. If they do not, the vernier may be moved by loosening the capstan-headed screws which hold the vernier to the standard. The zero-lines are brought together and the screws tightened.

83. ELIMINATING FROM MEASURED ANGLES THE EFFECT OF LACK OF ADJUSTMENT OF TRANSIT. A transit that has been recently adjusted may still be out of adjustment due to some unknown cause. Furthermore, there are inherent defects in instruments, which cannot be corrected by any adjustment. Consequently in exact work it is necessary to operate the instrument in such a way as to eliminate the errors due to non-adjustment, as well as those which are inherent in the instrument.

The errors referred to above are due to the following causes: (1) the verniers on opposite sides of the circle are not exactly 180° apart, (2) the center of the inner circle which carries the verniers and the center of the outer circle which carries the graduations were not manufactured so as to coincide, i.e., the centers are eccentric, (3) the axis of the plate level is not perpendicular to the spindle (vertical axis), (4) the cross-hairs do not define a line of sight that is exactly perpendicular to the horizontal axis of the telescope, and (5) the horizontal axis of the telescope is higher at one end than at the other.

These errors can be eliminated by taking certain precautions. If the two verniers are not set exactly 180° apart, this error can be eliminated by reading the same vernier in all readings concerned with the same angle. If the centers of the circles are eccentric, the resulting error will vary in different parts of the circle; in modern transits reading to single minutes this error of eccentricity is almost always too small to detect. It can be eliminated by reading the verniers on opposite sides of the circle and by using the mean of these readings. The effect of poor adjustments of the plate levels may be eliminated by leveling, reversing the bubble by turning the instrument 180° on its vertical axis; and if the bubble moves from the central position it can be brought half-way back to the center by means of the leveling screws, as explained in Art. 55. This will make the plates truly horizontal. Errors of the line of sight and of the horizontal axis may be eliminated by performing the work with

the instrument in its normal (direct) position; repeating the process with the instrument in the reversed position (telescope upside down) and taking the mean of the results, as explained in Arts. 57, 59 and 64.

FIELD EXERCISES

1. (a) Set up a transit over a point, select several objects such as church spires and cupolas, and measure the horizontal angles between consecutive points, resetting the circle reading at 0° before measuring each angle; "close the horizon" and check the sum against 360° . The magnetic bearings of these directions may be read from the compass needle, giving a rough check on the measured angles.

(b) Repeat this exercise, using points only a few feet away from the instrument.

2. Measure the interior angles of a triangle by repeating each angle 2 or 3 times. (Art. 59.)

3. Repeat problem (1), but sight on points not over 30 ft. from O . It may be necessary to drive stakes in the ground for this purpose. If so, the points on the separate stakes may be indicated by a pencil mark or by a tack in the top of the stake. For showing sights, use a pencil. Place the pencil point on the mark or tack and hold it vertically. If the pencil is not high enough to be seen through the transit, hold a plumb-bob over the point and sight on the plumb-line. Keep in mind that when a pencil or lining-pole is held on a point it is advisable to sight the object as near the actual mark as practicable, i.e., as low as can be seen on the pole or pencil.

4. Measuring the interior and deflection angles of a closed traverse. Set five stakes to represent the corners of a five-sided polygon. Arrange the stakes so that one angle will be re-entrant (interior angle over 180°). Measure the interior angles and check their sum with $540^\circ 00'$. Measure the deflection angles. Check the angles by taking the difference of the sums of the right and left deflection angles and seeing if the difference is $360^\circ 00'$.

5. Repeat (4), but double the angles by using the telescope first in the direct and then in the reverse position when measuring the angles. For the adopted angle, take half the second reading. Reject any means which do not agree reasonably well with the first reading (say within $01'$), and measure the angle over again.

6. Lay off from line AB an angle of 60° at A . Set the transit at A and level it. Set the vernier to read 0° and sight on B . Unclamp the upper motion and set the index (zero-point) of the vernier on $60^\circ 00'$. Set a stake C in line with the line of sight and mark a pencil point, or set tack, on the top of the stake exactly in line with vertical cross-hair. Repeat the angle BAC four times and take the mean. The difference of the mean and the angle 60° is considered to be the error in the angle laid out. Next, measure the distance AC and compute the distance the point C must be moved perpendicular to AC so that the angle will be precisely 60° . Measure this distance on the stake and set the final exact point on top of the stake.

7. Measuring a Vertical Angle.

Set the transit over a point and level it. Sight the top of a church steeple with the middle horizontal hair and read the vertical angle. Lower telescope until the bubble of the telescope level is in the center. Read index correction and apply it to the observed vertical angle.

8. Prolonging a Line.

Choose a point A on top of a knoll and a point B at a lower elevation. It is desired to set point C in prolongation of line BA . B and C are not intervisible. Set transit at A , backsight on B , plunge the telescope and set a point C on a stake in line with the vertical hair. Turn telescope (still inverted) about the vertical axis 180° and again sight point B . Plunge the telescope and note whether the vertical hair bisects C . If it does not, place another mark C' on the stake beside C where the vertical hair now cuts it. Mark a point D midway between C' and C . Point D is on the line BA prolonged. When high precision is desired, repeat one or more times and adopt the mean position found.

9. To Set a Point on the ground Vertically under a High Point.

To do this the high point must be an overhanging point, like the corner of a building cornice. If O is the high point, select two points A and B on the ground so that the horizontal angle AOB will be about 90° . Set over A and level the instrument. Sight O , lower telescope and set two points m and n in line with vertical hair so that they will straddle the line from B to O . Repeat with telescope inverted, and if line of sight does not intersect m and n , points m' and n' midway between the line of sight and m and n are the points desired. Mark the positions of m' and n' on the stakes. Set the transit at B and repeat the above operation obtaining the points p' and q' . The intersection of strings stretched from m' to n' and from p' to q' will define a point vertically under O .

10. Set two points about 150 ft. apart on a steep slope and measure the distance between them to .01 ft. with horizontal tape "breaking" the tape where necessary, i.e., in several lengths less than a tape-length. Measure the line first down the slope and then back up the slope, and compare the results. Which result would you consider more reliable and why? Then set up a transit over one end of the line and measure the slope distance and vertical angle directly from the horizontal axis of the transit to the point marking the other end of the line. A 200-ft. tape will be required for this exercise. Reduce the slope distance to horizontal and compare with results obtained by horizontal tape measurements. If a 200-ft. tape is not available, set the transit on line about midway between the two ends of the line and measure slope distances and vertical angles to each end of the line with a 100-ft. tape. Reduce these measurements to horizontal, add them and compare with results obtained by other methods.

11. To Lay Out a Rectangular Building Parallel to a Street Line.

Assume that the required rectangle is to be 120 ft. \times 45 ft. and that the long side is to be parallel to, and 20 ft. back from, a street line (Fig. 44).

Set the transit at point E on the street line and sight at F , the circle reading 0° . Then turn off 90° and measure $EG = 20$ ft. Set the transit at G ; measure 20 ft. from F with the tape held horizontally, and with 0-ft. mark at F swing an arc FH of 20-ft. radius. That position of the tape which throws point H farthest from line EF is the true perpendicular. After sighting on H , plunge the telescope and set point A on line at whatever distance is required to place A in the desired position, say, 8.0 ft. Plunge again and set B on line and 120 ft. from A . To establish the back corner D , set up at A , sight B , and turn 90° , setting D at a distance of 45 ft. from A . Test the angle by doubling it, and correct if necessary (Art. 59). At B , set C by the same process. Verify all tape measurements by measuring twice, in opposite directions if possible. To check the rectangle, measure DC , and the diagonals AC and BD . If all these measurements check within, say, 0.01 ft., the rectangle is accurate. The lengths of the diagonals give a particularly strong check; they can only be equal when $ABCD$ is a rectangle.

Whenever a point is set by backsighting and foresighting, as at *A*, the point should be set twice; once with the telescope direct, and once with it reversed, unless the instrument has been very recently tested and is known to be in perfect adjustment.

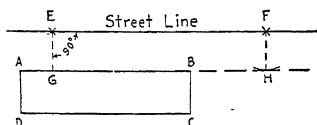


FIG. 44

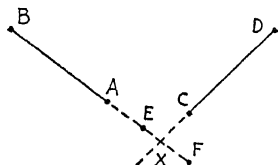


FIG. 45.

12. To Intersect Two Transit Lines.

If two transits are available the intersection is set in a direct manner, using one transit to sight along one line and the other transit to sight along the other. Usually but one transit is available; in place of the other, a "stretching line" is used. With the transit at *B* (Fig. 45), sight *A* and set *E* and *F* far enough apart to include the intersection point, but not so far as to cause difficulty in stretching the string. The string should consist of 10 or 15 ft. of stout plumb-bob string (fish line). With the transit at *D*, sight *C* and set a point where the vertical cross-hair cuts the string at *X*.

13. To Lay Out an Angle within 10'', with a half-minute transit.

Lay off 90° , or whatever the required angle may be, in the usual manner and mark the point. Repeat the angle several (3 to 6) times and divide the sum angle by the number of repetitions. It is advisable to read both verniers at the beginning and at the end of the series and to use the mean. Suppose the final reading (6th repetition) is $179^\circ 59' (+ 360^\circ)$. The accurate value of the angle is $89^\circ 59' 50''$. The point set must then be moved so that the angle is increased $10''$. An angle of one minute ($1'$) is equivalent to a distance of nearly 0.03 ft. as seen from a distance of 100 ft. An angle of $10''$ corresponds to one-sixth of this, or 0.005 ft. If the point being established is 200 ft. away from the transit, the amount that the preliminary point must be set over is 0.01 ft.

PROBLEMS

1. A transit is sighting toward *B* from a point *A*. In setting up the transit at *A*, it was carelessly set 0.01 ft. directly to one side of *A*, at a point *A'*. What would be the resulting error, i.e., the difference in direction (in seconds) between *AB* and *A'B* (1) when *AB* = 50 ft., (2) when *AB* = 500 ft., and (3) when *AB* = 1000 ft.?

2. An angle of 90° is laid off to the right with a "one-minute" transit, and the angle then determined by six repetitions, the final reading being $180^\circ 04' + 360^\circ$. The point sighted is 135 ft. from the transit.

(a) Compute the offset (to 0.001 ft.) to be laid off in order to correct the first angle. Should offset be laid off to the left, or to the right, from the point first set?

(b) If in the process above the angle was doubled and found to be $180^\circ 0'$ instead of measured six times, what would be the difference (to 0.001 ft.) between the offsets computed from the six repetitions and doubled angle respectively?

3. Design a vernier to read to $5'$ for a circle divided into degrees.

4. Design a vernier to read to $30''$ for a circle divided into $20'$ spaces.

5. Design a vernier to read to $10''$ for a circle divided into $10'$ spaces.

6. A transit is set up at a point B and a backsight taken on a point A , the telescope being in the direct position. The telescope is then plunged and a point set at C , 250 ft. from B . The instrument is now sighted on A with the telescope inverted and the line AB again prolonged by plunging the telescope. This time the point falls at C' , which is 0.18 ft. to the left of C . What would be the error in seconds of angle if the line were prolonged by only one plunging of the transit?

7. A line AK is to be laid out about 3000 ft. long, in 300-ft. intervals. At B , the end of the first 300-ft. interval, the transit is set up, sighted on A with telescope direct, and the telescope then plunged to give line for setting point C , 300 ft. in advance of B . In exactly the same way the entire line is laid out, the instrument being set up at every 300-ft. point. All the points are at approximately the same elevation. The transit has a collimation error of $1' 00''$ to the left, in the direct position of the telescope.

(a) Compute the angular divergence between the last 300-ft. portion (JK) of the line and the first 300-ft. portion (AB).

(b) Compute the right-angle offset between the true prolongation of AB and the end of the line at K (to 0.01 ft.).

(c) What procedure should have been followed to eliminate the divergence found above, i.e., to produce AB to K in a straight line?

8. Suppose angle ABC is measured when both of the lines AB and BC are 1500 ft. long, and the sights are taken on lining-poles held at A and C , respectively. Only the top foot of these poles can be seen; both may be 0.2 ft. out of plumb.

(a) What maximum error in angle would this make?

(b) Suppose that the same conditions exist except that both of the lines AB and BC are about 200 ft. long. What is the maximum error in angle due to poor plumbing of both poles?

(c) Suppose that the same conditions exist, except that line AB is 200 ft. and line BC is 1500 ft. long. What is the maximum error in angle due to poor plumbing of the lining-poles?

CHAPTER V

THE ENGINEER'S LEVEL AND ITS USE

84. LEVELING. Leveling is the process of determining differences in the elevation of points, i.e., the vertical distances between them. The instruments employed are the engineer's level and a graduated rod which is held vertically and is read to feet and tenths; to feet, tenths, and hundredths; or even closer if the work warrants greater precision. Leveling is used for determining the exact elevation (vertical distance) of definite points, called bench marks, above a datum (reference plane); these bench marks are used for establishing grades for highways, sewers, buildings, and for other construction work. Leveling is also employed in obtaining data for plotting the profile of an established line or for obtaining data to depict the shape of the earth's surface by means of lines of equal elevation (called contour lines).

Unlike the transit, the level does not have to be set up over a definite point; it is set up at any place from which the rod may readily be seen and read. Therefore no chain with hook is provided for attaching a plumb-bob.

85. THE ENGINEER'S LEVEL. There are two types of engineer's level; one is known as the *Wye Level* (Fig. 46) and the other, as the *Dumpy Level* (Fig. 47). The former is so named because the telescope is held in Y-shaped supports.

In general the level of either type has a telescope, mounted on two short vertical supports which are attached to the ends of a horizontal bar. This bar is parallel to the telescope. At the middle of the bar, and attached perpendicular to it on the under side, is a vertical conical axis. This axis fits into a conical socket in the leveling head, which acts as a bearing in which the axis can turn. The leveling head is similar to the leveling head of a transit. When in use, like the transit, the level is placed on a tripod (Fig. 46). Attached and parallel to the telescope is a sensitive spirit level for leveling the instrument. When the level

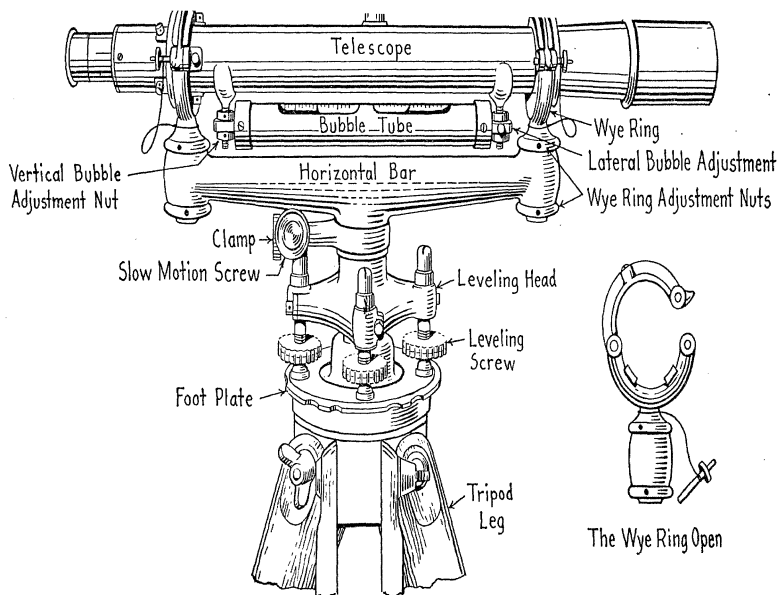


FIG. 46. THE WYE LEVEL.

is in adjustment the axis of the bubble tube and the line of sight are parallel and both are perpendicular to the vertical axis.

86. THE WYE LEVEL. In the wye level the bubble tube is attached to the telescope by capstan nuts in such a manner that one end of it may be adjusted vertically and thus bring the bubble axis parallel to the line of sight, and the other end may be adjusted laterally and thus bring its axis into the same vertical plane as the line of sight. Attached rigidly to the telescope tube are two annular rings, or collars, which rest in the Y-shaped supports and are held in place by stirrups, or clamps, which fit over the collars (Fig. 46). The Ys are held in place at the ends of the level bar (horizontal bar) by means of capstan nuts, and at least one is adjustable vertically. The instrument is so constructed that the telescope may be changed end-for-end in the Ys and thus be readily reversed for adjusting.

87. DUMPY LEVEL. In the dumpy level the vertical axis, the horizontal bar, the vertical supports of the telescope, and the telescope barrel are so rigidly fastened together that they form a single unit. This instrument will hold its adjustment for a long time and will withstand hard usage. No adjustments of these parts therefore are possible. The only adjustments which can be made in the dumpy level are the adjustment of the cross-hair ring (or diaphragm) and the vertical adjustment of one end of the level tube.

88. Testing the Sensitiveness of the Spirit Level. The principle of the level bubble has been described in Art. 51 and illustrated in Fig. 24. The bubbles used on levels are similar to those on the transit but are more sensitive. To test the sensitiveness of the spirit level, set up the level as explained in Art. 95, bring the bubble to the middle of the scale, and take a rod-reading at the horizontal cross-hair. Turn those opposite leveling screws which are nearer in line with the telescope, and thus make the bubble move to a new position, say, 5 divisions from the first one, and read the rod again. The change in the rod-reading divided by the horizontal distance from the level to the rod is the tangent of the vertical angle through which the telescope moved. Find the angle corresponding to the tangent and divide it by the number of divisions that the bubble has moved; this gives the angular

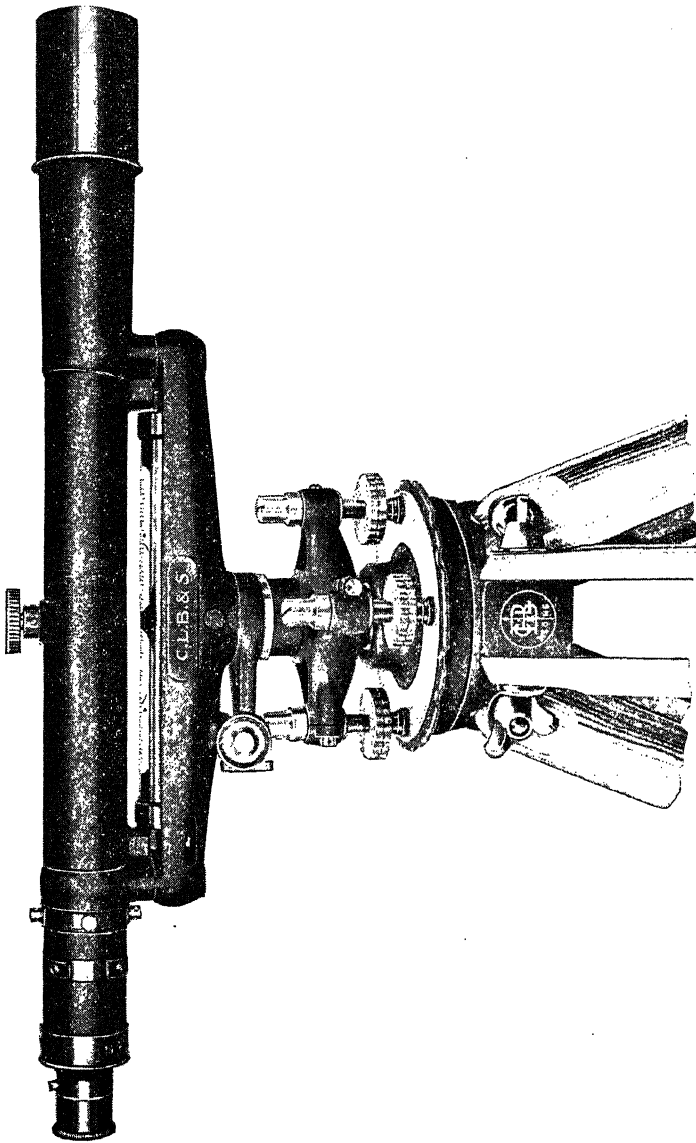


FIG. 47. THE DUMPY LEVEL.

movement corresponding to a single division of the bubble tube, i.e., the sensitiveness of the bubble.

89. THE LOCKE HAND LEVEL. The hand level, used for approximate leveling only, is a metal tube with a plain glass cover at each end and with a spirit level fastened to the top of the tube (Fig. 48). When looking through the tube, the bubble and the cross-wire (reflected by a prism) are seen in the left-half side of the field of view, and the landscape is seen in the right-half

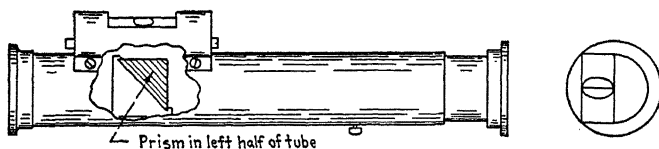


FIG. 48. LOCKE HAND LEVEL.

side. In order that the bubble and the landscape may be seen simultaneously, a half-lens is placed in a sliding tube to permit focusing on the bubble. The instrument is held by hand at the eye and the farther end is raised or lowered until the bubble is in the center of the tube, at which instant the horizontal wire will intercept the rod, or any other object, at a point which is at the elevation of the observer's eye. The cross-wire which defines the level line may be adjusted by means of two opposing screws.

90. The Clinometer (Abney Hand Level). The clinometer (Fig. 49) may be used for leveling or for measuring vertical angles. When the vernier index is set to 0° on the vertical scale, the instrument may be used for leveling in the same manner as in the Locke hand level (Art. 89). For measuring vertical angles the horizontal hair is set on the object. While still looking through the tube at the object, the control screw to which the vernier and bubble are rigidly attached is turned until the bubble appears to be bisected by the cross-hair. The angle is then read on the vertical arc.

The upper scale on the arc reads in per cent of grade and the lower scale in degrees of vertical angle.

91. LEVELING RODS. Leveling rods are classified as *Target* rods and *Self-reading* rods. Target rods are read by the rodman,

whereas self-reading rods are read directly by the levelman as he looks through the telescope. Some rods may be used either as a target or as a self-reading rod.

Most leveling rods are graduated in hundredths of feet, but some are graduated in half-tenths only. Self-reading rods may be read by estimation closer than the smallest division, i.e., readings on rods graduated to hundredths may be read to half-hundredths. The target used on some rods carries a vernier by means of which readings may be made to thousandths of feet (Art. 52).

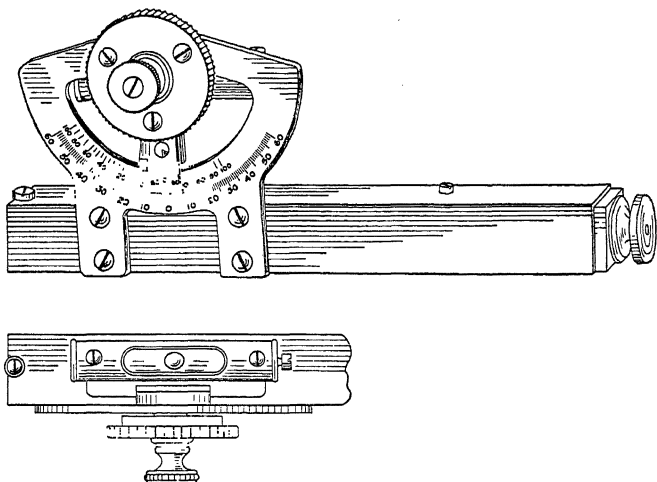


FIG. 49. ABNEY HAND LEVEL AND CLINOMETER.

The rod most commonly used is the Philadelphia rod. This rod is both a target and a self-reading rod. It is made of wood and may be obtained as a single strip of wood or as two or more strips which slide one over the other; the latter is called an extension rod. Although these rods may be obtained in various lengths, the one most commonly used is 12 to 13 ft. long; when extended, its parts are held in position by clamps. Referring to Fig. 50, which shows several types of Philadelphia rods, A is a one-piece rod $12\frac{1}{2}$ ft. long and is graduated throughout its length into hundredths of feet, zero being at the bottom of

the rod; B shows this rod in more detail. The face of the rod is painted white and on it the graduations, 0.01 ft. wide, are painted in black, and are spaced 0.01 ft. apart. Thus each alternate black and white space is 0.01 ft. high; the tops of the black graduations are the even hundredths. The large numbers, usually in red, indicate the foot-marks; and the smaller numbers, usually in black, the tenths of feet.

When used as a self-reading rod the levelman observes the point where the horizontal wire intersects the face of the rod. Thus the reading of the lower wire, shown in Fig. 50B, is 4.240 ft., and by estimation the upper wire reading is 4.545 ft.

Fig. 50C shows the front face of a two-piece Philadelphia rod with target attached at the 7-ft. mark. When this rod is used as a target rod and the readings are less than 7 ft., the target is moved up or down at the direction of the levelman until it is bisected by the horizontal cross-hair of the level. It is then clamped to the rod, as shown in the lower half of Fig. 50D, at the point where the reading is 5.386. The reading is obtained by means of a vernier (sometimes by a scale divided into 20ths of a foot) which is attached to the target. For readings higher than 7 ft. the center line of the target (zero of target vernier) is clamped at the 7-ft. mark, and the rear strip of the rod is raised until the target is bisected by the horizontal cross-hair. The two strips are then clamped together and the reading is obtained by reading the graduations and vernier on the rear face of the rod, as shown in the upper half of Fig. 50D, where the reading is 8.238. This collapsible rod may also be used as a self-reading rod; it is then extended to its full length for readings greater than 7 ft. When the rod is fully extended, the graduations on the front of the rod read continuously from the bottom to the top of the rod.

92. The New York rod is a two-piece rod which is essentially a target rod. It is graduated into hundredths of feet by fine lines, too fine to be read by the levelman except at close range. It is used and read in the same manner as is the Philadelphia rod when it is used as a target rod.

The Architect's rod is similar to the New York rod except that its graduations are in eighths of inches, and by means of verniers it may be read to $\frac{1}{64}$ of an inch.

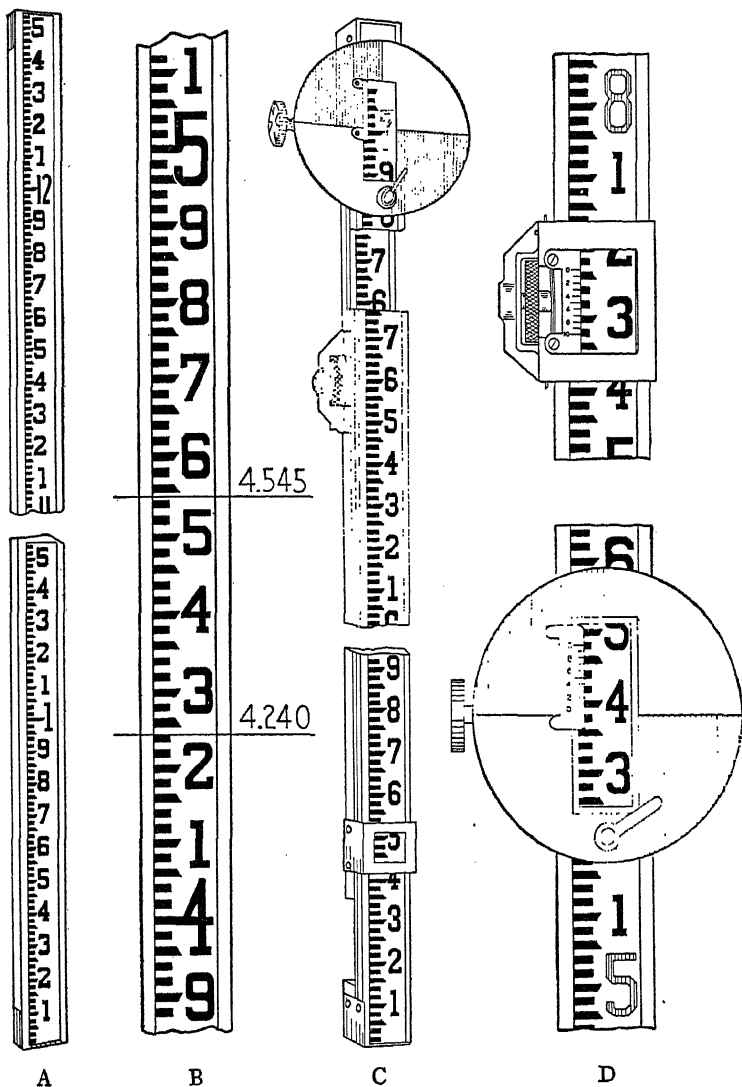


FIG. 50. LEVELING RODS.

93. Experience shows that as precise results may be obtained with a self-reading rod as with a target rod. The target will be found most useful, however, in dense woods, in dark places, in high winds, for exceptionally long sights, and for setting several points on the same straight grade line.

The exactness to which readings are made depends on the requirements of the project. For some work, the nearest tenth of a foot is sufficient, while in other kinds of work the nearest hundredth is required. To obtain the latter precision in the final result, a self-reading rod should be read to the nearest 0.005 ft. on intermediate readings; if a target rod is used it is customary to record the reading to the nearest 0.001 ft.

Under favorable conditions a self-reading rod graduated to hundredths can be read by estimation to the nearest half-hundredth at a distance of 300 ft. from the instrument.

The value of one division of the ordinary level bubble is about 20"; this corresponds to 0.03 ft. in 300 ft. The distance between the graduations on the level vial is about 0.1 in. It is reasonable to assume that the bubble is not centered closer than a fifth of a division ($\frac{1}{50}$ of an inch) at the instant the rod-reading is taken. At 300 ft. this discrepancy corresponds to a rod interval of 0.006 ft.; at 200 ft., 0.004 ft.; and at 100 ft., to 0.002 ft. The error in reading a rod by estimation to the nearest 0.005 is 0.0025 ft., which is the error to expect in reading a self-reading rod. When using a target rod, values read and recorded to the nearest 0.001 ft. often give a false impression of the precision actually obtained.

94. **The Tape Rod.** This rod consists of a strip of wood about 10 ft. long with a small roller at each end over which a flexible graduated strip of metal about 20 ft. long can move. The metal strip is graduated like a self-reading rod, except that the numbers read from the top toward the bottom.

In using this rod the instrument is set up at some convenient point, and the rod is held on a point of previously determined elevation, say, 114.32. The tape is moved until the 4.32-ft. mark is bisected by the horizontal wire and then clamped. If the elevation of an unknown point is desired, the rod is held on the point and the rod is read. Suppose the reading is 2.15; then the elevation of this point is $110.00 + 2.15 = 112.15$.

USE OF LEVEL

95. SETTING UP THE LEVEL. The method used in setting up a level is similar to that employed in setting up the transit, except that the level does not have to be centered over a definite point. By holding a leg in each hand as illustrated in Fig. 32, the third leg is placed on the ground and acts as a pivot, while the other two legs are moved so as to make the foot plate (circular plate at base of instrument) nearly horizontal. Without altering the horizontal position of the foot plate, the two legs are lowered to the ground. Pressure is then applied to all three legs to insure a stable set-up. On turf, this may be done by applying the pressure near the top of the tripod to all three legs; this can be done while the instrumentman stands in one position. The more nearly leveled the foot plate is kept, the less the leveling screws will have to be turned.

The instrument is then leveled by means of the four leveling screws. To center the bubble, turn the telescope until it is parallel to a pair of **opposite** leveling screws; turn this pair of screws, either both in or both out, until the bubble comes to the middle of the tube, as described in Art. 55. After the level is centered in the first position, the telescope is turned to a position at right angles to the first position (parallel to the other pair of screws) and the bubble is again centered by using this second pair of leveling screws. It is better not to try to center the bubble **exactly** over the first pair of screws until it has been centered over the second pair. The first centering will be usually within, say, $\frac{1}{4}$ division of the bubble. The second one (at right angles) will be, say, within $\frac{1}{2}$ division of the bubble. The third centering (the second time over the first pair of screws) should be exact. Two levelings over each pair of screws should center the bubble almost perfectly.

The leveling screws should feel neither tight nor loose, but should have a firm, even bearing. If the instrument is in complete adjustment, the bubble will remain central in any position of the telescope. If the bubble is not in perfect adjustment, it will move from the center of the bubble tube when the telescope is turned in any other direction than over the leveling screws.

This movement on a well-kept instrument may be as much as a division on the tube; if this is found to be so, by **slightly** turning **one** of the leveling screws the bubble can be brought exactly to the center of the tube just before a rod-reading is taken.

96. TAKING A ROD-READING. Before taking any sights the cross-hairs should be focused, by means of the eyepiece. To do this, point the telescope at some light surface such as a white house or the sky. Move the eyepiece **slowly** in or out until the most distinct position of the cross-hairs is found. Then turn the telescope to the rod and focus on it by means of the objective. To determine whether this focusing has been properly done, move the eye slowly up and down and observe whether the horizontal cross-hair apparently moves over the face of the rod. If motion of the cross-hairs is detected, parallax is present and this indicates that the cross-hairs are imperfectly focused and that further focusing is necessary. If the hairs do not appear to move on the rod, the cross-hairs are properly focused. It is important to keep in mind that when the eyepiece is once adjusted in such a way that the cross-hairs are properly focused, no further adjustment is necessary so long as the instrument is used by the same individual, for the eyepiece is then in the correct position for all sights irrespective of their length.

After focusing on the rod, observe whether the bubble is still in the center of the tube. If it is only a division or two off, center it by tightening **one** of the pairs of opposite leveling screws which lies nearer in line with the telescope; if it is more than two divisions off, relevel the instrument, as explained in Art. 95. When the bubble is exactly in the center, read the rod directly (if a self-reading rod is being used). Do not forget that **the rod must be held vertically**.

If the target is being used, direct the rodman to raise or lower the target on the rod until it is bisected by the horizontal hair. Before accepting any rod-reading, check the bubble to see that it has not moved from its central position. Wind pressure, stepping too close to the tripod, touching the tripod with the hand, and even changes in the position of the levelman's feet are likely to change the position of the bubble at any moment. Although

the level telescope may be held in position by means of a clamp with its slow-motion screw acting on the vertical axis, this clamp is rarely used except in a high wind when the telescope is likely to be blown from the desired pointing.

97. Signals. Although the rodman is seldom very far away from the levelman in this work, it is often convenient (in noisy city streets, for example) to use hand signals. The following signals are commonly used in leveling.

"Up" or "Down." The levelman motions to the rodman by raising his arm above his shoulder for an upward motion and dropping his arm below his waist for a downward motion. A slow motion indicates that the target should be moved a considerable distance, and a quick motion indicates a short distance.

"All Right." The levelman extends both hands horizontally and moves them up and down, or, if not far away, he merely shows the palms of both hands.

"Plumb the Rod." The hand is extended vertically above the head and moved slowly in the direction in which it is desired to have the rod plumb.

"Take a Turning Point." The arm is swung slowly in a circle above the head.

"Pick up the Level." When a new set-up of the level is desired, the chief of party signals the levelman by extending both arms downward and outward and then raising them quickly.

Some surveyors use an improvised system of signals for communicating the rod-readings, but mistakes are liable to be made unless great care is used.

98. Holding the Rod. It is extremely important that the rod be held in a vertical position when readings are being taken; otherwise the correct vertical distance between the bottom of the rod and the point where the line of sight intersects it will not be obtained. By comparing the rod with the vertical hair in the level, the levelman is able to tell whether the rod is vertical in the plane of the line of sight; if it is not, he can direct the rodman how to plumb it. He cannot, however, tell whether or not the rod is plumb in the direction of his line of sight. Some of the methods used by rodmen to insure verticality are:

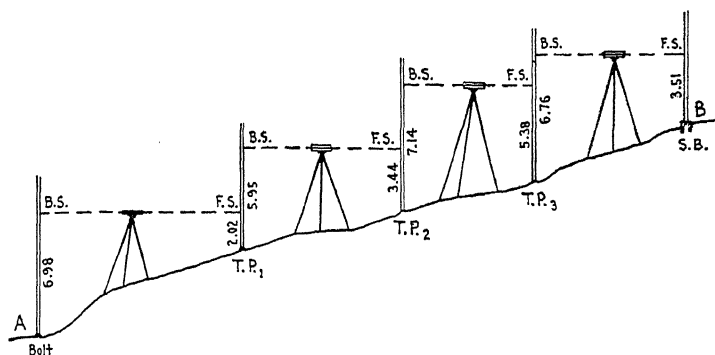
- (1) balancing the rod between the fingers; this method is excellent when the wind is not blowing.
- (2) comparing the rod with vertical objects such as the corners of buildings, lamp posts, and trees.
- (3) using a rod level. A rod level consists either of a watch level (circular bubble) or of two spirit levels placed at right angles to each other. Both types are mounted in such a way that they may be fastened to, or held on, the edge of the rod by the rodman. When the bubble, or bubbles, are centered the rod is vertical.
- (4) waving the rod. This method consists of swinging the rod slowly back and forth on its base as a pivot **keeping it always in the plane of the line of sight**, and observing the minimum reading. This reading is the true vertical reading required. This method is frequently used by some surveyors when the wind is strong and when the readings are near the top of the rod. Waving the rod evidently consumes time.

Some instrumentmen instruct the rodman to stand **behind** the rod, i.e., facing the instrument, and to plumb the rod by one of the methods previously mentioned. In this position the rodman can readily observe the levelman's signals. Other instrumentmen instruct the rodman to stand **at the side** of the rod. In this position the rodman can better decide when the rod is perpendicular to the level line of sight. In either case the levelman can tell whether the rod is in the plane of the vertical hair.

99. DIFFERENTIAL LEVELING. The elevation of any point is its vertical distance above a datum plane, such as Mean Sea-Level. If two points *A* and *B* (Fig. 51) are so situated that they cannot be sighted from a single position of the instrument, their difference of elevation may be determined from a series of differences as follows: The level is set up and a reading is taken with the rod held on the initial point *A*. This reading is called a *plus (+) sight*, or a *backsight (B.S.)*. **A backsight is a rod-reading taken on a point the elevation of which is known.** The rod is next held on some convenient point ahead (called a *turning point (T.P.)*) and a reading is taken and recorded. This is called a *minus (-) sight*, or a *foresight (F.S.)*. **A foresight is a rod-**

reading taken on a point the elevation of which is to be determined. In leveling, the terms backsight and foresight do not relate to the directions in which the sights are taken.

The level is then moved ahead to a new position from which a backsight can be taken on this **same** turning point. When this backsight rod-reading has been taken, the rod is carried ahead to



Point	B.S.	F.S.	Remarks
A	6.98		Bolt in S.W. Abutment High St. Bridge
T.P. ₁	5.95	2.02	
T.P. ₂	7.14	3.44	
T.P. ₃	6.76	5.38	
B		3.51	N.E. corner stone bound in sidewalk at #32 High St
	26.83	14.35	
	14.35		
Diff.	12.48		B above A

FIG. 51. DIFFERENTIAL LEVELING.

a second turning point and a foresight is taken on it, the level still remaining at its second set-up. These turning points must, of course, be such that their elevations will not change during the progress of the work. The leveling is continued in this manner until the final point is reached. The process is indicated in Fig. 51, and the form of record in the note-book is given beneath the figure.

Since the backsights are the vertical distances measured from the known points **up** to the plane of the cross-hair, and the foresights are the vertical distances measured from the plane of the cross-hair **down** to the unknown points, the sum of all the back-

sights minus the sum of all the foresights is the required difference in elevation. The result will be negative if point *B* is at a lower elevation than point *A*.

In practice the notes are arranged somewhat differently from those in Fig. 51, as shown in Fig. 54, but the principle of differential leveling described above is basic in all leveling work.

100. CURVATURE AND REFRACTION. A level surface is an imaginary curved surface which in plane surveying may be assumed to be parallel to sea-level. It is obvious that there are an infinite number of such level surfaces between sea-level and any point above or below it. If an instrument is set up and leveled,

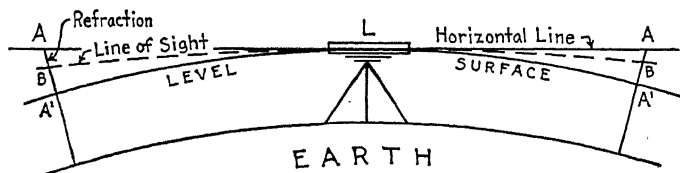


FIG. 52. EFFECT OF EARTH'S CURVATURE AND OF REFRACTION.

one of these (imaginary) curved level surfaces must pass through the telescope, tangent to the line of sight. The distance between such a level surface and the horizontal line tangent to it at the instrument is the "correction for curvature," such as *AA'* in Fig. 52. The amount of correction will vary depending on the distance the point is from the point of tangency. Assuming the earth to be spherical (radius = 3960 miles), the correction is easily calculated to be about 8 in. = 0.66+ ft. in a distance of one mile. (See Art. 281.)

The position of a point as seen by the observer is also affected by the refraction of the air, the point observed at the horizontal cross-hair being really a little below it, as at *B* in Fig. 52. The result is that curvature correction (0.66 ft. for 1 mile) must be diminished by about one-seventh part in order to obtain the net "correction for curvature and refraction," *A'B* in Fig. 52. This combined correction is 0.57 ft. for one mile; 0.08 ft. for 2000 ft.; 0.02 ft. for 1000 ft.; 0.005 ft. for 500 ft.; and about 0.001 for 200 ft. Whenever we assume that the point sighted by the horizontal cross-hair is at the same elevation as the instrument, the rod-

reading is in error by one of the amounts indicated. The amount varies as the square of the distance between the instrument and the point in question. It is only on very long individual sights such as across a river that the correction must be applied. It is usually eliminated in leveling practice, as described below.

The combined effect of curvature and refraction makes the cross-hairs appear to lie on the rod at a point slightly higher than a level surface reading would give. This error, however, is exactly the same amount for a 200-ft. backsight as for a 200-ft. foresight; the rod-readings on the backsight and on the foresight will be about 0.001 ft. greater in each case but it will not affect the **difference** in elevation. Hence, in any series of levels, the effect of curvature and refraction can be completely eliminated by **making the distance from the level to the point where the backsight reading is taken equal to the distance from the level to the point where the foresight reading is taken**. In fact, it is approved practice to endeavor to make the distances from any set-up of the level to the backsight and to the foresight equal.

101. BENCH-MARK LEVELING. A permanent point the elevation of which above some definite or assumed datum is known is called a *bench mark*. The top of an outcropping ledge, a stone or concrete monument, the foundation of a bridge or a building, or any other point not likely to undergo a change in elevation, is suitable for a bench mark. Its elevation should be determined with accuracy, and a careful description of its location should be recorded for future reference.

The level surface to which all elevations are referred is called the *datum*. Mean sea-level is the most common datum surface for leveling. Under some circumstances the elevations of conveniently established bench marks can be obtained from the U. S. Coast and Geodetic Survey, from the U. S. Geological Survey, or from state, city, or private engineers. As a temporary expedient it is always possible to assume an elevation for a bench mark and to proceed as if it were a known elevation; this is common practice when established bench marks are not nearby. The datum used, or assumed, should be carefully noted and recorded in order to prevent errors.

The purpose of bench-mark leveling is to establish the elevation

of new bench marks from those which are already in existence. It is rarely possible to establish a new bench mark from an old one by one set-up of the level. Consequently, the elevation has to be transferred through an intermediate point known as a *turning point*. A turning point should be a firm point which is unlikely to change its elevation during the leveling process, and it should be so chosen that the bottom of the rod will not change in elevation when the rod is faced in different directions; it should also present a definite point of support for the bottom of the rod. This last provision is especially important, since the same point should be recovered without any question if the rod is taken off the point and then replaced.

After the level has been set up, observations are taken in the following order: A backsight (B.S.) is first taken on the bench mark to determine the *elevation of line of sight*, or *height of instrument* (H.I.), i.e., vertical distance of the line of sight above the datum. Since the backsight is added to the elevation of the bench mark to obtain the H.I., it is usually called a *+sight*. A turning point is next selected and a *foresight** is taken on the rod which is held on the turning point. This sight is called a *-sight*, since it is subtracted from the H.I. to obtain the elevation of the turning point.

For example (Fig. 53), B.M.₁ has an elevation of 89.020 ft. above mean sea-level. From the instrument set-up, the B.S. is 4.125 ft. Therefore, the H.I. is $89.020 + 4.125 = 93.145$ ft. The F.S. on T.P.₁ is 6.195; this establishes the elevation of T.P.₁ as $93.145 - 6.195 = 86.950$ ft.

It would obviously be confusing to keep a record of the observations as a series of sketches like Fig. 53. The readings are therefore kept in note form as shown in Fig. 54. All the rod-readings taken in connection with a given point (a B.M. or a T.P.) are kept on the same horizontal line, but the foresights,

* It is customary to call the foresights "minus sights" because they are subtracted, although the sight itself is not essentially negative. There are cases, however, like a reading downward from an overhead point in roof of a tunnel, in which the sight itself is a negative quantity, since it is measured in the opposite direction from the other foresights.

or -sights, are subtracted from the previous H.I. to obtain the elevation of the point in question. Thus from the second set-up

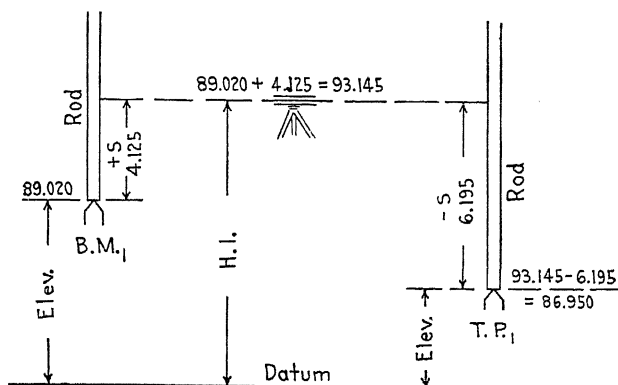


FIG. 53. BENCH-MARK LEVELING.

of the level, a B.S. is taken on T.P.₁ of 3.665, making the line of sight through the instrument at an elevation of 90.615. The F.S.

Left-hand Page					Right-hand Page	
B.M. Leveling for Flint St. Sewer					Party	Date
Sta.	B.S. or +S	H.I.	F.S. or -S	Elev.	Remarks	
B.M. ₁	4.125	93.145		89.020	Top S.E. cor. of Stone Bound cor. Flint and 71st Streets	
T.P. ₁	3.665	90.615	6.195	86.950	Right outer corner lowest concrete step entrance to #125 Flint St. N. nut top of hydrant opp. #147 Flint St.	
T.P. ₂	4.030	89.635	5.010	85.605		
B.M. ₂	3.905	86.165	7.375	82.260		
B.M. ₃	3.510	83.050	6.625	79.540	Top extreme outer cor. granite underpinning bank bldg., N.E. cor. Flint and 74th Streets	
T.P. ₃	6.015	80.345	8.720	74.330		
B.M. ₄			9.080	71.265		
	25.250		-43.005			
			+25.250			
		B.M. ₁	-17.755			
			=89.020			
		B.M. ₄	71.265 (check)			

FIG. 54. BENCH-MARK LEVELING NOTES.

(5.010) on T.P.₂ is placed on the same horizontal line as T.P.₂, but the F.S. is subtracted from the preceding H.I. (90.615) to

obtain its elevation (85.605). This process may be continued indefinitely until the bench mark B.M.₄ to be established is reached. When a bench mark is to be established, a foresight should be taken upon it exactly as on the T.Ps. If several bench marks are to be established as parts of the same line of levels, the intermediate bench marks should **always** be used as turning points.

The fieldwork of establishing bench marks should be checked by running the levels back to the point of origin, or to another bench mark the elevation of which has been previously and definitely established and which relates to the **same datum**. Thus any mistake in the observations may be detected.

It was shown in Art. 99 that the sum of the backsights minus the sum of the foresights equals the difference in elevation between two points. It is therefore possible to apply to the notes shown in Fig. 54 an independent arithmetical check by subtracting from the sum of the backsights the sum of the foresights and then testing to see whether their difference is numerically equal to the difference in elevation between the starting and the final bench marks. In Fig. 54, the difference between the backsights and foresights is -17.755 ft. (i.e., B.M.₄ is 17.755 ft. below B.M.₁). This value subtracted from the elevation of B.M.₁ (89.020) gives an elevation of 71.265 for the elevation of B.M.₄, which checks the value carried through the notes. In the event that a long series of bench-mark leveling is being undertaken, it is often well to apply this check several times during the running of the levels. This will prevent accumulation of arithmetical mistakes. Note that this check prevents arithmetical blunders, but does not check any mistakes in the fieldwork.

In all kinds of ordinary leveling the maximum length of sight, i.e., distance from the instrument to the rod, should not exceed about 300 ft. where elevations to the nearest 0.01 ft. are desired. It will be found that in good atmospheric conditions a newly painted self-reading rod can be read to the nearest half-hundredth at a distance of about 300 ft. It is therefore undesirable to make the B.Ss. or F.Ss. in bench-mark leveling greater than about 300 ft.

102. Double-Rodded Lines of Levels. A check on the fieldwork may be obtained as the work proceeds by selecting two

separate T.Ps. between each set-up of the level and recording backsights and foresights on each. In this way two independent determinations are obtained for each H.I. If these do not agree within the expected precision an error is indicated. This process is called running a "*double-rodDED*" line of levels. It is particularly useful in running a long line of levels that does not close on a previously established B.M. To insure against the possibility of reading the rod a foot off on both of a pair of T.Ps., the two T.Ps. should be so chosen as to be more than a foot apart in elevation.

103. PRECAUTIONS IN BENCH-MARK LEVELING. In leveling for bench marks it is important to equalize the distances from the level to the rod when taking the foresights and backsights, in order to eliminate so far as possible errors such as those caused by a lack of proper adjustment of the instrument. It is not safe to assume that the level is always in adjustment. If the line of sight dips downward from a level line and if the distances to the backsight and foresight are alike, the same error is made in the rod-readings; since one rod-reading is $+$ and the other $-$, the effect of the error is eliminated.

It is also important to examine the rod for error in length; a rod which is perfect when manufactured may change its length after a time. If a careful comparison with a steel tape shows an appreciable error, the rod should be discarded. The errors and mistakes inherent in leveling are given in Arts. 107-8. These errors **must be guarded against in all leveling work.**

104. Distributing the Error in a Level Circuit. When a line of levels starts from a bench mark of known elevation and terminates on the same bench mark, or terminates on another bench mark of known elevation, some difference is sure to be found between the previously recorded and the presently determined elevations. This discrepancy, called the *error of closure*, is the algebraic sum of all the errors in the circuit. Such errors should be distributed by some acceptable mathematical method, in order to make the elevations determined along the circuit consistent among themselves and to avoid having more than one value for the elevation of a given bench mark.

If the leveling has been performed with proper regard to equal-

izing backsight and foresight distances, the errors left to be corrected will be chiefly of the kind that tend to compensate. Such errors do not increase directly as the number of set-ups of the level, but rather as the square root of the number of set-ups (principle of Least Squares). In practice, these small errors of closure are distributed inversely proportional to the distance run.

Obviously if the line has been run in duplicate, forward and back, the best difference in elevation is the mean of the two results, but the error should be distributed among the intermediate bench marks, as will be later described.

The following method is simple and applicable to ordinary surveying work. Assume that levels have been run from B.M.₁ to B.M.₂ over three routes of different lengths, say, 1 mile, 1½ miles, and 2 miles. The best determination should be that of the one-mile line and it should therefore be given greater weight than the result of either of the other two. The least common multiple of 1, 1½ and 2 is 6. The one-mile line, therefore, should be given a weight of 6; the 1½-mile line, 4; and the 2-mile line, 3. The "weighted mean" difference of elevation is obtained by multiplying each observed difference in elevation by its weight, adding the products, and dividing the sum of the weights as follows:

If the observed differences of elevation between B.M.₁ and B.M.₂ along the three routes are 17.45 ft., 17.44 ft. and 17.50 ft. respectively, the weighted mean difference of elevation is computed as follows:

LINE	DISTANCE	=	WEIGHT <u>6</u> DISTANCE	OBS. DIFF. OF ELEV.	WEIGHT × OBS. DIFF. OF ELEV.
	1 mile		6	17.45 ft.	104.70
	1½ miles		4	17.44 ft.	69.76
	2 miles		<u>3</u>	17.50 ft.	<u>52.50</u>
			Sum 13		Sum 226.96

$$\text{Weighted mean difference of elevation} = \frac{226.96}{13} = 17.46 \text{ ft.}$$

By omitting the whole number of feet, since they are all the same, the arithmetical work may be considerably shortened. The calculation then becomes

$$\frac{6 \times 0.45 + 4 \times 0.44 + 3 \times 0.50}{6 + 4 + 3} = 0.458$$

$$\text{Weighted mean difference of elevation} = 17 + 0.46 = 17.46 \text{ ft.}$$

In establishing the elevations of intermediate bench marks, the error in the level line is proportioned directly as the distance.* For example, suppose that an intermediate bench mark is to be set half-way along line 3 at a point where the difference in elevation from B.M.₁ is 22.79 ft. The total error in line 3 between B.M.₁ and B.M.₂ is $17.50 - 17.46 = 0.04$ ft. Half-way along line 3 the error is assumed to be one-half the total error, or 0.02 ft. The corrected difference in elevation at the intermediate bench mark is, therefore, $22.79 - 0.02 = 22.77$ ft.

105. PROFILE LEVELING. In many engineering projects such as sewer, railroad, and highway construction, it is customary to stake out a traverse line on the surface of the ground with a transit and tape. The traverse line, for instance, may represent the proposed center line of the highway. In staking out such a traverse, the usual procedure is to start at the beginning of the line with station 0 and set stakes (stations) every 100 ft. along the line irrespective of whether the line is a continuous straight line, a broken line, or a curved line, as explained in Art. 34.

When the traverse line has been established as above, a profile of the line is next obtained.† A profile is the line of intersection of a vertical plane and the surface of the ground.

The data for plotting the profile of the traverse line are obtained by using the level and rod in a manner similar to that used in B.M. leveling (Art. 101). In addition, rod-readings are taken on the ground at each full station and at all points between stations where there is any change in the slope of the ground. The maximum length of sight should not exceed about 300 ft. The notes are kept in a form like that used for B.M. level notes, but there will be many more F.Ss. than B.Ss. because the B.Ss. are taken only on B.Ms. and T.Ps.

To illustrate the process of profile leveling, let the top part of Fig. 55 represent the plan of a traverse such as the center line of

* For more complicated problems of this nature, see Higher Surveying by Breed and Hosmer, Vol. II, Appendix C.

† When the traverse is of sufficient length the usual procedure is to establish bench marks every half mile or so along, or near, the line before the profile is run, so that the profile party may check its work as it progresses.

a preliminary survey for a highway, and the lower part of Fig. 55 represent the profile of the traverse line.

The position *A* is chosen for setting up the level. In choosing *A* the slope of the ground, as well as the visibility of those stations that can be seen in both directions from *A*, is taken into account. If a higher elevation had been chosen for *A*, the top of a rod held on Sta. 0 would have been below the line of sight. If the ground from Sta. 0 to Sta. 6 is fairly level, *A* could have been chosen farther along the line, near Sta. 3 or 4.

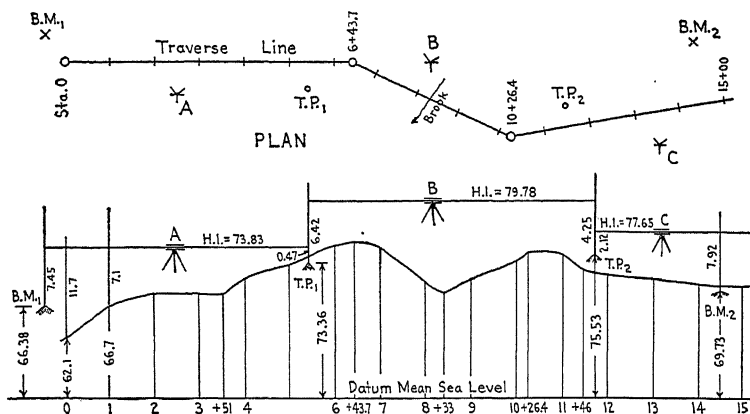


FIG. 55. TRAVERSE AND ITS PROFILE.

When the instrument has been leveled at *A*, the rod is held on B.M.₁ and a B.S. of 7.45 is observed and placed in the notes in the B.S. column, on the same line as B.M.₁ (Fig. 56). When the value is added to the elevation of B.M.₁, the H.I. of 73.83 is obtained. The rodman now proceeds to Sta. 0 and holds the rod on the ground at the foot of the stake. The levelman reads a F.S. of 11.7 ft., which is recorded in the foresight column, on the same line as Sta. 0. If this value is subtracted from the H.I., an elevation of 62.1 is obtained for Sta. 0. Readings on the ground are usually recorded to the nearest 0.1 ft. Since the slope from Sta. 0 to Sta. 1 is uniform, the rodman proceeds to Sta. 1, and holding the rod on the ground at the foot of the stake at Sta. 1, the levelman reads a F.S. of 7.1. This value is recorded in the

foresight column, on the same line as Sta. 1. When subtracted from the H.I. of 73.83, an elevation of 66.7 is obtained for Sta. 1. F.S. readings are next taken at Sta. 2 and Sta. 3 and the elevations of these stations are computed (Fig. 56). It will be noticed that the slope between Sta. 3 and Sta. 4 is not uniform. The rodman measures with his rod the distance from Sta. 3 to

Profile for Proposed Highway North St. to Martin's Corner Kanesville, Md.					Sept. 16, 1941	Jones Harlow $\overline{\wedge}$ Barnes
Sta.	B.S.	H.I.	F.S.	Elev.	Remarks	
B.M. ₁	7.45	73.83		66.38	Standard Bronze Disc in concrete Mon. North side North St. 150 feet West of Allen's Corner	
0			11.7	62.1		
1			7.1	66.7		
2			5.0	68.8		
3			4.9	68.9		
+51	6.42	79.78	5.2	68.6		
4			2.9	70.9		
5			0.9	72.9		
T.P. ₁			0.47	73.36		
6			4.1	75.7		
+43.7			3.4	76.4		
7			4.2	75.6		
8			9.3	70.5		
+33			11.1	68.7		
9			8.9	70.9		
10	2.12	77.65	6.2	73.6		
+26.4			4.4	75.4		
11			5.2	74.6		
+46			7.0	72.8		
T.P. ₂			4.25	75.53		
12			6.1	71.6	B.M. ₂ El. 69.71 Highest Point Ledge Outcrop 50 feet South of S.E. Corner Martin's Grist Mill	
13			6.3	71.4		
14			7.2	70.5		
B.M. ₂			7.92	69.73		
15			7.5	70.2		

FIG. 56. PROFILE LEVEL NOTES.

the point where the grades change; and lining himself in on the traverse line by eye, holds the rod on the ground at this point. If the distance is 51 ft., it is recorded in the notes as 3 + 51. The F.S. reading on this point is 5.2 and the elevation is 68.6. The rod is now held at Sta. 4 and at Sta. 5, and F.Ss. of 2.9 and 0.9

are observed at these stations, giving the elevations at Stas. 4 and 5, of 70.9 and 72.9 ft. respectively. The ground at Sta. 6 is above the line of sight of the instrument at *A*. Therefore we must establish a T.P. somewhere beyond Sta. 5. In the sketch it is marked T.P.₁. Note that it is not on the traverse but is at one side of it. The F.S. on the T.P.₁ is 0.47 and is recorded in the notes as shown, the corresponding elevation being 73.36. When the T.P.₁ has been established, the level is moved ahead to a new position, as *B*. A backsight of 6.42 is observed from *B* on T.P.₁, and an H.I. of 79.78 is computed. The rodman proceeds to Sta. 6, and the levelman observes a F.S. of 4.1; the corresponding elevation is 75.7. A foresight of 3.4 is next observed at the transit station (angle point) 6 + 43.7. F.Ss. are taken at Stations 7, 8, 8 + 33, 9, 10, 10 + 26.4, 11 and 11 + 46, just beyond which T.P.₂ is established. The level is now moved ahead to a position near Sta. 13, at *C*. A B.S. is observed on T.P.₂, and the above leveling process is repeated until the end of the line is reached. In the vicinity of Sta. 15 is B.M.₂; a reading is taken on it as a check.

The **elevations** should be carried out to the **same number of decimals** as the decimals of the foresights that **determine** them, i.e., in Fig. 56 ground elevations are recorded to tenths, and B.M. and T.P. elevations to hundredths.

When checking the computation of these notes, omit the F.Ss. that were taken on the ground (at the stations), since they are not used in computing the elevation of the last B.M. (Fig. 56). The elevations of stations may be checked by noting whether the differences in elevations between adjacent stations are the same as the differences in the F.Ss. that were taken from the same set-up of the level.

106. Plotting the Profile. The profile of any linear structure, such as a pipe line, is usually plotted to a horizontal scale of 20, 40, 80, 100 or 200 ft. per inch, and to a vertical scale of 4, 8, 10 or 20 ft. per inch. Thus the horizontal scale is usually 5 to 10 times the vertical scale. This distortion exaggerates the slope of the ground, but the exaggeration is desirable because vertical changes in the ground slope are so small relative to the horizontal scale that they cannot otherwise be readily or accurately scaled

from the profile. The profile is usually plotted on especially prepared profile paper with ruled lines. Each surface elevation is plotted, and these surface points are joined by straight lines drawn freehand between adjacent points, because if there had been any abrupt change in slope, another surface elevation would

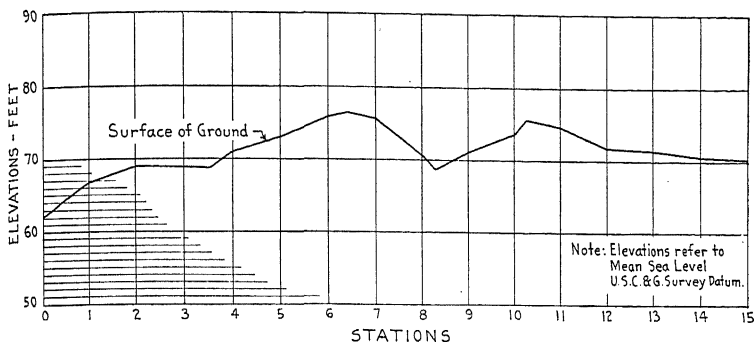


FIG. 57. PLOT OF PROFILE OF GROUND.

have been taken. Bench marks and turning points **are not**, of course, **points on the profile** and therefore do not appear upon it. Fig. 57 illustrates a profile of the center of a proposed street; it is plotted from the data in Fig. 56.

107. COMMON SOURCES OF ERROR IN LEVELING:

1. Improper focusing (parallax).
2. Bubble not in middle of tube at instant of sighting.
3. Rod not held plumb.
4. Foresights and corresponding backsights on turning points not equally distant from the instrument.
5. Poor turning points selected.
6. Erroneous length of rod.

108. COMMON MISTAKES:

1. Failing to take foresight and backsight on exactly the same spot on the T.P.
2. Neglecting to set target accurately when "long rod" is used.

3. In the use of the self-reading rod, neglecting to clamp the rod at the proper place when "long rod" is used.
4. Reading the wrong foot-mark or tenth-mark.
5. In keeping notes, entering F.S. in B.S. column, or vice versa.
6. In working up notes, adding F.S. or subtracting B.S.

ADJUSTMENTS OF LEVEL

109. ADJUSTMENTS OF THE LEVEL. A wye level is in adjustment when

- (1) the horizontal cross-hair is truly horizontal with the instrument leveled,
- (2) the line of sight coincides with the axis of the collars of the telescope, or is parallel to it,
- (3) the axis of the spirit level is in the same vertical* plane as the line of sight, or in a vertical plane parallel to a vertical plane through the line of sight,
- (4) the line of sight and the axis of the spirit level are parallel,
- (5) the axis of the spirit level is perpendicular to the vertical axis.

A dumpy level is in adjustment when

- (1) the horizontal cross-hair is truly horizontal with the instrument leveled,
- (2) the axis of the spirit level is perpendicular to the vertical axis,
- (3) the line of sight and the axis of the spirit level are parallel.

Items (2) and (3) under the wye level adjustments do not appear under the dumpy level adjustments. They are taken care of in the construction of the latter.

* In most levels the bubble is attached to the underside of the telescope; hence its axis should lie in the vertical plane passing through the line of sight. In some precise levels, however, the bubble is below, and to one side of, the telescope, in which case this plane is not vertical.

Because of use, handling, transporting, and temperature variations, the conditions for perfect adjustment will change; when tests show this change, the instruments should be adjusted.

110. ELIMINATING THE EFFECT OF ERRORS OF ADJUSTMENT. The two major adjustments of a level are: **the adjustment to make the line of sight parallel to the axis of the bubble tube; and the adjustment to make the bubble tube axis perpendicular to the vertical axis.** If the first one is made, error due to the second will always be eliminated if the rod-readings are taken only at the instant when the bubble is in the center of the tube; but centering the bubble will not eliminate the error due to the line of sight not being parallel to the bubble tube axis. In this case, the line of sight will fall below the true horizontal and thus give a rod-reading which is too small, or it will be above the true horizontal and give a rod-reading which is too large. In either case the amount of error in the rod-reading will be exactly proportional to the distance from the instrument to the rod. Consequently, such an error will be eliminated from the results if the sum of the distances to the backsights and the sum of the distances to the foresights are equal; these distances may be obtained by pacing. This is one of the most important principles to be observed in leveling work.

TESTING AND ADJUSTING THE WYE LEVEL

111. ADJUSTMENT OF CROSS-HAIRS. (a) **To Make the Horizontal Hair Truly Horizontal When the Instrument is Leveled.** The horizontal hair is usually assumed to be truly horizontal when the instrument is leveled, and rod-readings are made not only where the horizontal hair is crossed by the vertical but often a little to one side of the vertical hair. It is important therefore that the horizontal hair be truly horizontal, so it should be tested by leveling the telescope and moving it slowly right and left about the vertical axis, noting whether a point remains on the horizontal hair all the way across the field of view (Fig. 58). If it does not, the cross-hair ring must be rotated in its vertical plane. This necessitates loosening two capstan-headed screws, in order that the ring can be rotated until this test is satisfied.

112. (b) To Make the Line of Sight pass through the Centers of the Y-rings, or Parallel to Axis of Rings. The intersection of the cross-hairs is assumed to be in the center of the cylinder defined by the metal rings which bear on the Y-supports. This condition is tested by loosening the Y-stirrups and rotating the telescope in its bearings, about its own axis, noting whether the

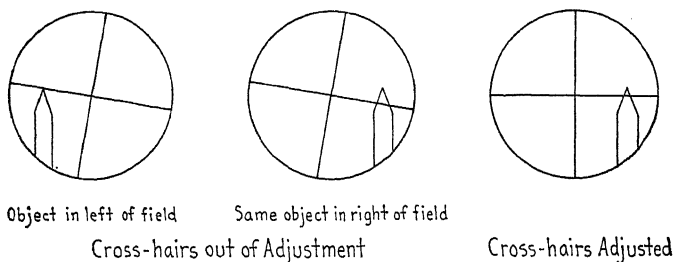


FIG. 58. TESTING THE HORIZONTAL CROSS-HAIRS.

intersection remains on the same point for the entire revolution. Mark a point on a wall at this intersection. Turn the telescope half-way round. Then each cross-hair must be moved **half-way** back to this mark. This is done by loosening one of the adjusting screws which hold the ring and tightening the opposite screw until the cross-hair is in the desired position. The two hairs must be adjusted separately.

113. ADJUSTMENT OF THE BUBBLE TUBE. (a) **To Make the Axis of the Bubble Tube and Line of Sight Lie in the Same Vertical Plane.** The test to determine whether the axis of the bubble and the line of sight are in the same vertical plane is made by rotating the telescope in the wyes a few degrees to one side, after the instrument has been carefully leveled, and observing the position of the bubble. If it remains in the center of the level tube, the axis of the bubble and the line of sight are in the same plane. If the bubble moves toward one end, they are not in the same plane, and the end toward which it moves is the higher. The bubble is brought to the center by means of the lateral adjusting screws at one end of the bubble case (Fig. 59). Repeat the operation if necessary. Test finally by rotating the telescope each way.

114. (b) **To Make the Axis of the Bubble Tube Parallel to the Line of Sight.** The adjustment of any spirit level is ordinarily tested by reversing it end-for-end. The principle is illustrated by the way in which a carpenter levels a timber. He places his level on it, raises or lowers one end of the timber until the bubble is in the center of its tube. To test the level, he **reverses** it. If the bubble then does not come exactly to the middle of the tube, he “splits the difference”; that is, he moves the end of the timber up or down until the level bubble comes **half-way** back to its first position. The timber is then level.

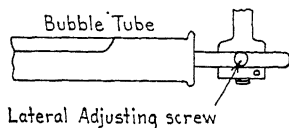


FIG. 59. LATERAL ADJUSTMENT OF BUBBLE TUBE.

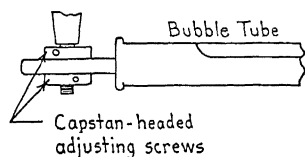


FIG. 60. VERTICAL ADJUSTMENT OF THE WYE LEVEL BUBBLE TUBE.

To test the bubble of a wye level to see if its axis is parallel to the line of sight, open the wye stirrups and center the bubble by means of the leveling screws; tighten the clamp to hold the level bar in position; lift the telescope carefully out of its wyes and, after turning it end-for-end, place it again in the supports; the eyepiece is now where the object glass was originally. If the bubble is not central, adjust it by moving the (vertical) adjusting screws at the end of bubble case until the bubble comes **half-way** back to the center (Fig. 60). Repeat the process until the bubble remains in the same (central) position whether the eyepiece is on the left or on the right.

115. **ADJUSTMENT OF YS—To Make the Axis of the Bubble Tube Perpendicular to the Vertical Axis.** If the axis of the level tube is not perpendicular to the vertical axis, it will be necessary for the levelman to re-level his instrument each time that he takes a sight. To test this condition, center the bubble and turn the telescope, about the vertical axis, through a horizontal angle of 180° . If the bubble does not remain in the center, raise or lower the vertical support at one end of the Y-bar so as to bring the bubble **half-way** back to the center. Notice that this does

not alter any of the adjustments already made. After re-leveling, the bubble should remain in the center as the telescope is turned right or left about the vertical axis.

TESTING AND ADJUSTING THE DUMPY LEVEL

ADJUSTMENT OF THE CROSS-HAIRS. The test of the horizontal hair of the dumpy level is made in exactly the same manner as that described for the wye level. (See Fig. 58).

116. ADJUSTMENT OF BUBBLE TUBE — To Make the Axis of the Bubble Tube Perpendicular to the Vertical Axis. The bubble of a dumpy level cannot be tested by such a simple process as that described for the wye level. The level axis can be made perpendicular to the vertical axis by exactly the same test as that already described, namely, by centering the bubble with the leveling screws, turning the telescope 180° , and bringing the bubble half-way back, if necessary, by means of the adjusting screws (Fig. 61). But the test of the parallelism of the level axis and the line of sight requires

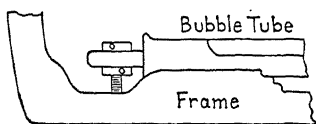


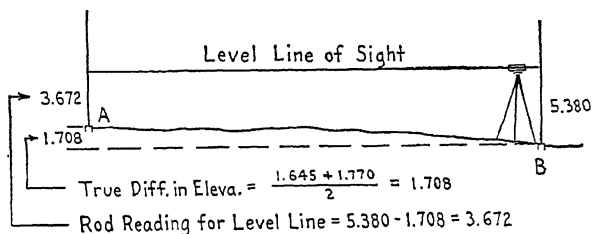
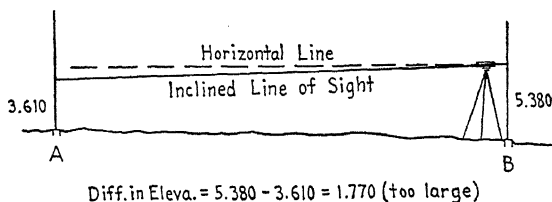
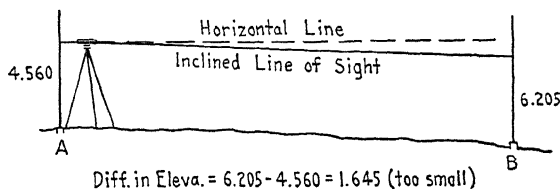
FIG. 61. VERTICAL ADJUSTMENT OF DUMPY LEVEL BUBBLE TUBE.

leveling between two turning points by a process known as the "*peg adjustment*" method.

117. PEG ADJUSTMENT — To Make the Line of Sight Parallel to the Axis of the Bubble. Select two points *A* and *B*, say, 200 ft. or more apart. Set up the level close to *A*, so that when a rod is held upon *A* the eyepiece will be about a quarter of an inch from the rod. Looking through the telescope **wrong-end-to**, sight at the rod and find the rod-reading at the cross-hair, if it is visible; otherwise, find the reading opposite the center of the field of view. After a little experience this can be done very accurately. Owing to the fact that only a small portion of the rod is visible, it will be found convenient to set a pencil-point on the rod at the center of the small field of view. Turn the telescope toward *B* and take a rod-reading on it in the usual way, being certain that the bubble is in the middle of the tube. Referring to Fig. 62, the difference between these two rod-readings is 1.645, but it is

smaller than it should be because the line of sight inclines downward.

The level is next taken to *B*, and the above operation is repeated. Since the line of sight inclines downward, as shown in Fig. 63, the difference in rod-reading, 1.770, is larger than it would



FIGS. 62, 63, AND 64. PEG ADJUSTMENT.

be if the line of sight were horizontal. The mean of the two results is the true difference in elevation of points *A* and *B*; it is, therefore, 1.708, as shown in Fig. 64. If the instrument is left in the same position at *B* and its line of sight is tilted upward so

that the reading $5.380 - 1.708 = 3.672$ is sighted on the rod held at *A*, the line of sight will be actually horizontal. This is done by bringing the bubble exactly to the center of the tube and raising or lowering the horizontal cross-hair, by means of the adjusting screws on the cross-hair ring, until the line of sight strikes the reading 3.672 on the rod held at *A*. In this method the small error due to curvature of the earth (nearly 0.001 ft. for a 200-ft. sight) has been neglected.

The peg method may also be used for adjusting the wye level or the transit, the difference being that in the dumpy level the axis of the bubble tube is first made horizontal and the line of sight is brought parallel to it, whereas in the wye level and in the transit the line of sight is first made horizontal and the axis of the bubble tube is made parallel to it. Consequently, in adjusting the dumpy level the cross-hair ring is moved, whereas with the transit, or the wye level, the adjustment is made in the bubble tube.

There are several other methods of applying the peg adjustment but the principle is the same as the one given above.

FIELD EXERCISES

1. Starting from some bench mark of known elevation (or an assumed elevation) run a line of levels establishing several new bench marks. Return to the initial bench mark to check the line of levels. Show in the notes the check of the calculations, and also the check in the redetermination of the elevation of the first bench mark. Mark and describe the new bench marks.

2. Select a line and measure it, in 100-ft. stations, for a profile, setting intermediate points at sharp changes of grade or at turns in the line. Determine the elevations of all stations. Check the calculations and plot the profile.

3. Carry a line of levels across a river or a ravine involving one long sight. Correct for curvature and refraction. Check the result if possible by means of a second determination, sighting back from the farther side of the river.

4. Determine sensitiveness of a level bubble in seconds per bubble division.

5. Starting from a bench mark of known elevation, determine the elevation of a point on the second story of a building about 1000 ft. away. Because of the topography the only set-up available near the point is about 6 ft. below it. Note: With the rod inverted, hold the bottom at the point and take rod-reading. Be careful to apply the reading to the H. I. correctly.

PROBLEMS

1. B.M. levels are run from elevation 67.01 to 143.20, using a rod on which the distance from the 1-ft. mark to the 11-ft. mark proves to be 10.005 when checked with a tape. What is the correct difference in elevation?

2. From the following notes compute the elevation of B.M.₂. Show check.

Sta.	B.S.	H.I.	F.S.	Elev.
B.M. ₁	7.850			169.875
T.P. ₁	9.125		3.645	
T.P. ₂	6.430		2.110	
T.P. ₃	8.975		5.565	
T.P. ₄	9.110		0.435	
T.P. ₅	3.710		1.210	
B.M. ₂			2.180	

Sta.	B.S.	H.I.	F.S.	Elev.
B.M. ₁				60.530
T.P. ₁				
T.P. ₂				
T.P. ₃				
T.P. ₄				
B.M. ₂				43.465

In the above form, fill in all data to show a complete set of bench-level notes that would be consistent in determining the elevation of B.M.₂ = 43.465 ft. from the elevation of B.M.₁ = 60.530 ft.

4. A dumpy level was tested by the "Peg Method" with the following results:

INSTRUMENT AT *A*

Rod-reading at *A* = 4.325 ft.

Rod-reading at *B* = 6.810 ft.

INSTRUMENT AT *B*

Rod-reading at *B* = 5.005 ft.

Rod-reading at *A* = 2.500 ft.

With instrument remaining at *B*, compute the rod-reading on *A* for a level line of sight. Is the line of sight inclined up or down? How much?

5. Sensitiveness of Bubble.

A level is set up at *A*, the bubble centered and a rod-reading of 5.980 ft. taken on a rod at *B*, 296 ft. from *A*. The levelman moves the bubble 4 divisions on the bubble tube and observes a reading of 6.100 ft. on the rod at *B*. Compute the angular value of one division of the bubble tube.

6. Curvature and Refraction.

A level is set up with the bubble carefully centered, and a B.S. of +4.780 is taken on a point 500 ft. away; then a F.S. of 7.965 is taken on a point 800 ft. away. Compute the curvature and refraction for each point and their difference in elevation.

7. Compute the elevations and plot the profile. Horizontal scale 1 in. = 100 ft.; vertical scale 1 in. = 10 ft.

Sta.	B.S.	H.I.	F.S.	Elev.
B.M. ₁	8.45			60.530
0			9.4	
1			7.2	
2			5.1	
+70			5.3	
3			3.0	
4			1.1	
T.P.	7.50		0.75	
+30			2.1	
5			4.0	
6			6.3	
+60			10.3	
7			7.4	
8			5.1	
T.P.	4.52		6.35	
9			3.8	
10			5.0	

8. The top of a 12-ft. level rod is 0.5 ft. out of plumb in the plane of the line of sight. What error would this cause if the rod-reading were 12 ft.? If the rod-reading were 2 ft.?

Sta.	B.S.	H.I.	F.S.	Elev.
B.M. ₁	8.750			60.895
T.P. ₁	9.625		3.355	
T.P. ₂	11.380		0.260	
T.P. ₃	10.585		1.100	
T.P. ₄	7.620		2.055	
B.M. ₂			4.310	

In the above set of B.M. Level Notes, compute the elevation of B.M.₂. Show check on the arithmetic.

10. Compute and fill in the missing values to complete the following set of B.M. Level Notes. Show check.

Sta.	B.S.	H.I.	F.S.	Elev.
B.M. ₁		104.25		100.00
T.P. ₁	6.84			96.93
T.P. ₂		96.13	9.85	
T.P. ₃	6.01		3.16	
B.M. ₂				95.38

11. Draw a sketch of a Philadelphia rod graduated to hundredths. Show target and vernier reading 4.075. Note: Only that portion of the rod in the vicinity of the target need be shown. Draw to scale; $\frac{1}{4}$ in. = .01 ft.

12. Put the following description in the form of profile level notes, and compute the elevations.

A level is set up and a reading of 9.82 ft. is taken on a B.M. the elevation of which is 39.78 ft. At the beginning of the line to be profiled the rod-reading is 8.6 ft.; 100 ft. from beginning it is 5.3 ft.; at 200 ft. it is 2.3 ft.; at 220 ft. and 270 ft. the rod-readings are 4.2 ft. and 2.5 ft. respectively. On a rock that is not on the line the rod-reading is 1.82 ft. The level is then moved ahead, set up, and a rod-reading of 6.40 ft. is observed, the rod still being held on the rock. The readings along the profile are then resumed:

300 ft.	from the beginning of the line	the rod-reading is	3.8 ft.
400 ft.	" " " " " " "	"	4.7 ft.
500 ft.	" " " " " " "	"	7.2 ft.

CHAPTER VI

TRAVERSES AND LAND SURVEYS

118. TRAVERSES. A traverse is a series of connected lines the lengths of which and the angles between which have been determined by measurement. They are of two kinds: (1) closed traverse, (2) open, or linear, traverse.

A *closed* traverse starts at a given point, runs in a circuit, and eventually returns to the point of beginning. An example of a closed traverse is the survey of the boundaries of a parcel of land; its boundaries form a polygon. If the sides and the angles in a closed traverse were determined exactly, the figure would mathematically close; i.e., the last point would mathematically fall on the first point. But, since in every survey there are small errors which are inherent in the process, the amount by which the last point fails mathematically to fall on the first point indicates at once whether a mistake has been made. This check is invaluable. Hence, whenever possible a survey should be **closed** on itself, or on previously established points the relative positions of which are known.

An *open* traverse does not return to the point of beginning. An example of this kind of traverse is a survey for a highway (*route survey*): in its simplest form no checks by calculations are available. If, however, such a traverse starts and ends on a triangulation point the coordinates of which are known, we can obtain the same sort of check as in the case of a closed traverse, because such a traverse will in fact constitute a closed traverse. Checks on the angles of an open traverse are often obtained by observing astronomically the true bearings of some of the traverse lines as the survey progresses (Arts. 129-32).

The lengths of the lines of any traverse may be checked by making two or more measurements of them in the field. When this procedure is followed, it is customary to make the measurements forward and backward along the line.

In closed traverses it is customary to record the measured length of each line, i.e., the length of each line from angle point to angle point. In many surveys, however, such as route surveys, the line is measured continuously from the starting point to the end point of the traverse. The starting point of the traverse is called "Station 0"; the next station 100 ft. ahead is "Station 1"; the next, "Station 2," etc. Every 100-ft. length is a full station and any fractional distance is called the "plus"; a station 540.1 ft. from Sta. 0 will be called "Station 5 + 40.1," as described in Art. 34.

Checks on the angles are made by doubling the angles or by repeating the angle a sufficient number of times to obtain the required precision (Art. 59).

There are several ways in which the angles are measured: (1) measurement of angles to the right, (2) measurement of the interior angles, (3) measurement of the deflection angles, (4) measurement of the azimuths. These are explained in Arts. 56-9.

Traverses are frequently run for the control of topography, i.e., the traverse serves as a skeleton from which measurements are taken for the purpose of locating physical features, such as buildings, roads, and property lines. In this type of survey the angle points (traverse stations) are so chosen that the traverse lines will pass near the physical objects. If contours are required, the points are placed in such positions that the surveyor will have a favorable command of the ground. The methods of locating buildings are given in Chap. XIV. In plotting the plan the traverse is first laid out and then distances and angles are plotted from the traverse to the physical features taken in the field, thus producing a map with all features in their correct positions.

If possible, the traverse should start and end at well-defined points, preferably at stations on a triangulation net, if the traverse is to be of any considerable length. Often such control points may not lie directly in the path of the traverse, but they are often so close to it that side loops of the traverse may be run to connect with them and thus serve as a most desirable check on the survey.

119. Referencing, or "Tying In," a Point or Station. All important points temporarily marked by stakes or nails should be "tied in," so that they may be readily found or replaced. At least three **horizontal** ties (measurements) to the nearest 0.01 ft. should be taken from three readily identifiable and permanent points to the station in question. These points may be corners of buildings, nails in trees, drill-holes in ledges, stone bounds, etc. All measurements should be carefully recorded, usually by means of a sketch, as in Figs. 65 and 66. Note that in the latter the ties are recorded along short, straight lines which are accurately run with a transit.

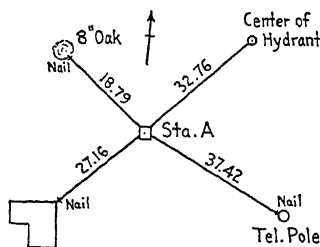


FIG. 65. TYING IN A STATION.

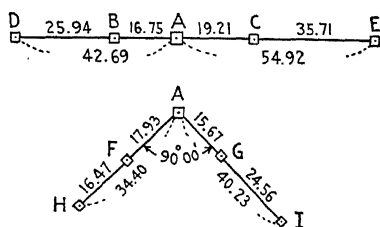


FIG. 66. REFERENCING IMPORTANT POINTS.

120. LAND SURVEYS. A property survey, sometimes called *Cadastral Survey*, is made usually for the purpose of furnishing a description of the boundaries for transfer of title or for record in Registry of Deeds, for determining the areas of property, and for obtaining data for making plans.

In making a survey of a parcel of land, a closed traverse should be run around the property, preferably exactly on the property lines. If possible the transit should be set at the actual corners of the property and the angles measured directly; the distances also should be measured directly along the property lines. If the corners of the property cannot be occupied by the transit, arbitrary transit stations are chosen and marked near the corners. The angles and distances of the traverse are then measured. To connect the property lines with this traverse, an angle and distance are taken from each arbitrary transit station to the

corresponding corner of the property. From these data the lengths and directions of the actual boundaries are computed.

In some surveys where the property corners cannot be occupied, it is convenient to lay out a traverse the lines of which are parallel to the property lines. The angles of such a traverse are equal to the actual angles of the property.

When making a survey that does not follow the property boundary lines, it is good practice when possible to measure the lengths of the property sides. A check is thus obtained on the computed lengths of the property lines.

Fig. 67 is the field-notes of a property survey made for the purpose of determining the area and making a plan. It is the type of survey that would be made in a town or in the outlying districts of a city.

The boundaries of the property are $ABCFG$; all are straight lines except AG , which is a curved stone wall. Corners A and B were marked by stone bounds; corners at C and F , by iron pipes; the corner G was determined by the intersection of the wire fence and center of the stone wall. The boundaries of most properties are not so well defined as this. Sound judgment and the knowledge of the recognized legal values of evidence as to property boundaries are required of the surveyor (Arts. 121-2).

Examination of this particular property showed that the stone bounds at A and B and the pipe at C could be occupied by the transit and that the property lines AB and BC could be measured directly. There was no obstruction along AB . Along BC , however, there was a low hedge, but it did not interfere with making the measurement or with sighting along this line. Corners F and G could not be occupied with the transit because of the height of the wire fence, so point D near F , and E near G were established, thus completing the closed traverse $ABCDE$.

In making the survey, the closed traverse $ABCDE$ was first run. Deflection angles were observed and checked by reversing the telescope and "doubling." The distances were checked by measuring the lines forward and backward. The values of the angles and distances are shown on the sketch. With these data the area of $ABCDE$ may be computed. At C , angle FCD and distance CF were measured; these were taken for a check.

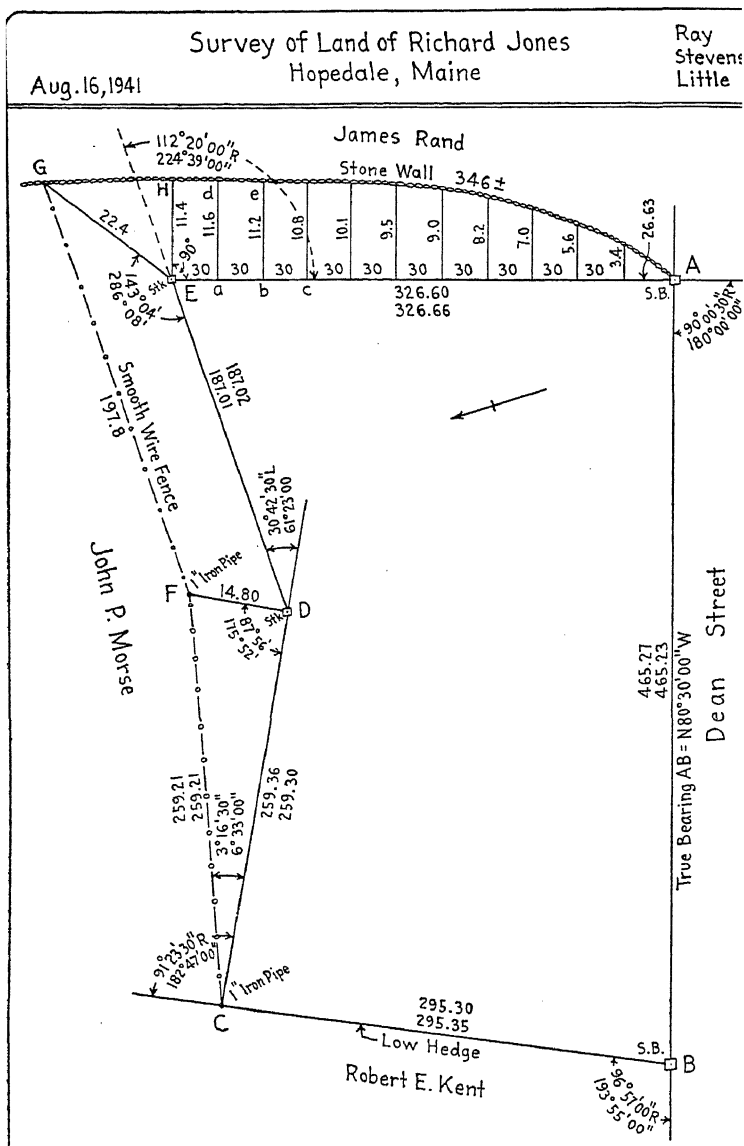


FIG. 67. PROPERTY SURVEY.

When D was occupied the angle and distance as shown were taken to the corner at F , and similarly at E the angle and distance were taken to the corner at G . These additional angles and distances were required for the purpose of computing the lengths and directions of CF and FG and determining the areas CDF and $DEGF$. Measurement FG was taken for a check.

In determining the area EGA it should be noted that the stone wall was not laid along a mathematical curve; had this been done the area EGA could be found exactly. Where the curve is irregular, as in this case, the field procedure is as follows: First, set stakes or pins at a, b, c , etc., along EA at a uniform distance apart; or if there is any apparent angle in the wall, set a stake opposite it. At right angles to EA , measure the distances EH, ad, be , etc., from EA to the center of the wall. These distances are called perpendicular offsets. It is not necessary to lay off 90° angles with the transit at E, a, b , etc. The 90° may be estimated by eye, because the offsets are measured to the nearest 0.1 of a foot only, the reason being that the exact center of the wall is not determinable any closer than that. The offsets as shown in this sketch were taken 30 ft. apart. The distance between them depends on the curvature, the idea being to choose them at such a distance apart that in each figure as $abed$, the line ed may be considered straight; figure $abed$ is therefore a trapezoid. From the data shown on the sketch the area of EHA may be computed as a series of trapezoids except the small area at A which is a triangle. In the triangle GEH it was found that GH was sufficiently straight to be considered a straight line. The necessary explanations and calculations for finding the area of $ABCFG$ will be found in Arts. 142-55.

121. Identifying the Property Corners. There should be no difficulty in identifying the exact corner points where property corners are marked by stone bounds or stakes. Lot lines are usually defined by physical objects, such as stone walls or fences, and the surveyor therefore must examine all the evidence as to the intended positions of these physical boundaries, in order to decide where the true corners are located.

Fences and walls along public ways should be so built that they will be entirely upon the property of the owner, and their faces

should be on the side lines of the highway. In country districts the center of the boundary wall of a road is frequently accepted as the side line of the road. Usually the descriptions of the public ways in towns and villages are so inaccurate that it is next to impossible to define them; for this reason the physical boundaries of the occupied way are often accepted in lieu of the loosely recorded descriptions. It is always well in re-running country road to **adhere to the width** given in the description and to make the alignment fit as nearly as possible the existing walls, fences, and other evidence of occupancy. This having been done, it is advisable to arrange with the town authorities to have the lines as then laid out re-established by proper municipal action.

The street lines in cities are usually well defined, as explained in Art. 267.

The legal boundary line between private owners is the center of a stone wall or a Virginia rail fence; it is the line between the bottom stringer and the **back side** of the boards or pickets of an ordinary fence **when the fence-posts are in a vertical position**, the posts being entirely on one side of the boundary line. In the built-up districts of cities the boundary is often the middle of a party-wall. When the deed of land reads "to thread of stream," the boundary is the line midway between the shores when the stream has its average regimen or flow.

The limits of woodlots are often marked by the blazing of trees, the blazing being done on the side of the tree nearer the boundary. When a tree lies directly "on line," it is blazed on both sides at points where the line strikes it. A common method of marking corners in woodlots is by means of round stakes driven into the ground, with small piles of stones laid carefully around and against them. Farmlands are often marked by substantial wooden stakes, sometimes with the surveyor's initials, or private mark, cut in their sides. If two or more markers are found for the same line, it is usually advisable to stick to the older of the two, if this fact can be determined.

122. Laws Relating to Boundaries. It is the surveyor's function to **develop evidence** of property ownership from his personal examination and measurements of the property. There are certain principles of priority of evidence which have been

established by legal decisions.* The surveyor should use these decisions in organizing the evidence relating to property lines. In this short treatise only a few basic rules will be mentioned.

It is distinctly his duty to find the position of the original boundaries of the property, he should **not attempt to correct the original survey**, even though he may be sure that an error exists in it. He must also fortify himself with evidence that will convince the judge or jury that he has **actually retraced the old boundaries** and that he has **followed the tracks of the original surveyor**.

Physical boundaries, provided that they are mentioned in the deed, have the legal force of monuments and therefore take precedence over courses and distances. These boundaries (or monuments) may be either "natural monuments," such as trees, lakes, boulders, ledges, etc.; or "artificial monuments," such as fences, stone walls, posts, stakes, pits, mounds, stone bounds, or other physical objects placed by man for boundary marks. Natural boundaries, because they are more permanent, take precedence over those that are artificial.

The testimony of old reliable witnesses may be resorted to when, after an exhaustive search, no physical boundaries can be found.† It is not uncommonly true, after all evidence furnished by persons familiar with the location of the lost lines has been analyzed, that one or more of the physical corner marks can now be found even though previous careful search had not disclosed it.

* Treatise on the Law of Surveying and Boundaries by Frank Emerson Clark, Esq., published by The Bobbs-Merrill Co., Indianapolis, Ind., 2nd Ed., 1939, treats of the usual legal decisions of different states and gives especial emphasis to those lands that come under the U. S. Public Land System.

Legal Elements of Boundaries and Adjacent Properties by Ray H. Skelton, published by The Bobbs-Merrill Co., Indianapolis, Ind., treats fully the technical and legal principles of boundary surveying.

† It must not be assumed that a boundary is missing because it is not at once visible. Stone bounds are often buried 2 or 3 ft. deep; the top of a stake soon rots off, but evidences of the existence of the stake are often found many years after the top has disappeared, and the supposed location should be carefully examined and then dug over to find traces of the old stake. Care should be taken, however, that the evidence is not destroyed in the process of digging. Traces of the old wood or charcoal may often be found if great care is used.

Sometimes only a spot of discolored earth at the place pointed out as the corner by some informed local person will reveal the point where a corner post or stake once stood.

The law also assumes that the deed was drawn by the grantor with honest intent to convey the property to the grantee. It shall therefore be interpreted if possible so as to make it effectual rather than void. The deed shall also be construed in the light of what was known at the time when the title was transferred. In its interpretation it is assumed that it was intended to convey property the boundaries of which will form a closed traverse. Therefore it is within the jurisdiction of the surveyor to reject any evident mistake in the description when running out the property line, e.g., a bearing that has obviously been recorded in the opposite direction or an entire side or tape-length omitted. When the area does not agree with the boundaries as described in the deed, then the boundaries control. All distances, unless otherwise specified, are to be taken as straight lines; but distances given as so many feet "along a wall" or "by a highway" are supposed to follow these lines even if they are not straight. When a deed refers to a recorded plan, the dimensions on that plan become a part of the deed description of the property. (See Arts. 123-4.)

Land that has been occupied openly and continuously for a long period of years may become by "*adverse possession*" the property of the person occupying it. In several states this period is twenty years. Property can be acquired in this manner even though a deed of excellent title to the property exists in the name of another person.

It is a sound general rule that any excess or deficiency found upon subsequent measurement to exist in a line marked by permanent monuments at its ends only, must be distributed among the several subdivisions of the line in proportion to the original measurements. For example, if a city block originally surveyed as 600.00 ft. long and permanently monumented was subdivided into twelve lots of 50.00 ft. frontage, and the block is subsequently found to measure 600.24 ft. between monuments, then each lot should be given a frontage of 50.02 ft. This rule cannot be followed blindly. For example, if the frontage on one of the

lots had been omitted from the plan or had been marked $50 \pm$ ft., then the whole excess of 0.24 ft. should go into that unmarked or approximately marked frontage. This general rule applies also to dimensions of irregular shaped lots and to deficiency or excess in areas, as when the original dimensions appear to leave a strip or to overlap where two lots are intended to abut upon each other and where it is intended to convey the whole tract.

In re-running old surveys from deeds it will usually be found that the distances stated in old deeds are shorter than those obtained in recent surveys. The tendency of the old surveyors was to give "full measure."

When using magnetic bearings as recorded on the plan, the surveyor should be on his guard for changes in declination, because the date of the plan may be several years subsequent to the date of the survey in which the bearings were determined. If only one boundary line can be identified and its bearing is given in the old deed, the difference between the declination of the needle at the time of the old survey and at the present time can readily be obtained. (Arts. 44-5.)

123. How to Look Up a Recorded Deed. In all the states of the Union the transfer of real property must be recorded in the respective county Registry of Deeds or in the office of the city or town clerk. At the Registry of Deeds is kept an exact copy of the deed, which can be examined by anyone. It is frequently necessary for the surveyor to use these Registry copies when it is not convenient to obtain the actual deed from the owner of the property.

In every Registry of Deeds is kept an index of deeds, which is divided into two parts, the *grantor* index and the *grantee* index; the grantor is the party who sells the land and the grantee, the one who buys it. These indexes are frequently filed chronologically and for this reason the surveyor should know not only the name of the party who bought or sold the property (both if it is convenient to get them), but also the approximate date of the transaction. With this information he can find in the proper index the name of the party, opposite which will appear the date of the transaction, the number of the deed book, and the page on which the copy of the deed is recorded. He then finds the deed

book, from which he can copy whatever data he desires; usually the description of the property is all that concerns the surveyor. In the deed book there is usually a reference number in the margin, or in the text of the deed, which refers to the next preceding transfer of the same property or to any attachments, assignments, and the like which may have been made on it. This method of indexing and filing deeds is used in the New England States and in many of the other states; the general principles are the same throughout the country, although the details may differ to some extent.

If a plan is referred to in a deed, then the plan becomes a part of the description and the filed plan should also be examined at the Registry.

Most Registries will furnish, for a small fee, a photostat copy of any deed or plan.

124. Deed Description. A deed description should give the name of the owner and the bearings (or angles) and distances along each property line; it should state whether bounded by a highway, stream, or private property. The description should run **to** and **along** lands of adjoining owners to aid in discovering mistakes which may have been made through the carelessness of the conveyancer or copyist. The following paragraph is a deed description of the property shown in Fig. 67. The true bearing of the traverse line AB was assumed correct and from it the bearings of the walls and fences bounding the property were computed. (See Fig. 78.)

“Beginning at a point in the northerly line of Dean Street in the town of Hopedale, Maine, at a stone bound set in the ground at the S.W. corner of land now or formerly belonging to James Rand, and running along said northerly line $N 80^{\circ} 30' W$ a distance of four hundred sixty-five and twenty-five hundredths (465.25) feet to a stone bound set in the northerly line of said Dean Street at the S.E. corner of property now or formerly belonging to Robert E. Kent; thence turning and running by land of said Kent $N 16^{\circ} 27' 30'' E$ a distance of two hundred ninety-five and thirty-two hundredths (295.32) feet to a one-inch iron pipe sunk in the ground at the S.W. corner of land now or formerly belonging to John P. Morse; thence turning and running

by land of said Morse S $75^{\circ} 25'$ E a distance of two hundred fifty-nine and twenty-one hundredths (259.21) feet to a one-inch iron pipe set in the ground; thence continuing along land of said Morse N $77^{\circ} 02'$ E a distance of one hundred ninety-seven and eight-tenths (197.8) feet to the middle of a stone wall at land now or formerly belonging to James Rand; thence turning and running in a southerly direction along the middle of said stone wall and by land of said Rand, a distance of about three hundred forty-six ($346 \pm$) feet to the point of beginning; all bearings referring to the true north and the parcel containing a calculated area of 133,300 square feet more or less."

Careful conveyancers prefer to write the bearings in words as well as in figures.

It is unfortunate that in the majority of cases the description of the property in deeds does not define the property in such a manner that it can be plotted from the description. Some descriptions are so loosely written that they contain nothing but the names of the owners of adjacent property, all bearings and distances being omitted.

125. ORGANIZATION OF TRANSIT PARTY. Transit surveys can be carried on readily by a party of three men. The note-keeper, who is in charge of the party, directs the entire work; the transitman, who has the instrument always in his care, sets it up where directed by the note-keeper, reads the angles and gives line when it is desired; the tapeman, generally acting as head-tapeman, and the note-keeper, as rear-tapeman, measure all distances. There is often economy, however, in having another tapeman, so that the note-keeper may be free to keep the notes and to organize and direct the survey. In places where much clearing of underbrush is required, an axeman is also a desirable or even a necessary addition to the party. In cutting out important boundary lines (state, county, or town lines) where a wide cut is required, several axemen may be needed.

The speed of the party depends upon the way in which the chief of party directs his men. He should so plan the work and train his men that every man will realize his responsibility and anticipate the particular function he is to perform; under such conditions there is little necessity for calling off orders. Some

surveying parties can perform in a day twice the work of others, owing solely to the fact that the chief has trained his men well and has insisted upon the kind of discipline that produces accurate results in a short time. The chief of party should check mentally, by estimate, all distances and angles measured, so that he may catch a large error the moment the dimension is called off to him. He should be **at the spot where important measurements are made**, so that he can check the readings himself if he has any doubt of their accuracy.

126. FIELD-NOTES. The notes are kept in a special notebook, which usually has ruled columns on the left-hand page and cross-section lines on the right-hand page. In general, the left-hand page is used for keeping the notes in tabular form, and the right-hand page is used for remarks, sketches, and notations relevant to the survey.

Note-keeping is a fine art. It is one of the most exacting jobs of the survey. All field data should be recorded **when taken**, and not left to be filled in from memory. Neatness and clearness are of the utmost importance, and great care should be exercised so that there will be no misunderstanding or misinterpretation of the notes. They should be taken and recorded in such a manner that anyone familiar with surveying will be able to interpret them readily. A sharp, hard pencil (3H or 4H) should be used. The sketches should be made with clear-cut lines; the straight lines should be ruled. Since clearness is essential, no attempt is made to sketch closely to scale. Lettering (Reinhardt's Style, Art. 353) is preferable to writing.

In addition to the measurements every set of notes should contain the following information: the name of the job, the location of the job, the character of the work, the date, the names of the field party, and sometimes also the names and numbers of the instruments, and the weather conditions.

The notes for some types of surveys are kept entirely in tabular form; survey data are on the left page, and descriptive remarks on the right page. Fig. 68 illustrates such a set of notes for the closed traverse portion of the survey shown in Fig. 67. This is like any survey in which the traverse was run on the actual boundaries. At each angle point the angle was "doubled" for a

check, and half the second reading was recorded for the angle. Distances were checked by at least two measurements, and the mean of the two was recorded. If the two agreed within $\frac{1}{5000}$, the mean was taken as the correct value. Bearings were observed as a rough check on the angles, although they frequently are not taken, especially in city surveys where the needle may be affected by electric currents. The distance recorded at any station is the distance from that station to the next station ahead. The

Survey of Land of Richard Jones Hopedale, Maine						Ray Stevens Little	B & B Tr. #1569	Aug 16, 1941
Sta.	Def Ang's	Mean Def.	Hor. Dist.	Mean Dist.	Mag Bear			
A	90°00'30R		465.27			Stone Mon.	True Bearing AB = N 80°30'00"W	
	180 00 00	90°00'00R	465.23	465.25	N 60°W			
B	96 57 00R		295.50		N 36½E	Stone Mon.	Computed Angles from Mag. Bearings A 90½° B 96½° C 91½° D 31½° E 113°	
	193 55 00	96 57 30R	295.30					
			295.36	295.33				
C	91 23 30R		259.35		S 52E	1" Iron Pipe		
	182 47 00	91 23 30R	259.30	259.32				
D	30 42 00L		187.01		S 83½E	2" Stake		
	61 23 00	30 41 30L	187.02	187.01				
E	112 20 00R		326.66		S 29¾W	2" Stake		
	224 39 00	112 19 30R	326.60	326.63				
	Sum R	390 40 30						
	Sum L	30 41 30						
	Diff.	359 59 00						

FIG. 68. FIELD-NOTES IN TABULAR FORM.

bearing recorded at any station is the bearing of the next station ahead. It will be observed that at station *B* three measurements were made of the line *BC*. The third measurement was necessary because the first two did not check within the precision required. The first measurement was finally abandoned. On completing the measurement of the angles, the difference in the sums of the right and left deflection angles was computed as a check. The discrepancy in this survey was 00° 01' 00".

For other types of surveys in which considerable detail must be shown, the entire information may be recorded on a sketch, as in Fig. 67. In keeping notes by sketches, care must be exercised not to crowd them; to avoid this it is frequently advisable to draw a supplementary sketch of the detail.

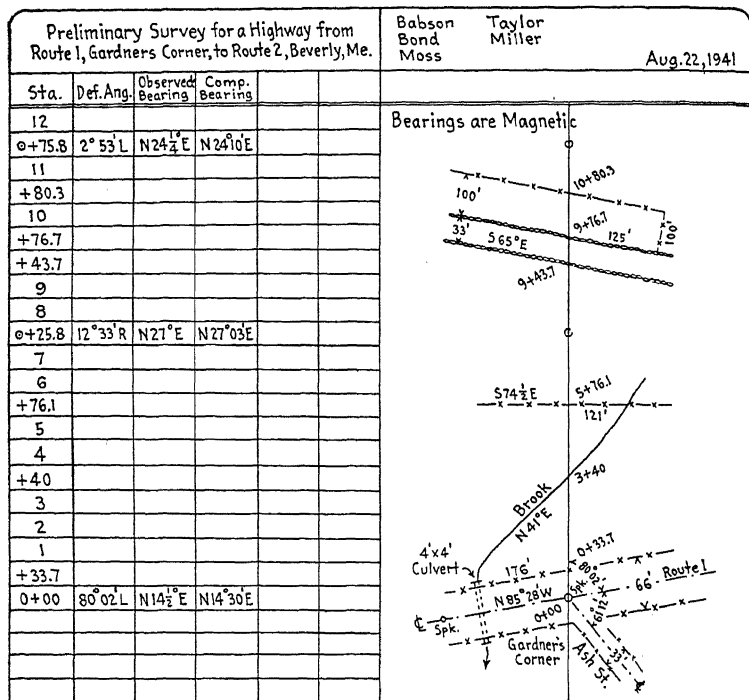


FIG. 69. FIELD-NOTES IN TABULAR AND SKETCH FORM.

If the required data along the traverse and in the vicinity of the angle points would cause the notes to be crowded when put on the sketch, the traverse data may be shown on the left page in tabulated form and the sketch may be placed on the right-hand page, as in Fig. 69. This arrangement is especially applicable for route surveys, such as railroads or highways; in these surveys the required data extend only a few hundred feet on each side of the traverse line. In notes of this kind it is customary to start

at the bottom of the page with both tabulated values and sketches and to work up the page. Stadia notes are commonly kept with tabular data on the left page and a sketch on the right page, as shown in Figs. 123 and 124.

127. Triangulation Stations of U. S. Coast and Geodetic Survey. The U. S. Coast and Geodetic Survey has covered a considerable portion of the country with a triangulation system of "first order" and "second order." Geodetic coordinates for these stations (latitude and longitude) expressed in degrees, minutes, seconds, and decimals of a second have been computed, and are on record with the Coast Survey. The intent has been to eventually locate points throughout the country at intervals of not more than five miles. There are some parts of the country where they are as close as that at the present time and other parts where they are spaced twenty to twenty-five miles.

128. Plane Coordinate State Systems. Plane coordinate systems have been prepared by the U. S. Coast and Geodetic Survey for all the states, and for this purpose the geodetic coordinates of triangulation stations and of other points such as traverse stations have been transposed into rectangular coordinates. In these systems, the X and Y coordinates are expressed in feet and hundredths. Thus surveys based on these coordinates may be carried out according to the principles of plane surveying without recourse to geodetic computations. (See Arts. 145-7.)

The plane rectangular coordinates for these state-wide systems are referred either to the *Transverse Mercator** projection (for states having an extent predominantly north-and-south) or to the *Lambert Conformal Conic** projection (for states having an extent predominantly east-and-west). An example of the former is the state of New Jersey; an example of the latter, the state of Massachusetts. Some states, such as California and Texas, have several zones within the state each having a separate projection plane. The type of projection is usually the same throughout a state, although New York, for example, has three Transverse

* See Higher Surveying by Breed and Hosmer, Vol. II, Chap. XII, Map Projections.

See also Horizontal Control Surveys to Supplement the Fundamental Net, Am. Soc. C.E. Manuals of Engineering Practice — No. 20.

Mercator projections for the mainland and a Lambert Conformal projection for Long Island.

The aim is to have points eventually located as close as a half a mile to a mile apart, so that local surveys may be based on the coordinates. When this is done, such surveys will then be based on the national triangulation system and their character will be improved.

In many states it is customary for highway departments to connect their surveys to coordinates of the state system. Harbor surveys, and in fact all public works surveys, are usually joined to the state coordinate system or to the city coordinate system which the larger cities have established. Lists of coordinates are now available at one of the administrative divisions of the state.

129. DETERMINING THE TRUE MERIDIAN BY OBSERVING POLARIS.* Although there are many ways in which a true meridian may be determined astronomically, the particular method and instrument suitable for this purpose will depend on the precision required. With the engineer's transit, it is comparatively easy to observe the sun or a star and obtain the true bearing of a line within less than one minute of angle. Many observations require a knowledge of astronomy and calculations that are not within the scope of this text, but there is one method that is simple and at the same time requires a minimum of calculation. It is known as the method of laying out a meridian by observing *Polaris* (Pole-Star) at *elongation*. Using the method here explained and an engineer's transit that is in good adjustment, the result should not be in error by an amount exceeding 10 or 15 seconds.

It is usually assumed that the earth is the center of a sphere of infinite radius called the celestial sphere (Fig. 70), upon the inner surface of which all celestial objects are projected. If the polar axis of the earth is produced, it will intersect the celestial sphere at *O*, called the celestial pole. Suppose that in the northern hemisphere a transit is set over stake *A* and that it were possible to

* See Elementary Surveying by Breed and Hosmer, Vol. I, Chap. VIII, for more detailed description of this subject.

sight on N , the north pole of the earth, or to sight on the imaginary celestial pole O ; the telescope would then be in the plane of the meridian, and to establish the meridian it would be necessary only to lower the telescope and set stake B . The line AB would be a true north-and-south line.

Since we cannot observe the earth's north pole or the celestial north pole, we make use of the star Polaris which, due to the motion of the earth on its axis, **appears** to move **counterclockwise** in a small circle around the celestial pole approximately every 24 hours, as indicated in Fig. 71. Polaris is readily identified from the constellation Ursa Major (the Great Dipper). (See Fig. 71.) A line joining the two stars called the "pointers" in the body of the "dipper" points to Polaris.

Another important constellation (shaped like the letter W) is Cassiopeia. (See Fig. 71.) This constellation is on the opposite side of the celestial

pole from the Great Dipper and on the same side of the celestial pole as Polaris; an imaginary line connecting δ Cassiopeia and ζ Ursa Major passes through Polaris and the celestial pole.

Because of the daily rotation of the earth on its axis, all constellations including Polaris **appear** to rotate about the pole in the direction indicated by the arrows, once in approximately 24 hours.

When the constellations are in the positions shown in Fig. 71, Polaris is above the celestial pole and is said to be at *Upper Culmination*. About 6 hours later Cassiopeia will appear to the left, and Ursa Major to the right, of the celestial pole, and when they are in these positions, Polaris is at western elongation (Fig. 71 viewed from the left side). When Cassiopeia is vertically below, and Ursa Major vertically above the pole, Polaris is at *Lower Culmination*; and when Cassiopeia is to the right, and

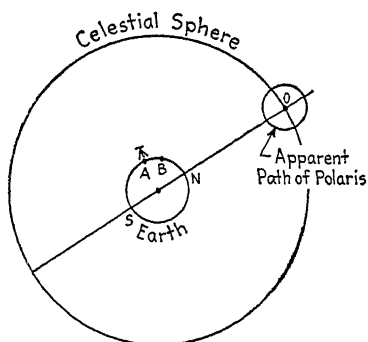


FIG. 70. RELATION BETWEEN THE EARTH AND POLARIS.

Ursa Major to the left, of the pole, Polaris is at eastern elongation. Fig. 71 shows these positions. From the foregoing, it is

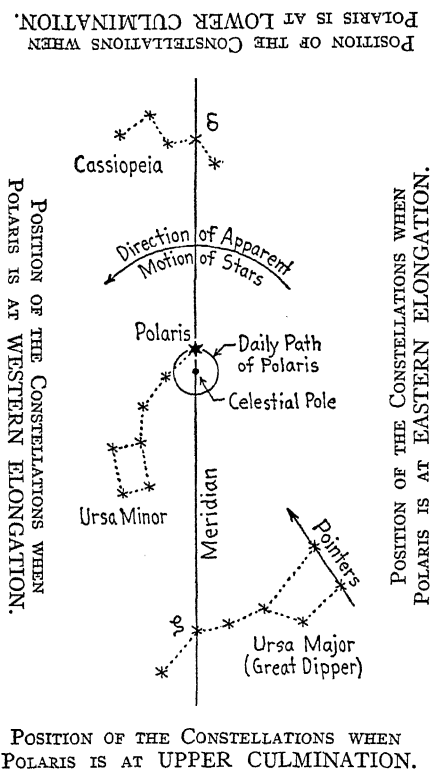


FIG. 71. RELATIVE POSITION OF THE CONSTELLATIONS NEAR THE NORTH POLE.

therefore possible for us to estimate roughly the time when Polaris will reach upper or lower culmination, or eastern or western elongation.*

If an observation is taken on Polaris when it is at western elongation and again about 12 hours later when it is exactly at

* The time of elongation can be found in American Ephemeris and Nautical Almanac, published by Nautical Almanac Office, Navy Dept., and sold by the Superintendent of Documents, Washington, D. C.

eastern elongation, the mean of these two directions is true north. It is only possible to use this method, however, when these elongations come during the night time, in the early morning, or in late afternoon. It is therefore seldom used.

The usual practice is to observe the direction of Polaris at the elongation at which it is visible, and to lay off from that direction an angle to establish the true meridian. The star should be sighted about 15 minutes before the expected time of the elongation. Especial attention should be given to leveling the plate, since any errors of adjustment might seriously affect the result of the observation. The desired result of this observation is to define the line through the star when at its farthest point east or west. Just before reaching elongation its path will appear to be nearly vertical; just at elongation it will appear to run in a vertical line for about 3 minutes, and for 10 minutes either side of elongation it changes about 5" of azimuth.

It is customary to observe Polaris during evening hours, preferably at eastern or western elongation.* Suppose western elongation has been observed and the stake *B* set on line (Fig. 72). Angle *DAB* then represents the true bearing of Polaris at western elongation. Similarly, angle *DAC* represents its true bearing at eastern elongation.

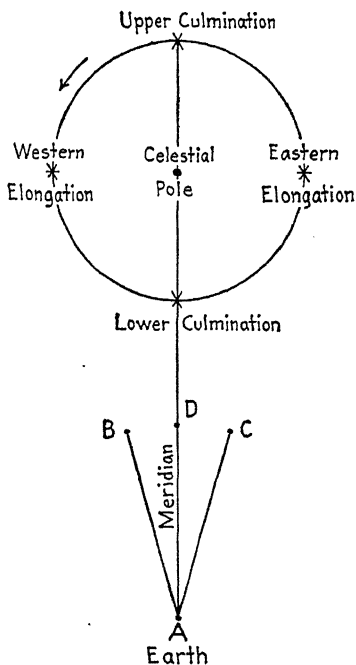


FIG. 72. POLARIS AT ELONGATION.

* In using a transit at night the cross-hairs must be illuminated. This may be done by holding a flashlight slightly to one side of the line of sight and near the objective; this will cast the light into the telescope.

130. Computing True Bearing from Observation on Polaris at Elongation. The true bearing of AB or of AC (Fig. 72) may be found by means of the following formula:

$$\text{Sin Star's True Bearing} = \frac{\text{sin Star's Polar Distance}}{\text{cos Latitude}}$$

The polar distance is the angular distance from the celestial pole to the celestial object; in this case, from the pole to Polaris. It varies slightly from day to day. For Polaris the values given in Table 2 may be used in the formula:

TABLE 2.
MEAN POLAR DISTANCE OF POLARIS

Year	Polar Distance	Year	Polar Distance
	° ' "		° ' "
1940	1 01 16	1945	1 01 16
1941	1 00 58	1946	0 59 28
1942	1 00 40	1947	0 59 10
1943	1 00 22	1948	0 58 52
1944	1 00 04	1949	0 58 34

The latitude of the place of observation may be scaled from a map. Maps to the scale of 1:62500, published by the U. S. Geological Survey, are convenient for this purpose.

EXAMPLE

Compute the true bearing of Polaris at western elongation Mar. 20, 1940. The latitude of the place of observation scaled from a map is $42^{\circ} 22' \text{ N}$. From Table 2 the polar distance is $1^{\circ} 01' 16'' = 1^{\circ} 01'.3$.

$$\log \sin 1^{\circ} 01'.3 = 8.25115$$

$$\log \cos 42^{\circ} 22' = 9.86855$$

$$\log \sin \text{Bearing} = 8.38260$$

$$\text{True Bearing} = 1^{\circ} 23'.0$$

Referring to Fig. 72, $1^{\circ} 23' = \text{angle } BAD$, which is the angle that must be laid off to the right of the line AB to obtain the true meridian AD . In laying off the angle, measure $AB = 500 \text{ ft.}$; then compute the offset $BD = 500 \tan 1^{\circ} 23' = 12.07 \text{ ft.}$ At B and perpendicular to BA , measure 12.07 ft. and set stake D ; then AD is the true meridian.

131. DETERMINING THE TRUE BEARING OR AZIMUTH OF A LINE BY AN OBSERVATION ON THE SUN. The engineer's transit is used in making the observation. It should be provided with a colored glass that can be attached to the eyepiece; this glass will protect the eye against injury when the sun is sighted. The following data are obtained from the field observation:

- (1) The horizontal angle between the traverse line and the sun's center. It is customary to measure this angle from the traverse line to the right.
- (2) The vertical angle to the sun's center, corrected for index error. (See Art. 70.)
- (3) The watch time of the observation.

Sun observations should not be made within 2 hrs. of noon nor when its altitude is less than 10° ; the former, because small errors in altitude and latitude cause large errors in the computed azimuth, and the latter because refraction correction is uncertain.

The actual fieldwork consists in setting up the transit at *A*, one end of the line *AB* the true bearing of which is desired, setting the index of the vernier to read 0° on the circle and sighting *B*. The upper clamp is then loosened, the colored glass placed in position on the eyepiece, and the telescope directed to the sun. Since we wish to obtain the horizontal angle from the traverse line and the vertical angle to the sun's center, and since there is no definite point at the sun's center on which to sight, the vertical and horizontal cross-hairs are brought **tangent** to two of the sun's limbs (lower and left) as at *C*, Fig. 73, i.e., the sun will lie in the upper right-hand quadrant with respect to the cross-hairs. The horizontal and vertical angles, the watch time of the instant of tangency, and the index error are recorded. As quickly as is consistent with good observing the sun's upper and right limbs are observed in the diagonally opposite quadrant of the cross-hairs (lower left), as shown at *D*. The horizontal and vertical angles, the watch time of the instant of tangency, and the index error are recorded. The mean of the readings taken at *C* and *D* gives the horizontal angle from the line to the sun's center, the vertical angle to the sun's center, and the time of the observation for a position, such as at *E*. The vertical

angle should be corrected, of course, for index error. When the pointings on the sun have been completed, the point *B* is again sighted using the upper-plate clamp and tangent screw. The horizontal circle should now read 0° . If it does not, it indicates that the lower plate has moved, in which case the observation should be discarded.

Because of the density of the colored glass neither cross-hairs nor terrestrial objects are visible when looking through the telescope at point *B*. When the telescope is pointed at the sun, only that portion of the cross-hairs directly in line with the sun's

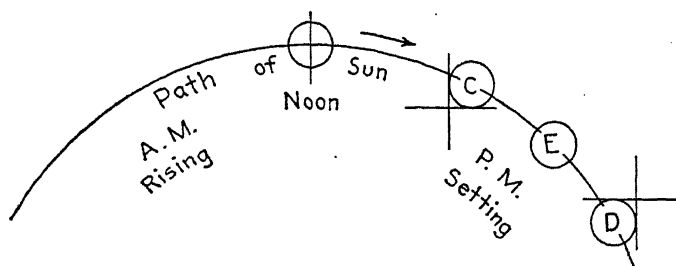


FIG. 73. AFTERNOON OBSERVATION ON SUN.

disc is visible when observed through the colored glass. It is necessary to have the cross-hairs in line with the sun in order to see them, but it is difficult, because the sun moves rapidly, to bring the sun tangent to the cross-hairs, as at *C*, Fig. 73, by turning the upper plate and the vertical motion tangent at the same time. Therefore, the following procedure is recommended, which necessitates the use of only one tangent screw, as explained below.

Fig. 74 illustrates an observation in the upper right quadrant. The arrow indicates the apparent motion of the sun. The cross-hairs are set, about as shown at *M*, i.e., the vertical cross-hair is set to the right of the sun's left limb, and the horizontal wire is kept tangent to its lower limb by continually turning the vertical motion tangent screw. As the sun moves to the right, it will soon become tangent to the vertical hair. The observer, of course, must watch for this instant, as well as keep the horizontal hair continually tangent to the lower limb. When the sun is tangent to both hairs as at *N*, the readings are taken and recorded.

In the diagonally opposite quadrant the horizontal hair is set just below the sun's upper limb, and the vertical hair is kept tangent to the sun's right limb by turning the upper plate tangent screw. Eventually the upper limb will gradually lower and become tangent to the horizontal wire. The simplest way of deciding which diagonally opposite quadrants to use is to sight at the sun, observe the direction in which it is moving, and choose those quadrants that will make it possible to eliminate the use of one of the two tangent screws. If the observation is made in the morning, the upper left and lower right quadrants will be used in the northern hemisphere. If the instrument has a full vertical circle, the telescope should be plunged between observations in the two quadrants.

Instead of making only one observation in each of the two diagonally opposite quadrants, it is good practice to make three consecutive observations in each quadrant, as shown in the following example.

Although it is not necessary to know the time of observation closer than 5 minutes, it is advisable to read the watch to seconds as a check on the intervals between the angle readings.

The mean values at bottom of field-notes are, horizontal angle between line *AB* and sun's center, observed vertical angle to sun's center, and time of observation. After the notes are recorded as above, they should be examined to see whether they are reasonably consistent.

We have assumed the path of the sun to be a straight line, whereas it is actually a curve; however, the error caused by this assumption is negligible if the elapsed time between the first and last observation does not exceed 10 minutes.

The difference in the readings of the horizontal and vertical angles should be proportional to the differences in time; on the field data (Fig. 75) these differences are shown by the small figures. In the second set of observations the differences in the time of the first and second, and the second and third observa-

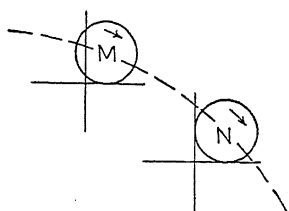


FIG. 74. SUN'S DISC BEFORE AND AT TIME OF AFTERNOON OBSERVATION.

tions, are 67 and 66 seconds; hence it is reasonable to expect that the differences in the corresponding horizontal angles and vertical angles should be equal. We find this to be true. In the

AZIMUTH BY SOLAR OBSERVATION

Inst. at Sta. A. Latitude = $43^{\circ} 00' .8$ N. Date: Sept. 26, 1940.
 Mark. Sta. B. Inst. B & B, No. 1990. Observer M.S. Row.

	Hor. Circle	Ver. Circle	Watch Time, P.M.	Remarks
Mark Sta. B	° ' '' 0 00	° ' '' 0 00	h m s	
$\frac{+}{0}$	245 25 $\frac{1}{2}$	31 36	2 24 41	No Index Corr. All Hor. Angles meas- ured to Right. Temp. 70° F.
$\frac{+}{0}$	245 42 $\frac{1}{2}$ ¹⁷	31 25 ¹¹	2 25 53 ⁷²	
$\frac{+}{0}$	245 54 $\frac{1}{2}$ ¹²	31 17 ⁸	2 26 43 ⁶⁰	
$\frac{0}{+}$	246 53 $\frac{1}{2}$	31 38	2 28 10	No Index Corr.
$\frac{0}{+}$	247 09 16 $\frac{1}{2}$	31 28 ¹⁰	2 29 17 ⁶⁷	
$\frac{0}{+}$	247 24 $\frac{1}{2}$ ^{15$\frac{1}{2}$}	31 18 ¹⁰	2 30 23 ⁶⁶	
Mark Sta. B	° ' '' 0 00	° ' '' 0 00		
Mean	246 24.9	31 27	2 27 31.2	

FIG. 75. FIELD DATA FOR SUN OBSERVATION.

first three observations the differences in time are 72 and 50 seconds. Therefore we would expect the difference between the second and third horizontal angle to be $\frac{50}{72} (17) = 11' .8$, or $12'$, and the difference in the vertical angles to be $\frac{50}{72} (11) = 7' .6$, or $8'$.

132. Computing True Bearing from Observation on Sun. Having satisfied ourselves that the field observations are correct the next step is to compute the azimuth of the sun. One of the many formulas for this purpose is

$$\cos Z_s = \tan h \tan L - \frac{\sin D}{\cos h \cos L}$$

in which

Z_s is the azimuth of the sun from the south. It is toward the east if the observation is made in the morning and toward the west, in the afternoon.

h is the true altitude (vertical angle) of the sun's center, i.e., the observed vertical angle from the horizontal, corrected for index error, refraction, and parallax.

L is the latitude of the place of observation.

D is the declination of the sun's center at the time of observation, i.e., it is the angular distance the sun is north or south of the equator.

The observed vertical angle is not the true altitude (vertical angle) even when corrected for index error. A ray of light coming from a celestial object appears higher than it actually is because of refraction; hence we apply a correction for refraction.* This correction is always subtracted. Another correction applied to the altitude is for parallax.* Parallax is the difference in direction of a celestial body as it would appear from the earth's center, instead of from the earth's surface. The parallax correction is always added and, in the case of observations on the sun, it never amounts to over 9 seconds. The actual values of refraction and parallax for different observed altitudes are given in Table 3. Sometimes the two are in separate tables. In the above example the

$$\begin{aligned}\text{Observed vert. angle} &= 31^{\circ} 27' \\ \text{Refraction and parallax} &= \underline{\quad - 1.4^{\dagger} \quad} \text{ (Table 3).} \\ \text{True altitude sun's center} &= 31^{\circ} 25'.6 = h.\end{aligned}$$

The latitude L in the formula may be scaled from a map, or it may be observed astronomically; the latter observation is not within the scope of this text. The latitude of the place of observation in the foregoing example $= 43^{\circ} 00'.8$ N.

* Tables of the sun's declination, refraction, and parallax, as well as other useful information, are published by the various instrument manufacturers and may be had on application.

The American Ephemeris and Nautical Almanac used chiefly by astronomers, and the American Nautical Almanac used by navigators and surveyors and obtainable from the Superintendent of Documents, Washington, D. C., also give the sun's declination, the latter publication for every 2^h of Greenwich Civil Time throughout the year.

† In this example the refraction correction was $-1'.5$ and the parallax correction was $+0'.1$.

The declination D is the angular distance of the sun north or south of the equator. It may have any value between 0° when on the equator and about $23\frac{1}{2}^\circ$ north of the equator, or between 0° and about $23\frac{1}{2}^\circ$ south of the equator. When *north* of the equator it is marked N or + and when south S or -. Values of the declination* are given for every day in the year in various publications. Since it is continually changing, it is given for the definite instant when the Greenwich Civil Time is 0^h , i.e., midnight. By interpolating in the tables we may find the declination for any instant of time. Naturally in the example given the declination is found for the mean instant of time of the observation, which is $2^h 27^m 31^s$ p.m.

Since the declination is given in the tables when the G.C.T. = 0^h , and since Standard Time is used in the United States, we must find the G.C.T. corresponding to the Standard time of the observation. If the observed time of the observation is *Eastern Standard*, the corresponding G.C.T. is found by adding 5^h if the observation is made in the a.m., and $5^h + 12^h = 17^h$ if the observation is made in the p.m.

Thus:

$$8:29 \text{ a.m. E.S.T.} = 8:29 + 5^h = 13^h 29^m \text{ G.C.T.}$$

$$3:15 \text{ p.m. E.S.T.} = 3:15 + 17^h = 20^h 15^m \text{ G.C.T.}$$

Central Standard time is found by adding 6^h instead of 5^h ; Mountain Standard time by adding 7^h , and Pacific Standard time by adding 8^h .

In our problem the E.S.T. of observation is $2^h 27^m 31^s$ p.m.; the corresponding G.C.T. = $2^h 27^m 31^s + 17^h = 19^h 27^m 31^s = 19^h.46$. Entering the declination table (in the Ephemeris), we find under the date of the observation that the

$$\text{Sun's decl. for G.C.T.} = 0^h \text{ on Sept. 26, 1940} = -1^\circ 05' 32''.$$

We also find the variation in declination per hour to be $-58''.5$. Hence for $19^h.46$ the change in declination is $19.46 \times -58.5 =$

* See footnote on previous page.

TABLE 3*
CORRECTION TO OBSERVED ALTITUDE OF THE SUN FOR
REFRACTION AND PARALLAX

APPT ALT. °	TEMPERATURE										APPT ALT. °
	-10° +14° F.	-5° +23° F.	0° +32° F.	+5° +41° F.	+10° +50° F.	+15° +59° F.	+20° +68° F.	+25° +77° F.	+30° +86° F.	+35° +95° F.	
10	5.52	5.42	5.30	5.20	5.10	5.00	4.92	4.83	4.75	4.67	10
11	5.02	4.92	4.82	4.73	4.63	4.55	4.47	4.38	4.32	4.23	11
12	4.60	4.50	4.42	4.33	4.25	4.17	4.10	4.03	3.97	3.88	12
13	4.23	4.15	4.07	4.00	3.92	3.85	3.78	3.72	3.65	3.58	13
14	3.92	3.83	3.77	3.70	3.62	3.55	3.50	3.45	3.37	3.32	14
15	3.65	3.58	3.50	3.43	3.37	3.32	3.25	3.20	3.13	3.08	15
16	3.43	3.35	3.30	3.23	3.17	3.12	3.07	3.00	2.95	2.90	16
17	3.22	3.15	3.10	3.03	2.98	2.92	2.88	2.82	2.77	2.72	17
18	3.02	2.95	2.90	2.85	2.80	2.75	2.70	2.65	2.60	2.55	18
19	2.83	2.78	2.73	2.68	2.63	2.58	2.53	2.48	2.43	2.40	19
20	2.68	2.63	2.58	2.53	2.48	2.43	2.38	2.33	2.30	2.27	20
21	2.53	2.48	2.43	2.38	2.35	2.30	2.27	2.22	2.17	2.13	21
22	2.38	2.35	2.30	2.25	2.22	2.18	2.13	2.08	2.05	2.02	22
23	2.28	2.25	2.20	2.15	2.12	2.08	2.03	1.98	1.95	1.93	23
24	2.17	2.13	2.08	2.05	2.02	1.98	1.93	1.88	1.87	1.83	24
25	2.07	2.03	1.98	1.95	1.92	1.88	1.83	1.80	1.77	1.75	25
26	1.99	1.95	1.90	1.87	1.83	1.80	1.75	1.72	1.70	1.67	26
27	1.88	1.85	1.82	1.78	1.75	1.72	1.68	1.63	1.62	1.60	27
28	1.80	1.77	1.72	1.70	1.67	1.63	1.60	1.57	1.53	1.52	28
29	1.72	1.68	1.65	1.63	1.60	1.57	1.53	1.50	1.47	1.46	29
30	1.65	1.62	1.58	1.57	1.53	1.50	1.47	1.45	1.42	1.40	30
32	1.53	1.50	1.47	1.45	1.42	1.38	1.35	1.33	1.30	1.28	32
34	1.41	1.37	1.35	1.32	1.30	1.27	1.25	1.23	1.20	1.18	34
36	1.30	1.27	1.25	1.22	1.20	1.18	1.15	1.13	1.10	1.08	36
38	1.20	1.18	1.15	1.13	1.12	1.10	1.07	1.05	1.02	1.02	38
40	1.11	1.10	1.07	1.05	1.03	1.02	0.98	0.97	0.95	0.93	40
42	1.03	1.00	0.98	0.97	0.95	0.93	0.90	0.88	0.87	0.87	42
44	0.96	0.93	0.92	0.90	0.88	0.87	0.85	0.83	0.82	0.80	44
46	0.89	0.88	0.87	0.85	0.83	0.82	0.80	0.78	0.77	0.75	46
48	0.83	0.82	0.80	0.78	0.77	0.75	0.73	0.72	0.70	0.68	48
50	0.77	0.75	0.73	0.72	0.70	0.68	0.67	0.67	0.65	0.63	50
55	0.63	0.62	0.60	0.60	0.58	0.57	0.57	0.55	0.53	0.52	55
60	0.52	0.52	0.50	0.50	0.48	0.47	0.47	0.45	0.45	0.43	60
65	0.42	0.40	0.40	0.40	0.38	0.38	0.37	0.37	0.35	0.33	65
70	0.32	0.32	0.32	0.30	0.30	0.30	0.28	0.28	0.28	0.27	70
75	0.23	0.23	0.23	0.22	0.22	0.22	0.20	0.20	0.20	0.18	75
80	0.15	0.15	0.13	0.13	0.13	0.13	0.13	0.12	0.12	0.12	80
85	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.05	0.05	0.05	85
90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	90

* From Principal Facts of Earth's Magnetism, U. S. Coast and Geodetic Survey.

$-18' 58''$. Hence the sun's declination at the instant of observation $= -1^{\circ} 05' 32'' + (-18' 58'') = -1^{\circ} 24' 30''$.

Substituting in the formula:

$$\cos Z_s = \tan 31^{\circ} 25'.6 \tan 43^{\circ} 00'.8 - \frac{\sin (-1^{\circ} 24'.5)}{\cos 31^{\circ} 25'.6 \cos 43^{\circ} 00'.8}.$$

In using the formula it is important to take the sign of the declination D into account in determining whether number (B) (see Example) should be added to or subtracted from number (A).

A sketch, such as Fig. 76, is helpful in determining how the azimuth of the sun should be applied to the horizontal angle in computing the bearing of the line.

$$\log \tan 31^{\circ} 25'.6 : 9.78607$$

$$\log \tan 43^{\circ} 00'.8 : 9.96986$$

$$\text{Sum} = 9.75593$$

$$\text{Number (A)} = 0.57008$$

$$\text{" (B)} = 0.03939$$

$$\text{Nat. cos } Z_s = 0.60947$$

$$Z_s = 52^{\circ} 26'.9$$

$$\text{Horizontal Angle} = 246^{\circ} 24'.9$$

$$193^{\circ} 58'.0$$

$$\text{Azimuth } AB : 166^{\circ} 02'.0$$

$$\text{Bearing } AB : N 13^{\circ} 58' W$$

$$\log \sin (-1^{\circ} 24'.5) = 8.39053 n$$

$$\log \sec 31^{\circ} 25'.6 = 0.06889$$

$$\log \sec 43^{\circ} 00'.8 = 0.13597$$

$$\text{Sum} = 8.59539 n$$

$$\text{Number (B)} = -0.03939$$

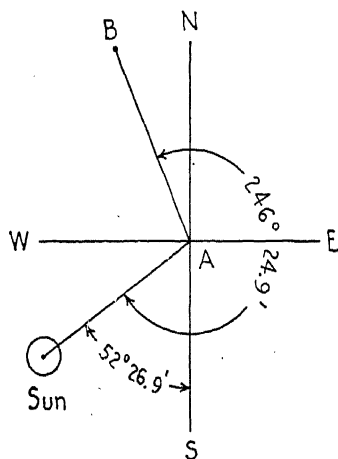


FIG. 76.

FIELD EXERCISES

1. Determine from the Ephemeris the times of eastern and western elongation of Polaris on a particular day. Select the time when it is convenient to observe the star at elongation. Set up over point *A* about 15 minutes before elongation, and level the instrument. Identify Polaris in the sky and sight on the star with the cross-hairs illuminated. Lower the telescope and set a stake *B* in line about 300 ft. away but do not mark it. About 3 minutes before the time of elongation bisect the star, clamp the instrument and lower the telescope to stake *B*, and mark a point on line. Reverse the telescope, bisect the star, lower the telescope, and mark another point in line on stake *B*. If the two marks on *B* coincide, place a tack in the stake at the mark; if the marks do not coincide, place the tack half-way between them. Repeat the process one or more times until 3 minutes have elapsed after greatest elongation, and use the mean of the position determined for point *B*.

Using the method of Art. 130, compute the true bearing of *AB*. Also compute the offset and its direction (E or W) from *B* to set a point *B'* that will be on the meridian through *A*. Set point *B'* by offset and check the angle *BAB'* by repetition.

2. To determine the bearing of a line *AB* from an observation on the sun, select a time of day when the altitude of the sun is between 20 and 40 degrees. Set up at *A*, level the instrument, set 0° on the plate, and sight *B* using the lower motion. Loosen upper clamp and turn toward the sun. Before sighting on the sun be sure that the eyepiece is covered with the colored glass provided for this purpose. Decide in which pair of quadrants the sun should be observed and the method in which the instrument should be manipulated to obtain tangency of cross-hairs with limbs of the sun. (See Art. 131.) Sight the sun, taking three sets of readings in one quadrant and three sets in the diagonally opposite quadrant. If instrument has a full vertical circle, reverse telescope between sets of readings. Each set of readings includes horizontal angle, vertical angle, index error, and watch time. (See Fig. 75.) Check the differences in the angle readings against time intervals to see that the readings are consistent. If an error is indicated the entire observation must be repeated.

Following the method given in Art. 132, compute the azimuth of the sun, using the average of the field observations for the sun's altitude. Find the sun's declination from the Ephemeris, and scale the latitude from U. S. Geological Survey map (scale $\frac{1}{62500}$). From the azimuth of the sun and the average horizontal angle, compute the bearing of the line *AB*.

PROBLEMS

1. Compute the true bearing of Polaris at eastern elongation on Aug. 5, 1941 at a place where latitude is $44^\circ 46'.8$ N.

2. With the instrument at *A* sighting $0^\circ 00'$ on *B* an observation was made on Polaris at eastern elongation. Date: May 27, 1941, Hor. angle = $101^\circ 15'.5$ (to right), Latitude $42^\circ 28'.9$ N. Find bearing of line *AB*.

3. Compute the azimuth of the line *AB* from the following observations made on the sun on April 18, 1941. Instrument is at *A*. No index correction. Hor. angle measured to the right. Temperature at time of observations 68° F.

PT. SIGHTED	HOR. ANG.	VER. ANG.	WATCH (E.S.T.)
Point B	0° 00'		
Left and	193 11	32° 55'	3 ^h 21 ^m 40 ^s P.M.
Lower Limbs	193 26	32 44	3 22 55
	193 40	32 33	3 23 56

Telescope reversed

Upper and	194 38	32 45	3 25 32
Right Limbs	194 50	32 36	3 26 32
	194 58	32 30	3 27 15

Latitude $42^{\circ} 21'.0$ N. Declination at 0^h Greenwich Civil Time = $+10^{\circ} 36'.6$ and the variation in declination per hour = $+52''.6$.

4. The azimuth angle from true south to the sun is computed to be $98^{\circ} 27'.6$ (A.M. observation). The average horizontal angle from the mark to the sun (clockwise) is $170^{\circ} 15'.5$. What is the bearing of the mark from the instrument?

5. Compute the bearing of line AB from the following observations on the sun made from Sta. A on August 22, 1941.

PT. SIGHTED	HOR. ANG. (To right)	VER. ANG.	WATCH (E.S.T.)
Sta. B	0° 00'		
Right and	76 41	36° 44'	8 ^h 17 ^m 03 ^s A.M.
Lower Limbs	76 21	36 58	8 18 30

Instrument reversed

Left and	76 44	37 43	8 19 42
Upper Limbs	76 31	37 53	8 20 41

Latitude $44^{\circ} 46'.8$ N. Declination at 0^h Greenwich Civil Time = $+11^{\circ} 59'.8$, and the hourly variation in declination = $-50''.1$. Index correction zero. Temperature 77° F.

CHAPTER VII

CALCULATION OF CLOSED TRAVERSES AND LAND AREAS

133. CHECKING ANGLES OF A TRAVERSE. A closed traverse is a polygon the angles and sides of which have been measured.

Before leaving the field a geometric check of the angles should be obtained. If the interior angles have been measured, their sum should equal $180^\circ (n - 2)$, in which n is the number of sides in the traverse. If the deflection angles have been measured, the difference in the sums of the right and left deflection angles should equal 360° . If the survey were based on azimuths, and if the azimuths were carried around the traverse and back to the first line, the two azimuths of the first line should check. Exact checks cannot be expected but any discrepancies should be small; in general, not over an amount equal to $\sqrt{\text{No. of angles} \times \text{allowable error per angle}}$. If the error is considerably greater than this, it indicates a mistake in one or more angles.

134. Balancing Angles. Any discrepancy between the actual and theoretical sum of the angles should be distributed among the angles so that they will add to the correct amount. Fig. 77 is a sketch of the closed traverse shown in Fig. 67, and Fig. 78 shows the form and calculations for computing the area by the D.M.D. method, which will be explained in detail in the following articles.

In this survey deflection angles were measured. The difference in the sums of the right and left deflection angles is $359^\circ 59' 00''$. The error of one minute is distributed among the angles. This is not done by dividing the error by the number of angles and applying the quotient to each angle. If this were done, each angle would have a correction of $01' \div 5 = 12''$, and when applied the angles would appear to have been measured to 1 second when they were actually measured to 30 seconds.

In making the survey there may have been good reason to believe that some of the angles were measured more accurately than

others. Some sights may have been on a pencil point held on the station, whereas others may have been measured to the top of a lining-pole held on the station. The latter sights probably were not so accurate as the former, since the pole may have been slightly out of plumb. Perhaps some sights were difficult to see. In such surveys the error should be distributed among the angles that were likely to have been the poorly measured ones. Instead of placing the whole error on one angle, it is better practice, in our example, to divide the error into units corresponding to the

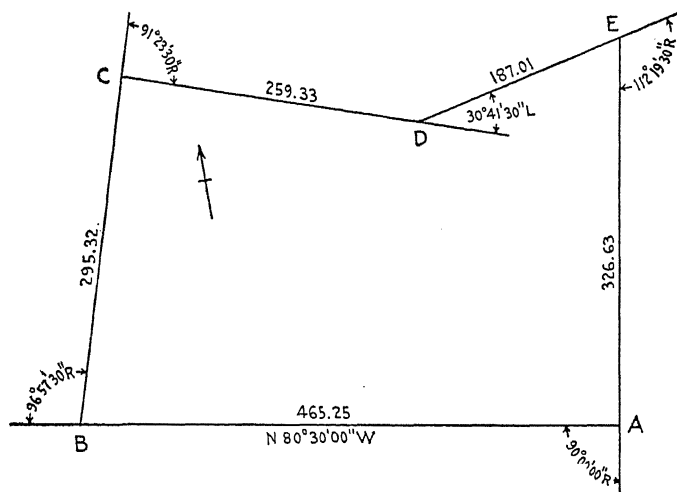


FIG. 77. CLOSED TRAVERSE.

smallest unit in the measured angles, i.e., 30 seconds, and correct two of the angles by 30 seconds, thus assigning the $01'$ error to two angles.

When the angles are measured with equal precision, i.e., when all angles are measured equally well, the error is divided as stated and the corrections are made on those angles at the ends of the shortest side. In Fig. 77, the shortest line is DE ; hence both angles D and E were arbitrarily corrected by 30 seconds. Since one of these angles is a right and the other a left deflection, 30 seconds were added to the angle E and 30 seconds subtracted

Computation of Area of Closed Traverse
Survey of Land Belonging to Richard Jones

Computed by _____
Checked by _____
Date _____

Sta.	Deflection Angles	Bearings	Dist.	Latitudes		Departures		Balanced		D.M.D.	Double Areas	
				N(+)	S(-)	E(+)	W(-)	Lat.	Dep.		+	-
A	90°00'00"R	N80°30'00"W	465.25	76.79	—	—	458.87	+76.80	-458.90	+458.90	35 243	—
B	96 57 30 R	N16 27 30 E	295.32	283.22	—	83 67	—	+283.25	+83.66	+83.66	23 697	—
C	91 23 30 R	S72 09 00 E	259.33	—	79.49	246.84	—	-79.48	+246.83	+414.15	—	32 917
D	30 41 20 L	N77 10 00 E	187.01	41.54	—	182.33	—	+41.54	+182.32	+843.30	35 031	—
E	112 19 30 R	S 9 30 00 W	326.63	—	322.15	—	53.91	-322.11	-53.91	+971.71	—	313 000

Sum R Defl. = 390°40'30" 1533.54 401.55 401.64 512.84 512.78 93 971 345 917
 " L " = 30 41 30 .55 .78 93 971
 Error = 359°59'00" .09 .06 2 251 946
 Diff. in " = 01'00" Error of closure = $\sqrt{(.09)^2 + (.06)^2} = 0.11$ Area 125 973 sf
 Adjust -30° at D, +30° at E Precision = $\frac{0.11}{1533.} = \frac{1}{13 900}$

Computation of Bearings

$$A-B = N80^{\circ}30'00"W$$

$$-96 57 30$$

$$B-C = N16 27 30 E$$

$$+91 23 30$$

$$C-D = S 72 09 00 E$$

$$+30 41 00$$

$$D-E = N 77 10 00 E$$

$$+112 20 00$$

$$E-A = S 9 30 00 W$$

$$+90 00 00$$

$$99 30 00$$

$$A-B = N 80 30 00 W \text{ Check}$$

Corr. to Lats.

$$\frac{.09}{803.19} = \frac{.01}{89}$$

$$A \quad .01$$

$$B \quad .03$$

$$C \quad .01$$

$$D \quad .00$$

$$E \quad .04$$

$$.09$$

Corr. to Deps.

$$\frac{.06}{1025.62} = \frac{.01}{171}$$

$$A \quad .03$$

$$B \quad .01$$

$$C \quad .01$$

$$D \quad .01$$

$$E \quad .00$$

$$.06$$

Computation of D.M.D.'s

$$B + 83.66$$

$$+ 83.66$$

$$+ 246.83$$

$$C \quad 414.15$$

$$+ 246.83$$

$$+ 182.32$$

$$D \quad 843.30$$

$$+ 182.32$$

$$- 53.91$$

$$E \quad 971.71$$

$$- 53.91$$

$$- 458.90$$

$$A + 458.90 \text{ Check}$$

Computation of Latitudes and Departures

	A - B	B - C	C - D	D - E	E - A
Latitude	76.79	283.22	79.49	41.54	322.15
Log Lat	1.88530	2.45212	1.90032	1.61845	2.50806
Log Cos Bear.	9.21761	9.98183	9.48647	9.34659	9.99400
Log Dist.	2.66769	2.47029	2.41385	2.27186	2.51406
Log Sin Bear.	9.99400	9.45228	9.97857	9.98901	9.21761
Log Dep.	2.66169	1.92257	2.39242	2.26087	1.73167
Departure	458.87	83.67	246.84	182.33	53.91

Computation of Double Areas

Log D.M.D.	2.66172	1.92252	2.61716	2.92598	2.98754
Log Lat.	1.88536	2.45217	1.90026	1.61847	2.50800
Log Double Area	4.54708	4.37469	4.51742	4.54445	5.49554
Double Area	35 243	23 697	32 917	35 031	313 000

FIG. 78. COMPUTATION OF CLOSED TRAVERSE BY D.M.D. METHOD.

from the angle D , as indicated in the second column of Fig. 78. With these corrections made, the difference in the sums of the right and the sums of the left deflection angles is 360° .

135. Computing Bearings. If it is desired to make a plan and show the direction of the cardinal points, the bearing of one side of the traverse must be **known**, and the rest can be calculated by using the measured angles. When only the area is required, the bearing of any side may be **assumed**, and the rest calculated. Since we desire a plan in addition to the area, the true bearing of the side AB was determined from coordinates starting with the coordinate of stations 75 and 76 in Fig. 84. It could have been

determined by astronomical observation.

The bearings of the other sides were computed from the known bearing of AB and the **corrected** deflection angles as shown in Fig. 78. Note that when the bearing of EA has been computed, we apply the deflection angle at A and compute the bearing of AB . This computed bearing of AB should check the original bearing of AB if no mistake has been made.

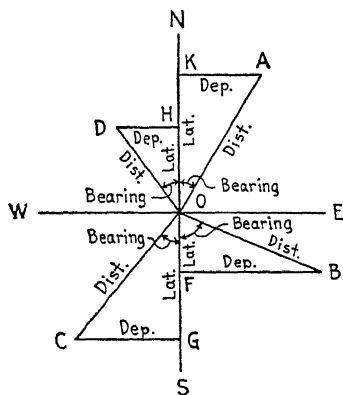


FIG. 79. LATITUDES AND DEPARTURES.

136. Latitudes and Departures. After the bearings have been computed, the *latitudes* and *departures* are computed. The latitude of any line is its projection on a north and south line, and the departure of the line is its projection on an east and west line. In Fig. 79 the latitude of OA is OK , of OB is OF , of OC is OG , and of OD is OH . The departure of OA is KA , of OB is FB , of OC is GC , and of OD is HD . North latitudes are designated N or $+$, south latitudes S or $-$, east departures E or $+$, and west departures W or $-$.

In Fig. 79 it is readily seen that the latitudes and departures may be computed from the bearing and length of a line by solving

a right triangle by trigonometry, i.e.,

$$\text{Lat.} = \text{Dist.} \times \cos \text{Bearing}$$

$$\text{Dep.} = \text{Dist.} \times \sin \text{Bearing}$$

For the line AB in Fig. 78

$$\text{Lat.} = 465.25 \cos 80^\circ 30' 00'' = 76.79$$

$$\text{Dep.} = 465.25 \sin 80^\circ 30' 00'' = 458.87$$

Since the bearing of the first line AB is $N 80^\circ 30' 00'' W$, 76.79 is placed in the north column and 458.87 in the west column, as

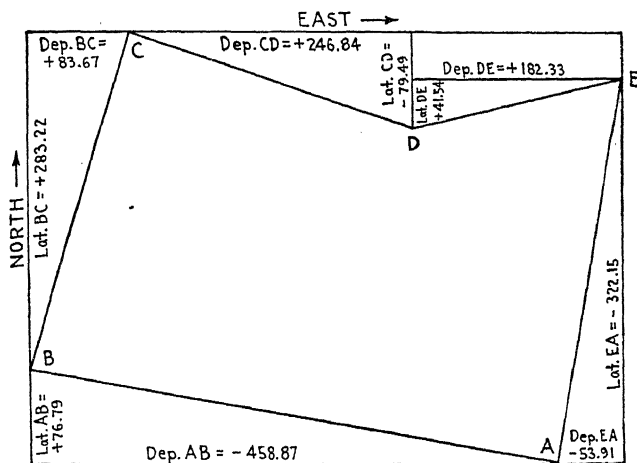


FIG. 80. LATs. AND DEPS. OF CLOSED TRAVERSE.

shown. In Fig. 78 the latitudes and departures have been computed for all lines and recorded in the proper column.

The values in the latitude and departure columns are then added and the sums put at the foot of the columns. Note that the north column values add to 401.55, the south column to 401.64, and that the values in the east column add to 512.84 and the west column to 512.78, i.e., the north and south column values differ by 0.09 ft., and the east and west values differ by 0.06 ft. If there were no errors in the angles or the distances of the survey, the sum of the north latitudes would equal the sum of the south latitudes, and the sum of the east departures would equal the sum

of the west departures. This is evident from an examination of Fig. 80, which is the same traverse as in Fig. 78, enclosed in a rectangle consisting of north and south, and east and west lines shown through the most northerly, southerly, easterly, and westerly points of the polygon.

The differences, 0.09 and 0.06, of latitudes and departures respectively, indicate some discrepancies in the measurements and show that if we started at *A* and could plot the traverse theoretically and accurately, the final point would not fall at *A* as it should, but would fall at some point, such as *M* in Fig. 81.

137. Error of Closure. The distance by which *M* fails to meet *A*, or *AM* in Fig. 81, is called the *error of closure*.

$$AM = \sqrt{(0.09)^2 + (0.06)^2} = 0.11 \text{ ft.}$$

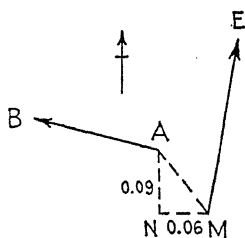


FIG. 81.
ERROR OF CLOSURE.

Comparing the error of closure with the perimeter of the survey will indicate the precision with which the measurements have been made. The perimeter (sum of the lengths of the sides) is 1533.54 ft. The precision is $\frac{0.11}{1533.54}$.

The usual way to express the precision is to make the numerator unity and change the denominator correspondingly.

$$\text{Thus } \frac{0.11}{1533.54} = \frac{1}{13900}, \text{ which means}$$

that for every 13,900 ft. measured the error is 1 ft.

In practice it is customary to specify the precision to be attained for different classes of surveys. When the precision is not specified, the surveyor should know from experience the precision required in the various classes, and how to carry on the survey so that the results will lie within those limits.

In discussing the "error of closure" and precision, there is one most important thing to keep in mind. Precision as used above means simply that the measurements are consistent but not necessarily accurate. Suppose that a survey for determining the area of a closed traverse is made with a tape exactly 100 ft. long and that the precision obtained is $\frac{1}{10000}$. Now assume

that a resurvey is made with a tape that has stretched uniformly one foot throughout its length, and that the computed precision is still $\frac{1}{10000}$. Notwithstanding that both precisions are the same, the area obtained when the first tape was used would be within reason, i.e., exact, whereas the area obtained from measurements made with the second tape would be considerably in error.

138. Balancing the Latitudes and Departures. Before any quantities such as area, final dimensions of the sides, or coordinates of the corners of a closed survey are calculated, the errors must be distributed so that a mathematically closed polygon is obtained. In calculating the latitudes and departures, we have seen that in a perfect polygon the algebraic sum of the Lats. should equal zero and the algebraic sum of the Deps. should equal zero. In Fig. 78 we found a difference in the Lats. of 0.09 ft., and in the Deps., of 0.06 ft. These differences are now distributed among the Lats. and the Deps. in such a manner as to make the algebraic sums zero.

There are several rules for distributing the discrepancies but only the two methods commonly used will be explained. Good tapemen appreciate that in measuring the lines of a traverse some lines are far more difficult to measure than others and that the accuracy of the measurements will vary. It is good practice to state in the note-book at the time the field measurements are made which of the lines are the least accurately measured.

139. The first method would then be to distribute the discrepancies among the Lats. and Deps. of the poorly measured lines, preferably **reducing the values in the columns the sums of which are the larger** wherever possible, because measured distances are liable to be too long rather than too short. In Fig. 78, if the field-notes indicated that lines *CD*, *DE* and *EA* were less accurately measured than *AB* and *BC*, we would distribute the error in latitude among the latitudes as follows: subtract 0.03 ft. from the latitude of *CD*, add 0.02 ft. to the latitude of *DE*, and subtract 0.04 ft. from the latitude of *EA*. If these values were applied to the values given in the columns, the algebraic sum of the latitudes would be zero. The error of 0.06 ft. would similarly be distributed among the departures. (Note that this method was **not** used in Fig. 78.)

140. The second method is based on the assumption that all lines are equally well measured. When this is the case, the errors are distributed among the latitudes and departures in accordance with the following rule:

The correction to be applied to the $\left\{ \begin{smallmatrix} \text{latitude} \\ \text{departure} \end{smallmatrix} \right\}$ of any course is to the total error in $\left\{ \begin{smallmatrix} \text{latitude} \\ \text{departure} \end{smallmatrix} \right\}$ as the $\left\{ \begin{smallmatrix} \text{latitude} \\ \text{departure} \end{smallmatrix} \right\}$ of that course is to the sum of all the $\left\{ \begin{smallmatrix} \text{latitudes} \\ \text{departures} \end{smallmatrix} \right\}$ without regard to the algebraic sign.

The foregoing rule means that the errors are distributed in proportion to the Lats. and Deps. For example, the correction to the latitude of the line AB would be

$$\text{Correction} = 0.09 \left(\frac{76.79}{803.19} \right) = 0.008 \quad 0.01$$

and the correction to the departure of the line AB would be

$$\text{Correction} = 0.06 \left(\frac{458.87}{1025.62} \right) = 0.027 \quad 0.03$$

All the other corrections would be computed in the same manner. It will be noted that the linear measurements of the traverse were taken to the nearest 0.01 ft., and the Lats. and Deps. were computed to the nearest 0.01 ft. Hence the corrections should be computed to the nearest 0.01 ft.

A simpler method of computing the corrections would be to find the distance in latitude and departure corresponding to 0.01 ft. and compute the corrections mentally. Thus if we wish to compute the corrections for the latitudes, we note that the error

is 0.09 ft. in a distance of 803.19 ft. or $\frac{0.09}{803.19} \quad \frac{0.01}{89.23}$ This means that for every 89.23 ft. of latitude the correction is 0.01 ft.

When the corrections have been computed, they are applied to the corresponding values, as in Fig. 78. Always remember to **add them to the small column** and **subtract them from the large column**. These corrections when applied will give the values in the columns headed "Balanced Latitudes" and "Balanced De-

partures." Note the + or - sign is also placed in these columns.

The foregoing calculations are utilized in computing the area by either the Double Meridian Distance Method or by the Coordinate Method.

141. COMPUTING AREA. The area of a closed traverse can be computed by dividing the polygon into triangles by drawing radial lines from one corner to all the others and computing the area of each triangle. That method is sometimes used, but it takes a longer time to make the computations and does not admit of desirable checks as do the methods to be explained.

There are two methods in common use for obtaining the area of a closed traverse: (1) Double Meridian Distance Method; (2) Coordinate Method.

142. Computing the Double Meridian Distances. If a meridian is drawn through any corner of a closed traverse and perpendiculars are dropped from the middle point of each line of the survey to that meridian line, these perpendicular distances are called *Meridian Distances*. Twice the length of any such line is called its *Double Meridian Distance, D.M.D.* In calculating the D.M.Ds. it is convenient to take the meridian through the most *westerly* point of the survey. In Fig. 82, which is the same traverse as in Figs. 77, 78 and 80, this point is *B* and the meridian is *MM'*. If *m, n, j, k* and *l* are the center points of the sides of the traverse, the distances *mh, nf, jc, kd* and *lg* are the meridian distances, and twice those values are the double meridian distances. Our problem is now to compute the values of the D.M.Ds. With reference to the meridian *MM'* the first distance, or course, is *BC*. Since $2nf = aC$, the D.M.D. of the first course equals the departure of the first course. To obtain $2jc$, the D.M.D. of the next course *CD*, note that $2jc = 2fn + 2nC + 2Ct = 2fn + aC + Cs$. Translated into words, this says that the D.M.D. of *CD* is equal to the D.M.D. of the previous course + the departure of the previous course + the departure of the course itself.

The D.M.D. of the next course *DE* is

$$2kd = 2cj + 2ts + 2sw = 2cj + Cs + sx,$$

which is in accordance with the above statement. The D.M.Ds. of *EA* and *AB* are similarly computed. The D.M.D. of the final

course is equal to the Dep. of the final course with opposite sign.
The D.M.D. of EA is

$$2lg = 2dk + 2wx - 2zl = 2dk + sx - qA,$$

and the D.M.D. of the last course AB is

$$2mh = (2gl - 2yA) - 2Ar = 2Ai - Ai = Ai.$$

From the foregoing it will be seen that the calculation of the D.M.Ds. follows definite rules and that the signs must be taken into account. The rules are:

- (1) The D.M.D. of the first course (starting from the reference meridian) equals the departure of the first course.
- (2) The D.M.D. of any course equals the D.M.D. of the preceding course plus the departure of the preceding course plus the departure of the course itself.
- (3) The D.M.D. of the last course equals the departure of the last course but with the opposite sign.

In Fig. 78 the D.M.Ds. have been computed with reference to the meridian through B . Notice that the D.M.D. values are all positive. Had the reference meridian been taken through the most easterly point E , all would have been negative; had it been taken through any point other than the most easterly or westerly, some of the D.M.Ds. would have been positive and some negative.

143. COMPUTING THE AREA BY THE DOUBLE MERIDIAN DISTANCE METHOD. It will be observed that the area of the traverse may be obtained by combining the areas of trapezoids and triangles. Referring to Fig. 82 we note that the

$$\text{Area } ABCDE = bEai - bEDe + aCDe - aCB - BAi$$

$$\begin{aligned} \text{But area } bEai &= bi \times gl; & bEDe &= be \times dk; \\ aCDe &= ae \times cj; & aCB &= aB \times fn; \\ BAi &= Bi \times hm; \end{aligned}$$

in which ae , be , bi , aB and Bi are the balanced latitudes and cj , dk , gl , fn and hm , the meridian distances. Hence if we multiply the balanced latitudes by the corresponding D.M.Ds. we obtain the double areas of the trapezoids and triangles that make up the area of the traverse $ABCDE$. Some of the double areas will be positive and some negative according to their signs. The prod-

ucts are placed in the proper columns in the computation form. The difference in the sums of the + double areas and - double areas divided by 2 is the area of the closed traverse.

It sometimes happens that the sum of the negative double areas is greater than the sum of the positive double areas. The dif-

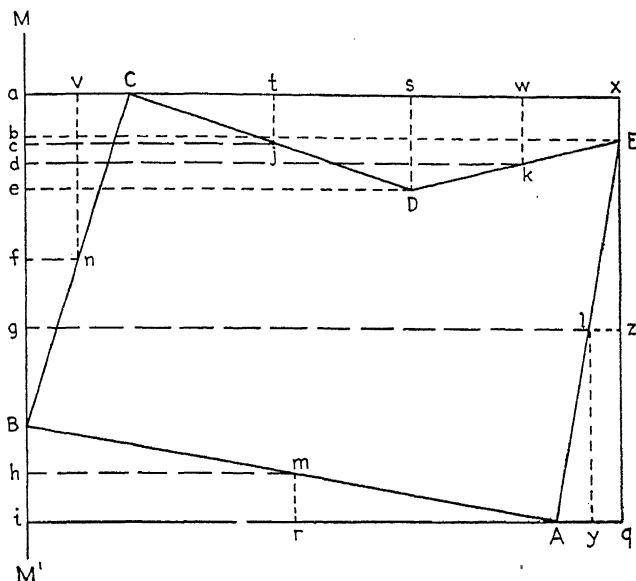


FIG. 82. DOUBLE MERIDIAN DISTANCES.

ference of the sums divided by 2 is the final area. The negative sign has no significance in the final result for area.

The computation of the area in Fig. 82 is shown in Fig. 78.

144. CALCULATING MACHINES. The computation of traverses and their areas is usually made in the office, where a calculating machine is available. In this case the natural function of the bearing is set on the machine and multiplied by the several separate digits in the number representing the distance. This avoids all use of logarithms and shortens the time required to a minimum.

145. RECTANGULAR COORDINATES. These have been adopted in many of the larger cities; recently it has also been adopted

by some of the states (Arts. 127-8). For this system permanent points have been established and their X and Y coordinates have been determined with reference to two lines at right angles to each other. These lines, called the X and Y axes, are usually a meridian and a line perpendicular to it. Fig. 83 shows two rec-

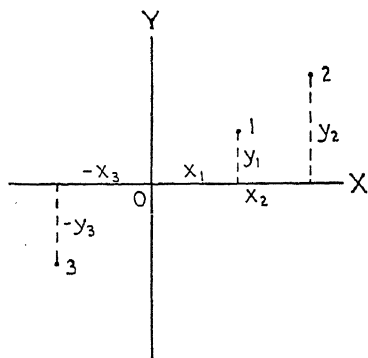


FIG. 83. COORDINATES.

tangular axes intersecting at O , called the *origin*. The meridian is the Y axis, and the east and west line the X axis. The coordinates of any point, such as 1, are the distances x_1 and y_1 ; of point 2, x_2 and y_2 ; and of point 3, $-x_3$ and $-y_3$. All x coordinates to the right (east) of the Y axis are considered **plus** and to the left (west), **minus**. All y coordinates above (north) the X axis are **plus**, and those

below (south), **minus**. Sometimes the axes are so chosen as to position that all the coordinates will have positive values.

146. JOINING A SURVEY TO A COORDINATE SYSTEM. In making a property survey based on a coordinate system, the surveyor would start at the nearest established coordinate point and run a traverse from that point to a corner of the property. He would then make a survey of the property, as previously described. The coordinates of the starting point being known, the coordinates of all the corners of the property can be computed from the survey, and with these values the area can be calculated.

Fig. 84 illustrates the procedure of tying in the survey $ABCDE$ to a coordinate system. Points 75 and 76 are two intervisible permanent points, the coordinates of which (referred to O and the X and Y axes) have been determined. In running a traverse from these points to A , the position of M is so chosen that 76 and A may be seen from it. The transit is set at 76, a backsight taken on 75, and an angle to the right and distance to M are measured. The transit is then set up at M , and an angle to the right and distance to A are measured. The next point occupied

is A . With a backsight on M , an angle to the right to B and distance AB are measured. The coordinates of 75 and 76, and the angles and distances measured at 76, M and A are given on the sketch.

147. COMPUTATION OF COORDINATES OF THE SURVEY. First compute the bearing of the line 76-75. The difference of the Y coordinates of these two points is the latitude of the line

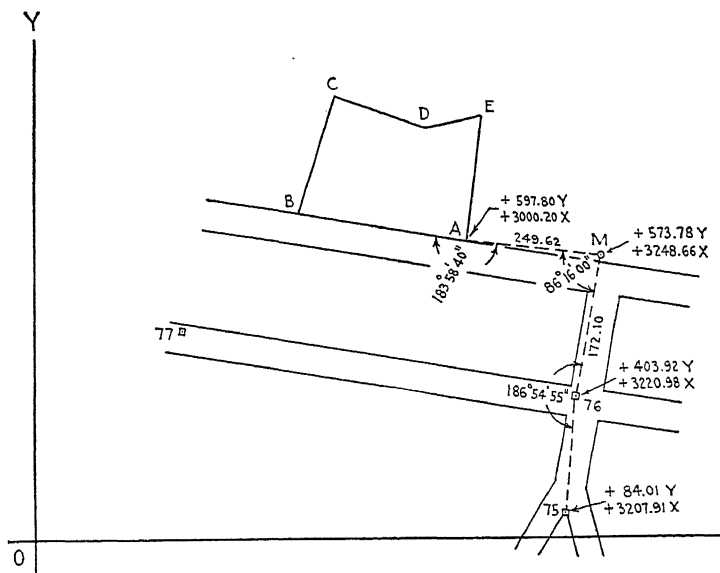


FIG. 84. USE OF RECTANGULAR COORDINATES IN CLOSED TRAVERSE.

76-75, and the difference of the X coordinates is the departure of the line.

When the latitude and departure of a line are known, its bearing may be computed. From Fig. 79,

$$\tan \text{Bearing} = \frac{\text{Dep.}}{\text{Lat.}}$$

$$\text{Dist.} = \frac{\text{Lat.}}{\cos \text{Bearing}} = \sqrt{(\text{Lat.})^2 + (\text{Dep.})^2}$$

$$\sin \text{Bearing} = \frac{\text{Dep.}}{\text{Dist.}}$$

In Fig. 84,

Coordinates

Sta.	Y	X	
76	403.92 N	3220.98 E	$\log 13.07 = 1.11628$
75	84.01 N	3207.91 E	$\log 319.91 = 2.50502$
Diff.	Lat. = 319.91	Dep. = 13.07	$\log \tan \text{Bear.} = 8.61126$
$\therefore \tan \text{Bear. } 76-75 = \frac{13.07}{319.91}$			Bear. = S $2^{\circ} 20' 25''$ W

Sta.	Angles to Right	Bearings	Dist.	Latitudes		Departures		Coordinates	
				N +	S -	E +	W -	Y	X
75	0 1 "	N $2^{\circ} 20' 25''$ E							
76	136 54 55	N $9^{\circ} 15' 20''$ E	172.10	169.86		27.68		+ 403.92	+ 3220.98
M	86 16 00	N $84^{\circ} 28' 40''$ W	249.62	24.02			248.46	+ 573.78	+ 3248.66
A	183 58 40	N $80^{\circ} 30' 00''$ W						+ 597.80	+ 3000.20

Computation of Coordinates

Sta.	X	
76	+ 403.92	+ 3220.98
	+ 169.86	+ 27.68
M	+ 573.78	+ 3248.66
	+ 24.02	- 248.46
	+ 597.80	+ 3000.20

FIG. 85. COMPUTATION OF COORDINATES.

Since the Y coordinate of 75 is less than the Y coordinate of 76, point 75 is south of 76; and since the X coordinate of 76 is greater than the X coordinate of 75, point 75 is west of 76. Hence the bearing of 76-75 is S $2^{\circ} 20' 25''$ W, and the bearing of 75-76 is N $2^{\circ} 20' 25''$ E.

The distance 76-75 is not required, but if desired it may be computed by one of the foregoing formulas.

Next compute, (1) Bearings of 76-M, MA, and AB; (2) Latitudes and Departures of the lines 76-M and MA; (3) Coordinates of M and A. These computations may be put in the form shown in Fig. 85.

It should be kept in mind that the linear traverse from 75 to A is not checked, and any error in it will cause an error in the coordinates of A and in all of the other corners of the closed traverse. Traverse 75 to A can be checked by re-running it or by continuing the traverse from A to some other coordinate station, such as 77.

148. AREA BY THE METHOD OF COORDINATES. For the principle of computing the area of a closed traverse when the coordinates of the stations (angle points) are known, reference is made to Fig. 86, in which X_A and Y_A are the coordinates of Sta. A; X_B and Y_B of Sta. B, etc.

The area of $ABCDE$ is made up of a series of trapezoids, i.e.,

$$\begin{aligned}\text{Area } ABCDE &= \text{trapezoids } A_{52}E + D_{31}C \\ &\quad - D_{32}E - A_{54}B - B_{41}C.\end{aligned}$$

The areas of the trapezoids in terms of their coordinates are

$$\begin{aligned}&\frac{1}{2}(Y_E - Y_A)(X_E + X_A) + \frac{1}{2}(Y_C - Y_D)(X_C + X_D) \\ &\quad - \frac{1}{2}(Y_E - Y_D)(X_E + X_D) \\ &\quad - \frac{1}{2}(Y_B - Y_A)(X_B + X_A) \\ &\quad - \frac{1}{2}(Y_C - Y_B)(X_C + X_B),\end{aligned}$$

or

$$\begin{aligned}&\frac{1}{2}(X_E Y_E - X_E Y_A + X_A Y_E - X_A Y_A + X_C Y_C \\ &\quad - X_C Y_D + X_D Y_C - X_D Y_D \\ &\quad - X_E Y_E + X_E Y_D - X_D Y_E + X_D Y_D \\ &\quad - X_B Y_B + X_B Y_A - X_A Y_B + X_A Y_A \\ &\quad - X_C Y_C + X_C Y_B - X_B Y_C + X_B Y_B).\end{aligned}$$

Cancelling the positive and negative like terms and rearranging, the equation becomes

$$\begin{aligned}\text{Area } ABCDE &= \frac{1}{2}[Y_A(X_B - X_E) + Y_B(X_C - X_A) \\ &\quad + Y_C(X_D - X_B) + Y_D(X_E - X_C) \\ &\quad + Y_E(X_A - X_D)].\end{aligned}$$

From the foregoing equation the following rules are obtained for computing the area of a closed traverse from the coordinates of its corners:

- (1) Letter or number the corners consecutively around the traverse;
- (2) Multiply each Y coordinate by the difference between the following and the preceding X coordinates, always subtracting the preceding from the following, or vice versa, and take half the sum of the products.

Fig. 87 shows the calculation of the area shown in Fig. 77 by the method of coordinates. Fig. 78 shows the calculation of the

same area by the D.M.D. method. The calculations in Fig. 87 are the same as those in the D.M.D. method up to and including the balanced latitudes and departures. After computing the balanced latitudes and departures, the coordinates are calculated. The computed coordinates of *A* as found in Art. 147 are entered in the form opposite *A*. Starting with these co-

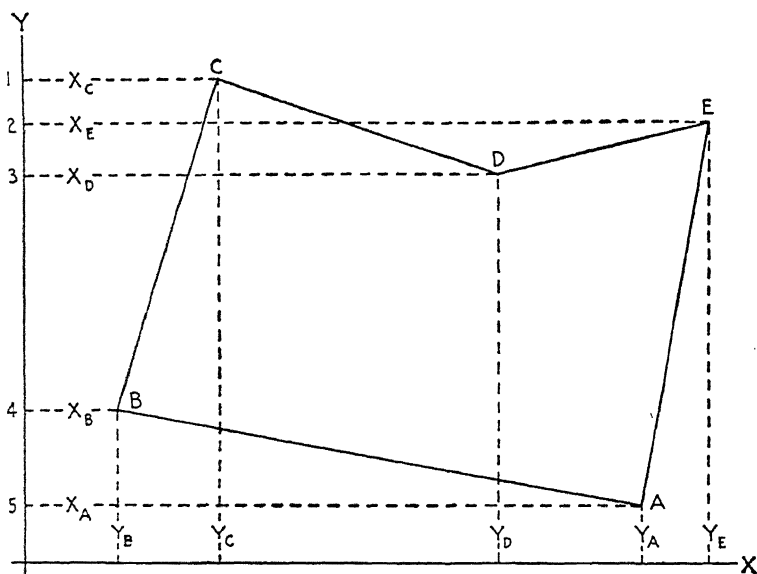


FIG. 86. AREA BY COORDINATES.

ordinates and applying the balanced latitudes and departures as shown, the *Y* and *X* coordinates of all the other corners are calculated. The values in the column headed "Difference of Adjacent *X*'s" are computed by rule (2). Thus $Y_B = 674.60$ is multiplied by the difference of $X_C - X_A = +2624.96 - 3000.20 = -375.24$, the result being placed on line with Sta. *B* in the tabulation. Similarly, Y_C is multiplied by the difference of $+2871.79$ and $+2541.30 = +330.49$. The other values are found in the same manner. As a check, note that the sum of the negative values and the positive values in the "Diff. of Adj. *X*'s" column have the same value. Multiplying the *Y* coordinates by

the corresponding differences in X 's gives the double areas, which may be either positive or negative in sign; these are placed in the proper area columns in accordance with the sign. The difference

Sta.	Bal. Lat.	Bal. Dep.	Coordinates		Diff. of Adj. X's	Double Areas	
			Y	X		+	-
A	+ 76.80	- 458.90	+ 597.80	+ 3000.20	- 512.81	—	306 560
B	+ 283.25	+ 83.66	+ 674.60	+ 2541.30	- 375.24	—	253 140
C	- 79.48	+ 246.83	+ 957.85	+ 2624.96	+ 330.49	316 560	—
D	+ 41.54	+ 182.32	+ 878.37	+ 2871.79	+ 429.15	376 950	—
E	- 322.11	- 53.91	+ 919.91	+ 3054.11	+ 128.41	118 130	—

Comp. of Coordinates			Comp. of Diff. of Adj. X's		811 640	559 700
Y	X		X _B + 2541.30		2) 251 940	
A + 597.80	+ 3000.20		- X _E - 3054.11		125 970	s.f. Area
+ 76.80	- 458.90		A - 512.81			
B + 674.60	+ 2541.30		X _C + 2624.96			
+ 283.25	+ 83.66		- X _A - 3000.20			
C + 957.85	+ 2624.96		B - 375.24			
- 79.48	+ 246.83		X _D + 2871.79			
D + 878.37	+ 2871.79		- X _B - 2541.30			
+ 41.54	+ 182.32		C + 330.49			
E + 919.91	+ 3054.11		X _E + 3054.11			
- 322.11	- 53.91		X _C - 2624.96			
A + 597.80	+ 3000.20		D + 429.15			
Check	Check		X _A + 3000.20			
			X _D - 2871.79			
			E + 128.41			

Computation of Double Areas					
	A	B	C	D	E
Log Diff. Adj. X's	2.70996	2.57431	2.51916	2.63261	2.10860
Log Y	2.77656	2.82905	2.98129	2.94367	2.96375
Log Double Areas	5.48652	5.40336	5.50045	5.57628	5.07235
Double Areas	306 560	253 140	316 560	376 950	118 130

FIG. 87. COMPUTATION OF AREA BY COORDINATES.

in the sums of the values in the plus and minus double area columns divided by 2 gives the final area of the closed traverse. This area is 3 sq. ft. less than that obtained by the D.M.D. method in Fig. 78; this difference, however, is in the sixth significant figure which is beyond the precision of the field measurements and the log tables which were good for five significant figures. These results therefore check each other.

149. Fractional Areas. Fractional areas are the additional areas that must be added to (or subtracted from) the area of the closed traverse in order to obtain the area within the boundaries of the property. They occur when the traverse does not follow the property line. Frequently these fractional areas are triangles and quadrilaterals, and their areas are found by the application of geometry or trigonometry. Certain formulas are available for computing the areas of irregularly shaped figures.

The fractional areas to be computed in the survey shown in Fig. 67 are reproduced in Fig. 88. These areas are: triangle *CDF*, quadrilateral *DEGF*, triangle *EHG*, irregular area *EAH*.

150. AREA OF TRIANGLE *CDF*. By trigonometry the area of any triangle may be computed by the formula

$$\text{Area} = \frac{1}{2} ab \sin C,$$

where *a* and *b* are two adjacent sides, and *C* is the angle between these sides.

A practical way in which to compute the area of an oblique triangle involves dividing it into two right triangles, as shown in Fig. 89. This method is also convenient for finding the length and bearing of line *CF*.

$$\begin{aligned} FK &= 14.80 \sin 87^\circ 56' = 14.79 \\ DK &= 14.80 \cos 87^\circ 56' = 0.53 \\ \text{Area} &= \frac{1}{2} (259.33 \times 14.79) = 1918 \text{ sq. ft.} \end{aligned}$$

In order to compute the bearing of *CF* from that of *CD*, the angle at *C* is required.

$$\begin{aligned} \tan FCD &= \frac{14.79}{259.33 - 0.53} = \frac{14.79}{258.80} \\ \text{Angle } FCD &: 3^\circ 16' 15'' \text{ (measured angle } 3^\circ 16' 30'') \\ \text{Bearing } CF &: S 72^\circ 09' 00'' E + 3^\circ 16' 15'' = S 75^\circ 25' 15'' E. \\ \text{Length of } CF &= \frac{CK}{\cos FCD} = 259.22 \text{ ft.} \end{aligned}$$

151. AREA OF QUADRILATERAL *DEGF*. The area is conveniently computed by dropping the perpendiculars shown in Fig. 90, and then computing the areas of two triangles and a trapezoid. The unknown sides and the areas of the two right triangles *DaF* and *EbG*, and the area of the trapezoid *abGF*, are computed. From these areas the area of *DEGF* is calculated as follows:

$$\begin{aligned} Da &= 14.80 \cos 61^\circ 22' 30'' & 7.09 & Fa = 14.80 \sin 61^\circ 22' 30'' = 12.99 \\ Eb &= 22.40 \cos 36^\circ 56' & 17.91 & Gb = 22.40 \sin 36^\circ 56' = 13.46 \\ ab &= DE + Eb - Da = 197.83 \\ \text{Area } DaF &= \frac{1}{2} (7.09) (12.99) = 46 \text{ sq. ft.} \\ \text{Area } abGF &= 197.83 \left(\frac{12.99 + 13.46}{2} \right) = 2616 \text{ sq. ft.} \\ \text{Area } EbG &= \frac{1}{2} (17.91) (13.46) = 121 \text{ sq. ft.} \\ \text{Area } DEGF &= DaF + abGF - EbG = 2541 \text{ sq. ft.} \end{aligned}$$

The first, known as the *Trapezoidal Rule*, assumes that the areas between the equally spaced rectangular offsets are trapezoids; i.e., that Hd , de , etc., are straight lines. The distance between the offsets, which depends on whether the curved boundary is flat or sharp, is so taken that the above assumption will cause no large error. The area is found by the following formula:

$$\text{Area} = d \left(\frac{h_e}{2} + \Sigma h + \frac{h_e'}{2} \right),$$

in which d is the distance between offsets, h_e and h_e' are the end offsets of the series of trapezoids, and Σh is the sum of the intermediate offsets.

The second formula, known as *Simpson's One-Third Rule*, assumes that the curved line is a parabolic curve. The formula derived on this basis is

$$\text{Area} = \frac{d}{3} (h_e + 2\Sigma h_{\text{odd}} + 4\Sigma h_{\text{even}} + h_e'),$$

in which d is the common distance between offsets, h_e and h_e' are the first and last offsets,

$2\Sigma h_{\text{odd}}$ = twice the sum of all the odd offsets, i.e., 3d, 5th, 7th, etc., from one end,

$4\Sigma h_{\text{even}}$ = four times the sum of all the even offsets, i.e., the 2d, 4th, 6th, etc., from the same end.

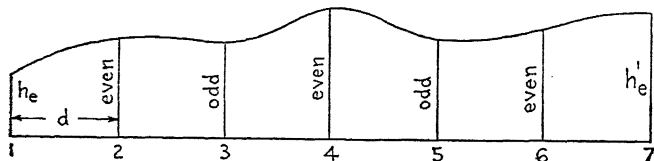


FIG. 88A. APPLICATION OF SIMPSON'S ONE-THIRD RULE.

This formula will apply only to an even number of strips or trapezoids (Fig. 88a). If the number of strips is odd, all but one strip may be computed by this rule, and the extra strip will be treated separately as a trapezoid.

154. In computing the area EHA , Fig. 88, by the above rules, note that the base xA of the triangle xyA is 26.63 ft. and must be computed independently; note also that since there are ten sections, an even number, Simpson's $\frac{1}{3}$ Rule will apply.

By Trapezoidal Rule:

$$\text{Area } EHyx = 30 \left(\frac{11.4}{2} + 11.6 + 11.2 + 10.8 + 10.1 + 9.5 + 9.0 + 8.2 + 7.0 + 5.6 + \frac{3.4}{2} \right) = 2712 \text{ sq. ft.}$$

By Simpson's $\frac{1}{3}$ Rule:

$$\text{Area } EHyx = \frac{30}{8} [11.4 + 2(11.2 + 10.1 + 9.0 + 7.0) + 4(11.6 + 10.8 + 9.5 + 8.2 + 5.6) + 3.4] = 10(11.4 + 74.6 + 182.8 + 3.4) = 2722 \text{ sq. ft.}$$

$$\text{Area of triangle } xyA = \frac{26.63 \times 3.4}{2} = 45 \text{ sq. ft.}$$

Hence the area of EHA by the trapezoidal rule is 2757 sq. ft. and by Simpson's $\frac{1}{3}$ rule 2767 sq. ft.

155. TOTAL AREA OF PROPERTY.

Traverse $ABCDE$	=	125973	sq. ft.	
Triangle CDF	=	1918	" "	
Quadrilateral $DEGF$	=	2541	" "	
Triangle GEH	=	110	" "	
Area $EHyx$	=	2712	" "	Trap. rule
Triangle Axy	=	45	" "	
Total	=	133299	sq. ft.	
Say		133,300	sq. ft.	

156. AREA OF ROUNDED CORNER LOT. At street intersections, Fig. 91, the corners are often rounded, the property ownership line conforming to the curve.

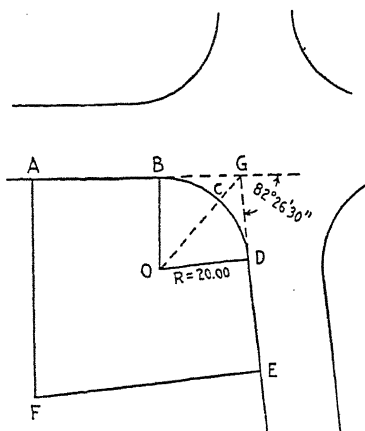


FIG. 91. ROUNDED CORNER LOT.

These curves are usually arcs of circles of definite radius. In making a survey of a corner lot such as $ABCDEF$, a point G at the intersection of the property lines AB and ED is first established. A survey of the closed traverse $AGEF$ is then made, and the area calculated. From this area, the area $BCDG$ must be subtracted. To find the area $BCDG$, the area $OBGD$ is first found and the area of the sector $OBCD$ is subtracted from it.

To compute the area of $BGDO$, draw GO which divides $BGDO$ into two right triangles. Then compute

$$BG = DG = 20 \tan \frac{1}{2}(82^\circ 26' 30'') = 17.52$$

$$\text{Area } BGDO = 17.52 \times 20 = 350 \text{ sq. ft.}$$

$$\text{The area of a sector of a circle} = \frac{RL_c}{2},$$

where R is the radius, and L_c the length of curve from B to D .

The length of curve may be found from the proportion

$$\frac{L_c}{2\pi R} = \frac{\text{angle } BOD}{360^\circ},$$

or by means of a table of circular arcs (Table VII). Using the latter method we find from Table VII that the length of the circular arc for radius unity is 1.4389, and arc $L_c = 20 \times 1.4389 = 28.78$.

$$\text{Area of the sector} = \frac{1}{2}(20 \times 28.78) = 288 \text{ sq. ft.}$$

$$\text{Area of } BGDC = 350 - 288 = 62 \text{ sq. ft.}$$

A simplification of the above gives the expression

$$\text{Area } BGDC = R \left(T - \frac{1}{2} L_c \right),$$

where $T = BG = GD =$ Tangent Distance of the curve. (Art. 270.)

$$L_c = BD = \text{Length of curve.}$$

By this method

$$\begin{aligned} \text{Area } BGDC &= 20(17.52 - 14.39) \\ &= 20 \times 3.13 = 62.6 \text{ sq. ft.} \end{aligned}$$

Since the tangent distance and length of curve are usually computed in connection with laying out the curve, the latter method is the more convenient to use in most cases.

157. MISSING SIDES OR BEARINGS OF A CLOSED TRAVERSE AND RANDOM TRAVERSES. If any two parts of a closed traverse are missing, i.e., have not been observed, the traverse may still be computed, and the missing parts found.

The missing parts may be:

- (1) the bearings of any two sides,
- (2) the lengths of any two sides,
- (3) the length of one side, and the bearing of another side,
- (4) the length and bearing of the same side.

The last is perhaps the most common and is illustrated by the following example, Fig. 92:

In a closed traverse $ABCD$ the only data observed are given on the sketch, which shows a portion of the same traverse as the one in Fig. 77. Note that no angles were measured at A and D , nor was the distance AD measured. Our problem is to find the length and bearing of the line DA . From the data given in Fig. 92 the latitude and departure of the lines AB , BC and CD are computed (see Fig. 78) and put in the form shown in Fig. 93. If a rectangle consisting of north and south and east and west lines be constructed through the most northerly, southerly, easterly, and westerly points of the traverse, as shown in Fig. 92,

it will be observed that $aB = \text{lat. } AB$; $Bb = \text{lat. } BC$; $dD = \text{lat. } CD$; and $eA = \text{lat. } DA$. Hence we may say that the

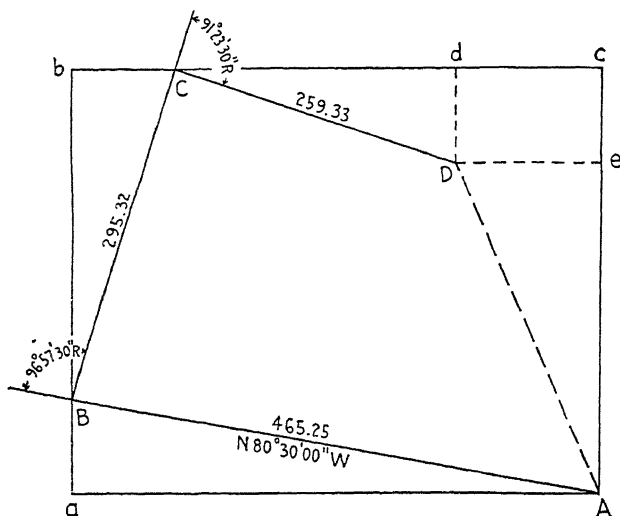


FIG. 92. MISSING SIDE IN CLOSED TRAVERSE.

Sta.	Deflection Angles	Bearings	Dist.	Latitudes		Departures	
				N +	S -	E +	W -
A	° ' "	N 80° 30' 00" W	465.25	76.79			458.87
B	96 57 30 R	N 16 27 30 E	295.32	283.22		83.67	
C	91 23 30 R	S 72 09 00 E	259.33		79.49	246.84	
D							
				360.01	79.49	330.51	458.87
				79.49			330.51
				Lat. DA = 280.52		Dep. DA = 128.36	

FIG. 93. COMPUTATION OF MISSING DATA.

Latitude of $DA = eA = aB + Bb - ce$; and similarly,
 Departure of $DA = De = -Aa + bC + Cd$

$$\begin{aligned}\tan \text{ Bearing } DA &= \frac{128.36}{280.52}, \\ \text{Bearing } DA &= \text{S } 24^{\circ} 35'.3 \text{ E}, \\ \text{Length } DA &= \frac{280.52}{\cos 24^{\circ} 35'.3} = 308.49 \text{ ft.}\end{aligned}$$

It will be observed that there is no geometric check on the angles and distances of the above traverse and no way to determine the error of closure. Any errors in the measured angles or distances would be reflected in the bearing and distance of DA .

Had the coordinates of A , B , C and D been computed, the differences of the Y and X coordinates of A and D respectively would give the latitude and departure of DA , from which the bearing and distance of DA could be computed.

A problem that involves the above principle, and one that frequently occurs in surveying, is to determine the distance and sometimes the bearing of a line, the ends of which are not inter-visible. This problem is solved by running a random traverse from one end of the line to the other end, and computing the required distance and bearing, as above. (See Art. 317.)

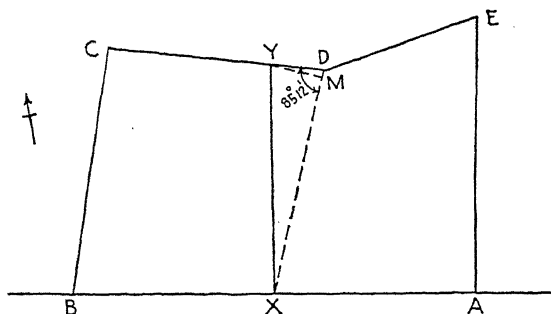
158. DIVISION OF LAND. There are many different problems that may arise in dividing land, but all may be solved by the application of the principles of geometry and trigonometry. The problems, commonly met, are to cut off a given area by

- (1) a straight line starting from a known point in the boundary,
- (2) a line running in a given direction.

As an example of the first case, assume that the traverse $ABCDE$, Fig. 94, had been run along the actual boundaries of a parcel of property and that it is desired to divide this area into two equal parts by a line starting at a point X , midway between A and B , thus giving equal frontage to both lots. The problem, then, is to find the distance that a point Y must be set along DC or DE , so that a line XY will divide $ABCDE$ into two equal areas.

The area $ABCDE$ is first computed. This area is 125,973 sq. ft. See Fig. 78. A construction line XD is drawn, and the area of $XBCD$ computed. The difference in one-half the area of $ABCDE$ and the area $XBCD$ is the excess or deficiency (in our

example the excess) that must be subtracted or added to $XBCD$ to equal one-half the area $ABCD$. Since point X is fixed, this



Sta.	Bearings	Dist.	Bal. Lats.		Bal. Deps.		D. M. D.	Double Areas	
			+ N	- S	+ E	- W		+	-
X	N 80° 30' 00" W	232.62	38.40			229.45	229.45	8 811	
B	N 16° 27' 30" E	295.32	283.25		83.66		83.66	23 697	
C	S 72° 09' 00" E	259.33		79.48	246.83		414.15		32 917
D				242.17		101.04	559.94		135 600
$ \begin{array}{r} 321.65 \\ - 79.48 \\ \hline 242.17 \end{array} \quad \begin{array}{r} 79.48 \\ 330.49 \\ - 229.45 \\ \hline 101.04 \end{array} $									$ \begin{array}{r} 32\ 508 \\ 168\ 517 \\ \hline 32\ 508 \\ 2\ 136\ 009 \\ \hline \text{Area } XBCD = 68\ 005 \text{ s.f.} \end{array} $
<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p>Tan. Bear. DX = $\frac{101.04}{242.17}$</p> <p>Bearing DX = S 22° 38.9' W or S 22° 39' W (To nearest min.)</p> <p>Length DX = $\frac{242.17}{\cos 22^\circ 38.9'} = 262.41$</p> <p>Total Area ABCDE = 125 973 (Fig. 78)</p> <p>Half " " = 62 986</p> <p>Area XBCD = 68 005</p> <p>Diff. = 5019 s.f.</p> <p>Altitude YM of $\triangle XYD = \frac{2(5019)}{262.41} = 38.25$</p> <p>Bearing DC = N 72° 09' W</p> <p>Bearing DX = S 22° 39' W</p> <p>Angle CDX = $180^\circ 00' - (72^\circ 09' + 22^\circ 39') = 85^\circ 12'$</p> </div> <div style="width: 45%;"> <p>In triangle YDM</p> <p>$DY = \frac{38.25}{\sin 85^\circ 12'} = 38.38 \text{ ft.}$</p> <p>$DM = 38.25 \cotan 85^\circ 12' = 3.21$</p> <p>In triangle XYM</p> <p>$XM = XD - DM = 262.41 - 3.21 = 259.20$</p> <p>$\tan YXM = \frac{38.25}{259.20}$</p> <p>Angle YXM = $8^\circ 23.7'$ or $8^\circ 23' 30''$ (to nearest $\frac{1}{2}$ min.)</p> <p>Bearing XY = Bearing XD - $8^\circ 23' 30''$</p> <p>$= N 22^\circ 39' 00'' E - 8^\circ 23' 30'' = N 14^\circ 15' 30'' E$</p> <p>$XY = \frac{259.20}{\cos 8^\circ 23.7'} = 262.01 \text{ ft.}$</p> </div> </div>									

FIG. 94. CUTTING OFF A GIVEN AREA.

excess is a triangle shown in the figure as the triangle XDY . Knowing the area of XDY and the base DX , we compute the

altitude YM . Knowing the bearings of CD and DX , we compute the angle YDM . In the right triangle DYM we know YM and angle YDM . Therefore we may compute the required distance DY .

If the length and bearing of XY are required, the triangle XDY may be solved for distance XY and angle YXD . The latter value applied to the bearing of DX will give the bearing of XY .

The calculations for finding the distance DY , with the exception of computing the area $ABCDE$, are shown in Fig. 94. In computing the area $XBCD$ either the D.M.D. or the coordinate method can be used. The former is used in this example. Many data for calculating this area (Fig. 94) are taken from the data in Fig. 78. Naturally the values taken from Fig. 78 will be the balanced latitudes and departures.

Since we do not know the length and bearing of DX , these values will be found from the latitude and departure of that line. To compute the latitude (and departure) of DX , add algebraically the latitudes (and departures) of the lines XB , BC and CD . The results are: the latitude of $DX = 242.17$ ft. and the departure of $DX = 101.04$ ft. In Fig. 94 these values are shown enclosed by rectangles. Using the latitude and departure of DX , we compute the length and bearing of that line and the area $XBCD$.

Art. 290 illustrates the subdivision of a block of land into building lots that would be suitable to the development of suburban property.

PROBLEMS

1. Referring to Fig. 78, calculate the distance between points A and C and the bearing of the line. (Use the balanced Lats. and Deps.)
2. Referring to Fig. 78, calculate the coordinates (x and y) of points B , C , D , and E , referred to point A as an origin.
3. In Fig. 78, calculate the new bearings (to nearest 5 sec.) and distances (to 0.01 ft.) consistent with the balanced latitudes and departures.
4. Following is a "closed" compass and chain survey. Calculate the area by the coordinate method and by the D.M.D. method.

Station	Bearing	Distance
1	S 40° W	17.50 chains
2	N 45° W	22.25
3	N 36½° E	31.25
4	North	13.50
5	S 81° E	46.50
6	S 8½° W	34.25
7	West	32.50

5. Following is a closed transit and tape traverse. Calculate the area by the coordinate method and by the D.M.D. method.

Station	Deflection Angle	Distance
A	52° 49' R	400.9 ft.
B	72 12 R	526.1
C	90 22 R	390.8
D	85 11 L	339.6
E	123 17 R	816.7
F	106 30 R	829.5

6. Closed traverse. Compute area by D.M.D. and by coordinate methods.

Station	Deflection	Bearing	Distance
1	91° 41' 30" R	N 65° 51' W	319.15 ft.
2	120 03' 30" R		168.40
3	11 14' L		498.80
4	151 23' R		394.18
5	8 07' R		229.35

In this problem 100-ft. tape is 0.067 ft. short at 68° F., 10 lbs. pull.

7. In the following traverse there are two mistakes. Find where they occur and determine their amounts.

Station	Observed Bearing	Deflection Angle	Distance (Feet)	Calculated Bearings	Remarks
A	N 34° E	164° 14' R	240.2	N 34° 00' E	CE = 188.1 BCE = 34° 14' DEC = 81° 25'
B	S 73½° E	62° 16' R	163.7		
C	S 10½° W	84° 22' R	207.6		
D	N 26½° W	142° 49' R	273.1		
E	S 52° W	103° 41' L	147.4		

8. In the following notes of a lot of land a straight line from *B* to *X* (on line *EF*) is desired to straighten the irregular boundary and still preserve the same area. Find the bearing and length of *BX* and the distance *EX*.

Station	Bearing	Distance (Feet)
<i>A</i>	S 89° 14' E	373.62
<i>B</i>	N 13° 10' E	100.27
<i>C</i>	N 0° 17' W	91.26
<i>D</i>	N 27° 39' E	112.48
<i>E</i>	N 72° 12' W	346.07
<i>F</i>	S 5° 07' W	272.42
	etc.	etc.

9. (a) In the lot of land, *ABCD*, the lines *AB* and *DC* both have a bearing of N 23° E; the bearing of *AD* is due East; *AD* is 600 ft., *AB* is 272.7 ft., and *DC* is 484.6 ft. Find the length of a line *EF* parallel to *AB* which will cut off an area *ABFE* equal to half an acre. Also find the length of the lines *AE* and *BF*.

(b) What is the area of *EFCD*?

10. Given the notes of a traverse which does not close, as follows:—

Station	Deflection Angle	
0		Find the length of a straight line from 0 to 20 + 64 and the angle it makes with the line from 0 to 6 + 40.
6 + 40	6° 17' L	
9 + 20	18° 43' L	
14 + 55	12° 47' R	
17 + 18	45° 24' L	
20 + 64	68° 06' R	

11. The line *Ax* has a bearing S 51° 10' E, and line *Ex* has a bearing N 21° 04' E. *A* and *E* are connected by the following traverse: *AB*, S 60° 51' W, 102.69 ft.; *BC*, S 0° 59' E, 298.65 ft.; *CD*, S 89° 01' E, 101.20 ft.; *DE*, N 80° 52' E, 148.28 ft. Compute the distances *Ax* and *Ex*.

12. Compute the area enclosed by the following traverse.

Station	Deflection Angle	Bearing	Distance
1	79° 13' R	North	1278.2 ft.
2	76° 21' R		230.6
3	17° 03' R		383.4
4	22° 00' L		500.2
5	113° 38' R		850.3
6	18° 58' L		881.1
7	114° 45' R		1241.5

13. A corner city lot $ABCDE$ has the following interior angles. $A = 7^\circ 15'$, $B = 94^\circ 50'$, $C = 91^\circ 22'$, $D = 120^\circ 27'$, $E = 226^\circ 06'$. The measured sides are $AB = 583.58$, $BC = 102.88$, $CD = 153.01$, $DE = 86.85$, $EA = 391.42$. Bearing of AB is $N 66^\circ 50' E$. At the corner B the lot is rounded by a circular curve of 20.00 ft. radius; point B is at the vertex of this curve. Compute the area of the lot.

14. Compute the area enclosed by the following traverse.

Station	Deflection Angle	Bearing	Distance
1	$99^\circ 49' R$	$S 1^\circ 39' W$	854.1 ft.
2	$100^\circ 06' R$		203.9
3	$19^\circ 47' L$		250.7
4	$22^\circ 10' R$		479.6
5	$32^\circ 54' L$		454.0
6	$112^\circ 06' R$		687.3
7	$78^\circ 32' R$		1341.6

CHAPTER VIII

THE UNITED STATES SYSTEM OF SURVEYING THE PUBLIC LANDS

159. THE SYSTEM. The United States System of Surveying the Public Lands, frequently referred to as the rectangular system, was planned in 1784 by the Continental Congress, and has been modified only slightly by subsequent acts of Congress. The law requires that the public lands "shall be divided by north and south lines run according to the true meridian, and by others crossing them at right angles so as to form townships six miles square," and that the corners of the townships thus surveyed "must be marked with progressive numbers from the beginning." It provides that the townships shall be subdivided into thirty-six sections, each of which shall contain six hundred and forty acres, as nearly as may be; the division to be by cardinal lines surveyed and marked at intervals of a mile.*

In order to make the adjustments in the survey to provide for the convergency of meridians, a unique system has been developed which applies uniformly and effectively throughout the whole of the public domain where the jurisdiction over the lands is vested in the Federal government.†

The work of the Public Lands Survey is carried on under the direction of the Commissioner of the General Land Office. Where more detail is required than given in this text recourse should be had to the Manual‡ of Instructions prepared by the General Land Office.

* Cardinal lines are those running either due north and south or due east and west.

† The surveys and the granting of the title to the lands within the New England and Atlantic Coast States (excepting Florida), and in the states of Pennsylvania, West Virginia, Kentucky, Tennessee, and Texas, are under the jurisdiction of the states.

‡ Manual of Instructions for the Survey of Public Lands of the United States, 1930, reprint, 1934; may be obtained from the Superintendent of Documents, Washington, D. C.

The law requires that measurements be true horizontal distances expressed in miles, chains, and links.* The directions of all lines are expressed as bearing angles, which are intended to refer to the true meridian.

The general system will be described in Arts. 160 and 161, followed by a detailed discussion of the several processes. The practice is applicable within twenty-nine of the states and in the Territory of Alaska, and applies to the retracement of old surveys as well as to their extension into new territory.

160. Outline of the System. It will be convenient to consider the process of subdivision as separated into several distinct operations, to be carried out in sequence. It must be understood, however, that one operation, for instance, the division of the area into 24-mile tracts, is rarely or never completed over the entire area to be covered before the next operation in order is begun; a single surveying group may be carrying on two or three different operations, such as running township exteriors and immediately afterward subdividing the townships into sections.

Briefly stated, the subdivision work is carried on as follows:

FIRST. The establishment of

(a) An *Initial Point* by astronomical observations.

(b) A *Principal Meridian* conforming to a true meridian of longitude through the Initial Point, and extending both north and south therefrom, and

(c) A *Base-Line* conforming to a true parallel of latitude through the Initial Point, and extending both east and west therefrom. This initial operation is indicated in Fig. 95.

The principal meridian will be marked out on the ground as a straight line, and the base-line will follow the curve of a due east and west line, being at every point at right angles to the meridian through that point.

* Chain = 66 ft.; link = .66 ft. These units were used in the early Colonial surveys of the English speaking settlers, and in the state grant surveys excepting in Texas, where the Spanish *vara* was employed; the early French settlers used the *arpent* unit. The arpent is a measure of area, one side of this square unit equals 191.994 ft. (Louisiana, Mississippi, Alabama and Northwestern Florida); 192.500 ft. (Arkansas and Missouri). The vara = 33.333333 in. (Texas); 33.372 in. (Florida); 32.99312 in. ("public domain" in Southwest).

SECOND. The division of the area to be surveyed into tracts approximately 24 miles square (Fig. 95) by the establishment of

(a) *Standard Parallels* conforming to true parallels of latitude through the 24-mile points previously established on the principal meridian, and extending both east and west therefrom, and

(b) *Guide Meridians* conforming to true meridians of longitude through the 24-mile points previously established on the

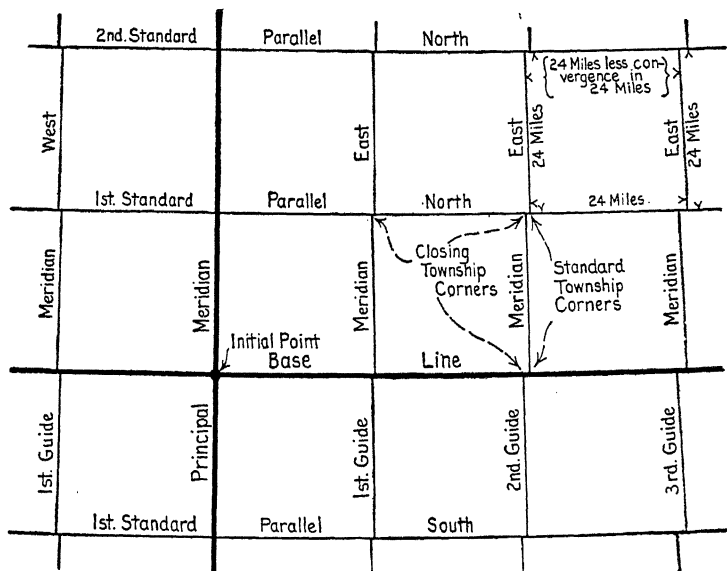


FIG. 95. SHOWING DIVISION INTO 24-MILE BLOCKS.

base-line and standard parallels, and extending north therefrom to an intersection with the next standard parallel or base-line.

Since the guide meridians converge, these 24-mile tracts will be 24 miles wide on their southern and less than this on their northern boundaries. Theoretically, both the east and the west boundaries should be just 24 miles in length, but, owing to discrepancies of field measurements, this is rarely or never the case.

THIRD. The division of each 24-mile tract into *Townships*, each approximately 6 miles square, by the establishment of

(a) Meridional lines, usually called *Range Lines*, conforming to true meridians through the standard township corners previously established at intervals of 6 miles on the base-line and standard parallels, and extending north therefrom to an intersection with the next standard parallel, or to the base-line, and

(b) Latitudinal lines, sometimes called *Township Lines*, joining the township corners previously established at intervals of 6 miles on the principal meridian, guide meridians, and range

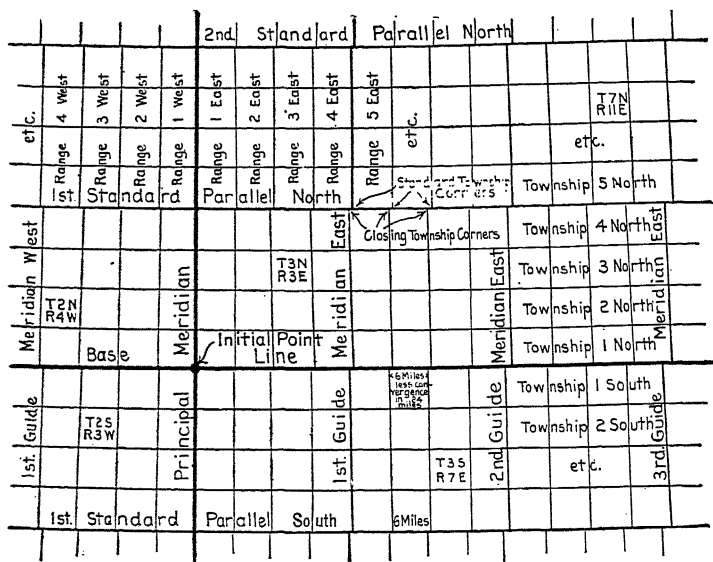


FIG. 96. SHOWING SUBDIVISION OF 24-MILE BLOCKS INTO TOWNSHIPS.

lines. The division resulting from the first three operations is indicated in Fig. 96.

Neglecting the effect of discrepancies and irregularities in measurement, both the east and the west boundaries of all townships will be just 6 miles in length, but the north and south boundaries will vary in length from a maximum at the standard parallel or base-line forming the southern limit of the 24-mile tract to a minimum at that forming its northern limit.

FOURTH. The subdivision of each township into *Sections*, each approximately 1 mile square and containing about 640 acres, by

the establishment of *Section Lines*, both meridional and latitudinal, parallel to and at intervals of 1 mile from the eastern and southern boundaries of the township.

Assuming all fieldwork to be done with mathematical exactness, this subdivision would result in sections exactly 80 chains (1 mile) on each of the four sides,* except the most westerly range of 6 sections in each township, which would be less than 80 chains in width by an amount varying with the distance from the southern boundary of the 24-mile tract. The extent to which this condition is realized in practice is indicated in Art. 171, wherein the usual field methods of subdividing a township are described in detail.

161. Methods of Designating Lines and Areas. The various principal meridians and base-lines of the Public Lands Surveys are designated by definite names or by number, as, for example, "The Fifth Principal Meridian and Base-Line," or "The Cimarron Meridian."[†]

The standard parallels are numbered in order both north and south from the base-line, and are so designated. The guide meridians are numbered in a similar manner east and west from the principal meridian. Fig. 95 illustrates the method.

Any series of contiguous townships or sections situated north and south of each other constitutes a *range*, and such a series situated in an east and west direction constitutes a *tier*.

The tiers of townships are numbered in order, to both the north and the south, beginning with number 1 at the base-line; and the ranges of townships are numbered to both the east and the west, beginning with number 1 at the principal meridian. A township is designated, therefore, by its serial number north or south of the base-line, followed by its number east or west of the principal meridian, as "Township 7 south, Range 19 east,

* These theoretical sections would not be exactly square, as may be readily perceived, but would be rhomboids.

[†] These bases and meridians are shown on the large wall map of the United States published by the General Land Office, on the various official state maps, and on a special map entitled "United States, Showing Principal Meridians, Base Lines and Areas Governed Thereby," Superintendent of Documents, Washington, D. C.

of the Sixth Principal Meridian." This is usually shortened to "T. 7 S., R. 19 E., 6th P.M."

The sections of a township are numbered as in Fig. 97. In all surveys of fractional townships the sections will bear the same numbers they would have if the township were complete.

By the terms of the original law and by general practice section lines are surveyed from south to north and from east to west, in order uniformly to place excess or deficiency of measurement on the north and west sides of the townships.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

FIG. 97. DIAGRAM OF A TOWNSHIP ILLUSTRATING METHOD OF NUMBERING THE SECTIONS.

162. FIELD METHODS. The work of subdivision of the Public Lands has already been largely completed, and the surveyor of today is usually concerned only with the retracing of old lines, the relocation of lost corners, or with the subdivision work that comes with increase in population. For all these, however, a thoroughgoing knowledge of at least the common field processes and methods that have been used in the original surveys is essential. Certain details of field practice have varied

somewhat from time to time, but the leading features have remained fairly constant for all those areas that have been surveyed since the system became well established.

In the following pages is given a somewhat detailed description of the methods commonly employed in carrying out the operations briefly indicated in the preceding articles.

163. Initial Points. No new initial points have been established in many years, excepting in Alaska. All of the thirty-four existing points were selected for the control of the surveys in agricultural areas to meet the demands for the settlement of the lands. After the selection of the point, the geographic position, in each location, was determined by field astronomical methods.

164. BASE-LINE — ESTABLISHING A PARALLEL OF LATITUDE. The base-line is a true parallel of latitude extending both east and west from the initial point.

There are three general methods for establishing a true parallel of latitude: (1) the solar method, (2) the tangent method, and (3) the secant method.

With the solar attachment to the transit, the direction of the meridian may be quickly found at every set-up when the sun is visible. A line which at every point is at right angles to the meridian will be a true parallel of latitude. If the sights between set-ups are not over 20 to 40 chains in length, the line so established does not differ appreciably from the true parallel of latitude.

The solar transit* is prescribed by the General Land Office for locating the meridian, particularly in forested regions, and in dense undergrowth where the sights are short. In the open country, and also when the sun is obscured, ordinary transit lines are run. The azimuths are determined and tested at intervals by observations on the sun, or observations on Polaris at elongation or at any hour angle. (See Arts. 129-32.) The instrument that is used is designed especially for the public-land surveying practice, it is equipped for making all necessary observations for time, latitude, and azimuth; the specifications require the best performance that ordinarily can be expected with a one-minute transit. The measurements are made with a long steel tape; slope distances are appropriately reduced to the true horizontal (Arts. 18-20).

When the sun is not visible or when an instrument without a solar attachment is used, the true latitude curve may be determined by offsets from a straight transit line. For this purpose the tangent method or the secant method† may be employed. The former is the one ordinarily employed in surveying practice.

165. Tangent Method. The tangent method of running the true latitude curve consists in establishing the true meridian at the point of beginning, from which a horizontal angle of 90° is turned to the east or west, as may be required; the projection of

* The solar transit has an auxiliary telescope which may be pointed at the sun in such a way that the direction of the meridian is found directly in the field without calculations. For this method, see *Elementary Surveying* by Breed and Hosmer, Vol. I, Arts. 240, 241.

† For secant method, see the *Manual, or Elementary Surveying* by Breed and Hosmer, Vol. I, Art. 159.

the line thus determined is called the tangent. The tangent is projected 6 miles as a straight line; at the distances for each corner the offsets are measured to the north (on meridional lines) from the tangent to the line of the true parallel, on which the monuments are established. The data for the azimuths of the line of the tangent, at mile intervals, and for the offsets from the tangent to the parallel, at one-half mile intervals, are shown in Tables 4 and 5. The offset method is illustrated in Fig. 98.

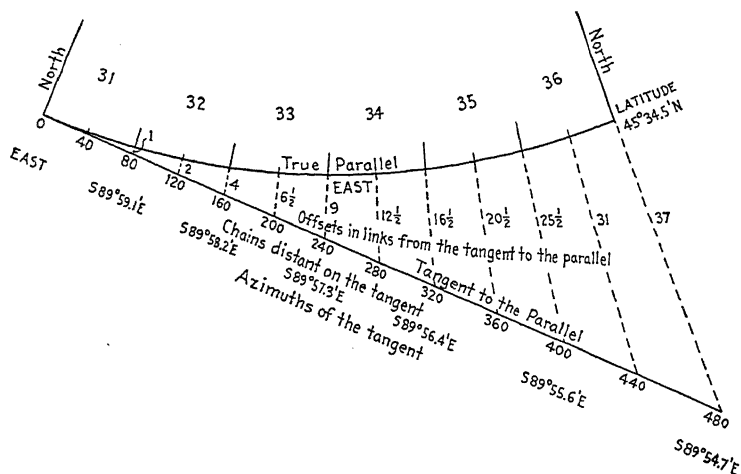


FIG. 98. LOCATING POINTS ON A PARALLEL BY TANGENT METHOD.

166. Running the Base-Line. In running the base-line great care should be taken to secure instrumental accuracy. In prolonging the line at each set-up, two double reversals of the instrument are made and the mean of the observations is used.

The direction of the meridian obtained with the solar attachment should be checked, whenever practicable, by an observation on Polaris; and in all cases where error is discovered, the line should be corrected before proceeding with the survey. All observations and adjustments should be fully described in the field-notes.

To insure accuracy in the measurements two sets of chainmen are employed, one set checking the other. Each measures the 40-chain intervals, and if the difference in results is within the

TABLE 4. AZIMUTHS OF THE TANGENT TO THE PARALLEL.

Lat.	1 ml.	2 ml.	3 ml.	4 ml.	5 ml.	6 ml.
25	89 59.6	89 59.2	89 58.8	89 58.4	89 58.0	89 57.6
26	59.6	59.2	58.7	58.3	57.9	57.5
27	59.6	59.1	58.7	58.2	57.8	57.4
28	59.5	59.1	58.6	58.2	57.7	57.2
29	59.5	59.0	58.6	58.1	57.6	57.1
30	59.5	59.0	58.5	58.0	57.5	57.0
31	59.5	59.0	58.4	57.9	57.4	56.9
32	59.5	58.9	58.4	57.8	57.3	56.8
33	59.4	58.9	58.3	57.7	57.2	56.6
34	59.4	58.8	58.2	57.7	57.1	56.5
35	59.4	58.8	58.2	57.6	57.0	56.4
36	59.4	58.7	58.1	57.5	56.9	56.2
37	59.3	58.7	58.0	57.4	56.7	56.1
38	59.3	58.6	58.0	57.3	56.6	55.9
39	59.3	58.6	57.9	57.2	56.5	55.8
40	59.3	58.5	57.8	57.1	56.4	55.6
41	59.2	58.5	57.7	57.0	56.2	55.5
42	59.2	58.4	57.7	56.9	56.1	55.3
43	59.2	58.4	57.6	56.8	56.0	55.2
44	59.2	58.3	57.5	56.7	55.8	55.0
45	59.1	58.3	57.4	56.5	55.7	54.8
46	59.1	58.2	57.3	56.4	55.5	54.6
47	59.1	58.1	57.2	56.3	55.4	54.4
48	59.0	58.1	57.1	56.2	55.2	54.2
49	59.0	58.0	57.0	56.0	55.0	54.0
50	59.0	57.9	56.9	55.9	54.8	53.8
51	58.9	57.9	56.8	55.7	54.6	53.6
52	58.9	57.8	56.7	55.6	54.5	53.4
53	58.8	57.7	56.6	55.4	54.2	53.1
54	58.8	57.6	56.4	55.2	54.0	52.8
55	58.8	57.5	56.3	55.1	53.8	52.6
56	58.7	57.4	56.2	54.9	53.6	52.3
57	58.7	57.3	56.0	54.7	53.3	52.0
58	58.6	57.2	55.8	54.5	53.1	51.7
59	58.6	57.1	55.7	54.2	52.8	51.4
60	58.5	57.0	55.5	54.0	52.5	51.0
61	58.4	56.9	55.3	53.8	52.2	50.6
62	58.4	56.7	55.1	53.5	51.9	50.2
63	58.3	56.6	54.9	53.2	51.5	49.8
64	58.2	56.5	54.7	52.9	51.1	49.4
65	58.1	56.3	54.4	52.6	50.7	48.9
66	58.1	56.1	54.2	52.2	50.3	48.4
67	58.0	55.9	53.9	51.8	49.8	47.8
68	57.9	55.7	53.6	51.4	49.3	47.2
69	57.8	55.5	53.2	51.0	48.8	46.5
70	89 57.6	89 55.3	89 52.9	89 50.5	89 48.1	89 45.8

TABLE 5. OFFSETS, IN LINKS, FROM THE TANGENT TO THE PARALLEL.

Lat.	1 ml.	1½ ml.	2 ml.	2½ ml.	3 ml.	3½ ml.	4 ml.	4½ ml.	5 ml.	5½ ml.	6 ml.
25	0	0	1	2	3	4	6	8	10	12	14
26	0	0	1	2	3	4	6	8	10	12	13
27	0	1	1	2	3	4	6	8	10	12	13
28	0	1	1	2	3	4	6	8	11	13	15
29	0	1	1	2	3	4	6	7	9	11	14
30	0	1	1	2	3	4	6	7	9	12	15
31	0	1	1	2	3	4	6	7	10	12	15
32	0	1	1	2	3	4	6	8	10	13	16
33	0	1	1	2	3	4	6	8	10	13	16
34	0	1	2	3	4	6	8	11	14	17	21
35	0	1	2	3	4	6	8	11	14	18	21
36	0	1	2	3	5	7	9	12	15	18	22
37	0	1	2	3	5	7	9	12	15	19	23
38	0	1	2	3	5	7	10	13	16	20	24
39	0	1	2	3	5	7	10	13	17	20	25
40	0	1	2	3	5	8	10	14	17	21	26
41	0	1	2	3	5	8	11	14	18	22	27
42	0	1	2	4	6	8	11	15	18	22	27
43	0	1	2	4	6	8	11	15	19	24	28
44	0	1	2	4	6	9	12	16	20	24	29
45	0	1	2	4	6	9	12	16	20	25	30
46	0	1	2	4	7	9	13	17	21	26	32
47	0	1	2	4	7	10	13	17	22	27	33
48	0	1	3	4	7	10	14	18	23	28	34
49	0	1	3	5	7	10	14	19	23	29	35
50	0	1	3	5	8	11	15	19	24	30	36
51	0	1	3	5	8	11	15	20	25	31	38
52	0	1	3	5	8	12	16	21	26	32	39
53	0	1	3	5	8	12	16	21	27	33	40
54	0	1	3	6	9	12	17	22	28	35	42
55	0	1	3	6	9	13	18	23	29	36	43
56	0	1	3	6	9	13	18	24	30	37	45
57	0	2	3	6	10	14	19	25	31	39	47
58	0	2	4	6	10	14	20	26	33	40	49
59	0	2	4	7	10	15	20	27	34	42	51
60	0	2	4	7	11	16	21	28	35	44	53
61	0	2	4	7	11	16	22	29	37	45	55
62	0	2	4	8	12	17	23	30	38	47	57
63	0	2	4	8	12	18	24	32	40	49	60
64	1	2	6	8	13	19	25	33	42	52	62
65	1	2	5	9	13	19	26	34	44	54	65
66	1	2	5	9	14	20	28	36	46	57	68
67	1	2	5	9	15	21	29	38	48	59	72
68	1	2	6	10	16	22	30	40	50	62	75
69	1	3	6	10	16	24	32	42	53	66	79
70	1	3	6	11	17	25	34	44	56	69	84

(From "Standard Field Tables," U. S. General Land Office.)

prescribed limits, the proper corner is placed midway between the end points of the two measurements. If the discrepancy exceeds 8 links on even ground, or 25 links on mountainous surface, the distance will have to be rechaind.

Regular quarter-section and section corners are established at intervals of 40 chains, and regular township corners at intervals of 480 chains along the base-line. Meander corners are set wherever the line crosses meanderable streams, lakes, or bayous. (See Art. 179.) When the corner falls in an inaccessible place, as on a cliff or in a lake, a witness corner is established. The monuments used to mark the positions of the township corners or section corners are marked with the letters SC, "standard corner," in addition to the other usual marks, for the purpose of easily distinguishing them from the "closing corners," to be set later on the same line.

167. Principal Meridian. The principal meridian is extended as a true meridian of longitude both north and south from the initial point. The methods used for the determination of directions, and the precautions observed to secure accuracy of measurement, are the same as those described in the preceding article, under the subject of "Running the Base-Line."

Also, as in the case of the base-line, all township, section, quarter-section, and other necessary corners are established in the proper places as the survey proceeds.

168. Standard Parallels. Standard parallels, which are also sometimes referred to as *correction lines*, are extended both east and west from every fourth township corner previously established on the principal meridian. Sometimes, however, the distance between them is more or less than 24 miles, depending upon the requirements of the particular survey in question. For example, in Kansas, the correction lines occur at regular intervals of 30 instead of 24 miles. Where standard parallels are at intervals of 30 or 36 miles and where present conditions require additional parallels for initiating new, or closing the extension of an old survey, an intermediate correction line should be established and a local name given to it, such as "Cedar Creek Correction Line"; and the same will be run, in all respects, like the regular standard parallels.

Standard parallels are established as true parallels of latitude, and are run in the same manner and with the same precautions for accuracy as in the survey of the base-line.

Appropriate corners are established at the proper intervals, and the township and section corners are marked *SC*, the same as those on the base-line.

169. Guide Meridians. Guide meridians are extended north from the base-line, or standard parallels, at intervals of 24 miles east and west from the principal meridian. They are run as true meridians of longitude, and are extended to an intersection with the next correction line north. At the point of intersection of the guide meridian with the correction line a *closing corner* is established, and the monument is marked with the letters *CC*, to distinguish it from the standard corners already in place. Also, the distance of the closing corner from the nearest standard corner is measured and recorded in the field-notes. This correction offset will vary with the latitude and with the distance of the corner from the principal meridian. At a distance of 15 or 20 ranges from the principal meridian it may be so great that the closing corner will be nearer to the adjacent quarter-section corner than to the standard township corner. Furthermore, it is obvious that the closing corners will be west of the corresponding standard corners on the east side of the principal meridian, and east of them on the west side.

The mile and half-mile distances on the guide meridians are made full 80 and 40 chains in length until the last half-mile is reached, into which all excess or deficiency due to discrepancies of measurement is thrown.

The general method of running the guide meridians is the same as that used in running the principal meridian, and all the provisions for securing accuracy of alignment and measurement, and for establishing corners, prescribed for the latter apply to the former also.

Provision is made for running guide meridians from north to south where existing local conditions require this departure from the usual practice. In such a case the closing corner is first established on the correction line by calculating the proper correction distance and laying it off from the standard corner; and

then the guide meridian is run due south from this point. This method may be used in case the standard corner from which the guide meridian would ordinarily originate is inaccessible, or for other adequate reasons.

The Manual also provides that where guide meridians have been placed at intervals exceeding 24 miles, and new governing lines are required in order to control new surveys, a new guide meridian will be established, and a local name may be assigned to the same, e.g., "Grass Valley Guide Meridian." These auxiliary guide meridians will be surveyed in all respects like regular guide meridians.

170. Township Exteriors. The usual method of subdividing a 24-mile tract into townships is, as follows (see Fig. 96):

Beginning at the standard corner at the southeast corner of the southwest township in the tract, the surveyor runs north on a true meridian of longitude a distance of 6 miles, setting all necessary corners by the way. From the township corner thus established he runs due west on a random line (Art. 317) to intersect the guide meridian (or the principal meridian, in case he is working in Range 1 East), setting temporary section and quarter-section corners as he goes. When he intersects the meridian, he notes the "falling"* of his random line, and, if this is within the limit prescribed, he then calculates the course of the true line joining the two township corners and runs back on it, setting permanent corners opposite the temporary ones previously set on the random line. In this way all the deficiency due to the convergence of the meridional boundaries of the township, and whatever excess or deficiency may arise from inaccuracies in measurement, is thrown into the most westerly half-mile of the latitudinal boundary.

The range line is now continued as a true meridian for another 6 miles, permanent corners being set as before. Then another random line is closed across to the western boundary of the range of townships, and is corrected back to the true line, in the same manner as that just described. This process is continued until the most northerly township in the 24-mile tract is reached,

* That is, the distance of the point at which the random line intersects the meridian from the objective corner.

when the range line is merely continued as a true meridian to an intersection with the correction line, at which point a closing township corner is established. The half-mile intervals on the range line are made full 40 chains for the entire 24 miles, except the most northerly half-mile, into which all excess or deficiency due to irregularities of measurement is thrown.

The two other range lines of the 24-mile block are run in a similar manner, the latitudinal township lines being extended to the westward at the proper intervals and made to connect with the township corners previously established. At the township corners on the last range line, however, random lines are run also to the westward from the guide meridian, and are then corrected back to the eastward on a true line between the township corners. This is done in such a way that the excess or deficiency of this line also is thrown into the most westerly half-mile.

In new surveys the township boundaries should be kept within 14 minutes of the cardinal direction in order to assure that the interior subdivision lines are kept within the prescribed limit of 21 minutes with the cardinal direction; but previously established township boundaries that come within the 21-minute limit will not be considered defective.

171. Subdivision of Townships. The subdivision of a township proceeds in the order indicated in Fig. 99. Starting at the southwest corner of Section 36 a line is run toward the north parallel to the east boundary of the township; quarter-section and section corners are set at 40 and 80 chains respectively. From

the section corner just set a random line is run eastward parallel to the south boundary of the township, a temporary quarter-section corner being set at 40 chains. The falling of this random

6	60	5	44	4	33	3	22	2	11	1
59	58	43	32	21	10					
7	57	8	42	9	31	10	20	11	9	12
56	55	41	30	19	8					
18	54	17	40	16	29	15	18	14	7	13
53	52	39	28	17	6					
19	51	20	38	21	27	22	16	23	5	24
50	49	37	26	15	4					
30	48	29	36	28	25	27	14	26	3	25
47	46	35	24	13	2					
31	45	32	34	33	23	34	12	35	1	36

FIG. 99. ORDER OF RUNNING SECTION LINES. THE NUMBERS ON THE SECTION LINES INDICATE THE NORMAL ORDER IN WHICH LINES ARE RUN.

line where it intersects the range line is noted, and also the distance by which it overruns or falls short of the length of the south boundary of the section. If the falling is not more than 50 links (33 feet, representing an angular deviation of 21 minutes), and if the distance overruns or falls short of the length of the southern boundary of Section 36 by not more than the same amount, a return course which will join the two section corners is calculated; this new line is then run toward the west, the permanent quarter-section corner being set at its middle point.

From the section corner just regained the survey is now continued north between Sections 25 and 26. At 40 and 80 chains on this line the quarter-section and section corners, respectively, are set, and from the section corner a random is run across to the range line, and a return course is calculated and run as before. This process is continued until five of the six sections in the series are enclosed. Then, if the north boundary of the township is not a base-line or standard parallel, from the section corner last established a random is run north to the township boundary, and from the data thus secured a true line is calculated and run from the section corner on the township line back to the initial corner. If the north boundary of the township is a base-line or standard parallel, however, the point at which the random intersects this boundary is established as a **closing corner** and its distance from the nearest **standard corner** is measured and recorded. In either case the permanent quarter-section corner is established at 40 chains north of the initial corner, the excess or deficiency being thrown into the most northerly half-mile. Here the meridional distance is checked by calculating the closure of the lines around the last section. The "limit of closure" is expressed by the fraction $\frac{1}{4\frac{1}{2}2}$, provided that the limit of closure in neither latitude nor departure exceeds $\frac{1}{8\frac{1}{4}0}$. An accumulative error of $12\frac{1}{2}$ links per mile of perimeter in either latitude or departure cannot be exceeded in an acceptable survey.

The latitudes and departures of a normal section shall each close within 50 links; of a normal range or tier of sections, within 175 links; and of a normal township, within 300 links. The boundaries of each fractional section including irregular claim lines or

meander traverses should close within the limit $\frac{1}{840}$ for latitude and for departure considered separately.

In a similar manner the succeeding ranges of sections are enclosed, randoms being run across eastward to the section corners previously established and true lines corrected back. From the fifth series of section corners thus established, however, random lines are projected to the westward also, and are closed on the corresponding section corners in the range line forming the western boundary of the township. In correcting these lines back, however, the permanent quarter-section corners are established at points 40 chains from the initial corners of the randoms, thereby throwing all fractional measurements into the most westerly half-miles.

Meander corners are established wherever the section lines intersect meanderable bodies of water. The boundaries of the fractional sections formed by meander traverses (Art. 179 and Fig. 105) and section lines are checked for closure by the usual surveying methods. (See Chap. VII.)

Table 6, taken from the Manual, gives (to the nearest whole minute) the angular convergency of meridians from one to five miles apart. The meridional section lines, therefore, by reason of being (theoretically) parallel to the range line on the east boundary of the township, will depart from true meridians by the amounts indicated in the table.

TABLE 6.
CORRECTIONS FOR CONVERGENCY WITHIN A TOWNSHIP

Latitude	Correction to be applied to bearing of range lines at a distance of				
	1 mile	2 miles	3 miles	4 miles	5 miles
	'	'	'	'	'
30 to 35	1	1	2	2	3
35 to 40	1	1	2	3	3
40 to 45	1	2	2	3	4
45 to 50	1	2	3	4	5
50 to 55	1	2	3	5	6
55 to 60	1	3	4	5	7
60 to 65	2	3	5	7	8
65 to 70	2	4	6	8	10

From a consideration of the foregoing it will be apparent

(1) That interior meridional section lines are 80 chains in length, except those next to the north boundary of the township; and that the south half of these is 40 chains in length.

(2) That interior latitudinal section lines are within 50 links of the length of the line forming the southern boundary of the range of sections, except those section lines next to the west boundary of the township; and that the east half of these is 40 chains in length.

(3) That meridional section lines are run straight for five miles and adjustments made in sixth mile; latitudinal section lines are run straight between section corners on meridional lines.

(4) That except in those section lines next to the north and west boundaries of the township, the quarter-section corners are placed equidistant from the two section corners on each side.

(5) That meridional section lines are intended to be parallel to the range line forming the eastern boundary of the township; and similarly, that latitudinal section lines are intended to be parallel to the township line forming its southern boundary.

(6) That the cumulative deficiency in latitudinal lines due to the convergence of the meridians is thrown into the most westerly half-mile of the township.

(7) That in the older practice, quarter-section closing corners were not always established on correction lines for the use of the sections south of these lines but these are established in the current surveys.

172. CONVERGENCE OF MERIDIANS.* The convergence of the meridians in a distance of 6 miles results in the north boundary of a township being shorter than the south boundary by an amount which must be taken into account in computing the closure of the township. The amount of the convergence varies in different latitudes. Table 7 shows the convergence of meridians 6 miles long, and 6 miles apart, in different latitudes. For shorter lengths the convergence is proportionately reduced.

* For method of computing convergence, see *Elementary Surveying* by Breed and Hosmer, Vol. I, Art. 178.

TABLE 7.

CONVERGENCE OF MERIDIANS, SIX MILES LONG AND SIX MILES
APART, AND DIFFERENCES OF LATITUDE AND LONGITUDE.

(From "Standard Field Tables," U. S. General Land Office)

Lat.	Convergency		Difference of longitude per range		Difference of latitude for —	
	On the parallel	Angle	In arc	In time	1 mi.	1 Tp.
°	<i>Lks.</i>	' "	' "	<i>Seconds</i>		
25	33.9	2 25	5 44.34	22.96	0.871	5.229
26	35.4	2 32	5 47.20	23.15		
27	37.0	2 39	5 50.22	23.35		
28	38.6	2 46	5 53.40	23.56		
29	40.2	2 53	5 56.74	23.78		
30	41.9	3 0	6 0.26	24.02	0.871	5.225
31	43.6	3 7	6 3.97	24.26		
32	45.4	3 15	6 7.87	24.52		
33	47.2	3 23	6 11.96	24.80		
34	49.1	3 30	6 16.26	25.08		
35	50.9	3 38	6 20.78	25.39	0.870	5.221
36	52.7	3 46	6 25.53	25.70		
37	54.7	3 55	6 30.52	26.03		
38	56.8	4 4	6 35.76	26.38		
39	58.8	4 13	6 41.27	26.75		
40	60.9	4 22	6 47.06	27.14	0.869	5.216
41	63.1	4 31	6 53.15	27.54		
42	65.4	4 41	6 59.56	27.97		
43	67.7	4 51	7 6.29	28.42		
44	70.1	5 1	7 13.39	28.89		
45	72.6	5 12	7 20.86	29.39	0.869	5.211
46	75.2	5 23	7 28.74	29.92		
47	77.8	5 34	7 37.04	30.47		
48	80.6	5 46	7 45.80	31.05		
49	83.5	5 59	7 55.05	31.67		
50	86.4	6 12	8 4.83	32.32	0.868	5.207
51	89.6	6 25	8 15.17	33.03		
52	92.8	6 39	8 26.13	33.74		
53	96.2	6 54	8 37.75	34.52		
54	99.8	7 9	8 50.07	35.34		
55	103.5	7 25	9 3.18	36.22	0.867	5.202
56	107.5	7 42	9 17.12	37.14		
57	111.6	8 0	9 31.97	38.13		
58	116.0	8 19	9 47.83	39.19		
59	120.6	8 38	10 4.78	40.32		
60	125.5	8 59	10 22.94	41.52	0.866	5.198
61	130.8	9 22	10 42.42	42.83		
62	136.3	9 46	11 3.38	44.22		
63	142.2	10 11	11 25.97	45.73		
64	148.6	10 38	11 50.37	47.36		
65	155.0	11 8	12 16.82	49.12	0.866	5.195
66	162.8	11 39	12 45.55	51.04		
67	170.7	12 13	13 16.88	53.12		
68	179.3	12 51	13 51.15	55.41		
69	188.7	13 31	14 28.77	57.92		
70	199.1	14 15	15 10.26	60.68	0.866	5.193

173. MARKING CORNERS. Corners are marked by various kinds of monuments. In the past, stone monuments were usually set where stone was plentiful. In timbered districts where suitable stones were difficult to find, wooden posts were used. In prairie regions, where neither stone nor timber were available, a mound of earth was commonly raised over the corner and a small marked stone, a charred stake, a quart of charcoal, broken glass, or other permanent distinguishable object was placed beneath it. Occasionally in timbered country when the corner fell on a spot occupied by a tree too large to be removed the tree itself was made the monument. The type of marking used on any corner should be thoroughly described in the notes.

If stones are set, they are marked with notches (or grooves) to indicate their respective positions. Section corners on range lines, including under this term principal and guide meridians, are marked with notches on their north and south edges, the number of notches being equal to the number of miles to the next adjacent township corner north or south. In a similar manner the section corners on the township lines are notched on their east and west edges. Township corners, being located on both range and township lines, are marked with the township and range numbers on each of the four sides. In addition to being *grooved* for the number of miles as just indicated, corners on correction lines are marked SC on their northern or CC on their southern faces, depending on whether they are standard or closing corners. Section corners in the interior of a township are given notches on their east and south edges corresponding to the number of miles to the east and south boundaries of the township. Thus, the corner common to sections 20, 21, 28 and 29 (Fig. 97) would have two notches on the south and four on the east edge. Quarter-section corners are marked with the fraction " $\frac{1}{4}$," those on meridional lines on their west, and those on latitudinal lines on their north faces.

In current practice an iron post is set, and the identifying marks inscribed on a bronze cap.

174. Witnessing Corners. Wherever possible the monument set at a corner is witnessed by several nearby objects, called "accessories," which may be easily found by anyone looking for

the corner itself, and which are comparatively permanent. In timbered country the stone or post is usually witnessed by "bearing trees" located near the corner. The process of establishing a witness tree is to take its bearing and distance from the corner, then to blaze off the bark from a short section of the trunk on the side facing the corner and to cut into the wood with scribing tools certain letters and numerals indicative of the section in which the tree is located. For example, the tree northeast from the corner mentioned above would be marked T 7 S R 15 E S 21 B T (written vertically), the marks being low on the trunk, the letters and figures being abbreviations of "Township 7 south, Range 15 east, Section 21, Bearing Tree." Usually one tree is marked in each of the sections to which the corner refers, provided suitable trees can be found within a reasonable distance of the corner.

In prairie regions small rectangular pits are dug near the corner, the earth taken from them being scattered so as not to refill the pits. Marks of this kind are of much greater value than might at first be supposed, for, although the sharp outlines are quickly worn away, the grass sod soon grows down into the pits and preserves them from entire obliteration.

175. Witness Corners. In case a regular corner falls in a creek, pond, or in any other place where it is impracticable to set or maintain a monument, a *witness corner* is set on one line leading to this corner. This is marked with the letters *WC* in addition to the markings that would be appropriate to the corner of which it is a witness. Witness corners are, in turn, referenced by bearing trees, pits, and other objects, the same as true corners.

176. Meander Corners. Where a surveyed line intersects a meanderable body of water (Art. 179), a *meander corner* is established. The distance from the nearest section or quarter-section corner is measured and recorded in the notes, and the stone or post set as a monument is marked *MC* on the side facing the water, and the point is referenced by bearing trees or by other accessories. If practicable, the line is then carried across the stream or other body of water by triangulation to another meander corner set in line on the further bank, and the survey is continued.

177. Index of Corner Markings. The following is an index of the ordinary markings common to all classes of corners and accessories:

MARKS	TO INDICATE	MARKS	TO INDICATE
A M C	Auxiliary meander corner.	R	Range.
A P	Angle point.	S	Section.
B O	Bearing object.	S	South.
B T	Bearing tree.	S C	Standard corner.
C	Center.	SE	Southeast.
C C	Closing corner.	S M C	Special meander corner.
E	East.	SW	Southwest.
M	Mile.	T	Township.
M C	Meander corner.	TR	Tract.
N	North.	W	West.
NE	Northeast.	W C	Witness corner.
NW	Northwest.	W P	Witness point.
P L	Public land (unsurveyed).	$\frac{1}{4}$	Quarter section.
		$\frac{1}{16}$	Sixteenth section

178. Marking Lines Through Timber. The act of May 18, 1796, which is still in force, requires the marking of trees along the surveyed lines as positively as the erecting of monuments. Those trees that are intersected by the line have two hacks or notches cut on each of the sides facing the line; these are called sight trees or line trees. Other trees that stand within 50 links of the line are blazed on two sides quartering toward the line.

179. MEANDERING. All navigable bodies of water and important rivers and lakes are segregated from the public lands at mean high-water elevation. The traverse of the margin of a permanent natural body of water is termed a meander line.

Numerous decisions in the United States Supreme Court and in many of the state courts assert the principle that meander lines are not boundaries defining the area of ownership of tracts adjacent to waters. The general rule is set forth in the decision (10 Iowa, 549) that in a navigable stream, as the Des Moines River in Iowa, high-water mark is the boundary line. When by

action of the water the river bed changes, and therefore high-water mark changes, the ownership of the land also changes.

Mean high-water elevation is found at the margin of the area occupied by the water for the greater portion of each average year; at this level a definite escarpment in the soil will generally be traceable, at the top of which is the true position for the meander line.

In the survey of lands bordering on tide waters, meander corners are set at the intersection of the surveyed lines with the margin of mean high tide. When the exact corner cannot be readily monumented, a witness corner is set at a secure point near the true point for the meander corner.

All meander courses are referred to the true meridian. For convenience the courses of meanderlines are taken to the nearest quarter degree, with angle points at distances of whole chains, or at multiples of 10 links, and with odd links only in the final course.

Following a river downstream, the bank on the left hand is termed the left bank and that on the right hand, the right bank. Navigable rivers and bayous, and all rivers not embraced in the class denominated "navigable" the right-angle width of which is 3 chains and upward, are meandered on both banks at the ordinary mean high-water mark by taking the courses and distances of their sinuosities. In general, rivers not classed as navigable are meandered only to the point where the average right-angle width is less than 3 chains. However, when such streams are so deep, swift, and dangerous as to be impassable, they may be meandered where good agricultural lands along the banks require separation into fractional lots for the benefit of settlers.

Tidewater streams, whether more or less than 3 chains wide, are meandered at ordinary high tide and as far as tidewater extends.

All lakes of 25 acres and upward in area are meanderable. The section and quarter-section lines are projected theoretically across such lakes, and meander corners are located wherever the shore is intercepted. If a meanderable lake lies wholly within a quarter-section, an "auxiliary meander corner" is established at some suitable point on its margin, and a connecting line is run to a regular corner on the section boundary.

A connecting traverse line will be recorded, if one is run, but it will also be reduced to the equivalent direct connecting course and distance, all of which will be stated in the field-notes, and the course and length of the direct connecting line will be shown on the plat of the survey.

Islands above the mean high-water elevation of any meanderable body of water are located by triangulation or by any other available method, and the section and quarter-section lines projected thereon. If the island lies wholly within a quarter-section, an auxiliary meander corner is set on its margin and tied in to a regular corner on the mainland. In the case of both islands and lakes, the point set at the intersection of a projected quarter-section line and the high-water line is called a "special meander corner."

180. Topography. Generally the most essential topographic data only are shown upon the plat. Distances to stream crossings, roads and trails, springs, notable changes in the ground slope or elevation, entering and leaving timber, and similar important items of topography, are recorded when running the true lines. The development of the map features within the sections varies greatly in accordance with the needs; this development may be carried out by sketching, or with the use of air-photos; sometimes it is done by traversing, or by a plane-table survey. The intersections are noted at the lines of mineral and other irregular surveys; connecting lines are run to the mineral-location monuments, and to all bench marks and geodetic stations.

181. Field-Notes. The field-notes are the written record of the survey. All are permanently filed for reference purposes; the notes are accessible for examination, and copies may be secured. This technical record shows the appropriate identification of the lines established by prior surveys, from which the new surveys may be extended, and the older lines upon which they may close. This is the base record for all directions and lengths of lines, for description of monuments and the "calls" that refer to natural objects, line trees, and items of topography, such as distances to stream crossings. Fig. 100 shows a typical page of field-notes in proper form.

SUBDIVISION OF T. 15 N., R. 20 E.

Chains.	I commence the subdivisional survey at the cor. of secs. 1, 2, 35, and 36, on the S. bdy. of the Tp., which is a sandstone, 8 x 6 x 5 ins. above ground, firmly set, marked and witnessed as described in the official record.
	N. 0° 01' W., bet. secs. 35 and 36.
	Over level bottom land.
20.00	Enter scattering ash and cottonwood.
29.30	SE. cor. of field; leave scattering timber.
31.50	A settler's cabin bears West, 6.00 chs. dist.
39.50	Set an iron post, 3 ft. long, 1 in. diam., 27 ins. in the ground, for witness $\frac{1}{2}$ sec. cor., with brass cap mkd.
	<div style="text-align: center;"> WC $\frac{1}{2}$ S 35 S 36 1925 </div>
	18 x 18 x 12 ins., N. and S. of post, 3 ft. dist. dig pits,
	Enter an ungraded road, bears N. along section line, and E. to Mound City.
40.00	True point for $\frac{1}{2}$ sec. cor. falls in road.
	Deposit a sandstone, 14 x 8 x 5 ins., mkd. X, 24 ins. in the ground.
50.50	NE. cor. of field.
51.50	Road to Bozeman bears N. 70° W.
57.50	Enter heavy ash and cottonwood, and dense undergrowth, bears N. 54° E. and S. 54° W.
72.00	Leave undergrowth.
80.00	Set an iron post, 3 ft. long, 2 ins. diam., 27 ins. in the ground, for cor. of secs. 25, 26, 35, and 36, with brass cap mkd.
	<div style="text-align: center;"> T 15 N R 20 E S 26 S 25 S 35 S 36 1925 </div>
	from which
	A green ash, 13 ins. diam., bears N. 22° E., 26 lks. dist., mkd. T 15 N R 20 E S 25 B T.
	A green ash, 23 ins. diam., bears S. 71 $\frac{1}{4}$ ° E., 37 lks. dist., mkd. T 15 N R 20 E S 36 B T.
	A green ash, 17 ins. diam., bears S. 64° W., 41 lks. dist., mkd. T 15 N R 20 E S 35 B T.
	A cottonwood, 13 ins. diam., bears N. 21 $\frac{1}{4}$ ° W., 36 lks. dist., mkd. T 15 N R 20 E S 26 B T.
	Land, level bottom; northern 20 chs. subject to overflow.
	Soil, alluvial, silt and loam; 1st rate.
	Timber, green ash and cottonwood; undergrowth, willow.

FIG. 100. SPECIMEN OF FIELD-NOTES.

182. TOWNSHIP PLATS. The township plats are developed from the field-notes; copies may be secured from the General Land Office. For many years each plat has carried a certificate that reads substantially as follows:

“ The above plat of Township , Range , of the Meridian, is strictly conformable to the field-notes of the survey thereof which have been examined and approved.”

There are many supplemental plats showing new or additional lottings within one or more sections; these supersede the original lottings. There are also many plats of the survey of islands, and other fragmentary parcels of public land where such areas were surveyed after the completion of the main parts of the township. These supplemental plats should be considered locally in tracing descriptions that are based upon the new lottings.

183. Computation of Areas. The meridional section lines (except in the sections along the north boundary of the township) are normally surveyed as 80.00 chains in length; the latitudinal section lines (except in the sections along the west boundary) will vary from 79.50 to 80.50 chains (called “within limits”) and are regarded as being 80.00 chains for the purpose of showing the official area; these sections are treated as containing an exact 640 acres. There are twenty-five of these regular sections in every normal township (the numbers, 8 to 17, 20 to 29, and 32 to 36). The *aliquot parts* are the regular quarter-sections of 160 acres and the quarter-quarter-sections of 40 acres each, including the regular portions of the eleven sections (1 to 7, inclusive, and 18, 19, 30 and 31) which have the *fractional lottings* along the north and west boundaries of the township.

The deficiency in area that results from the convergence of meridians is placed normally in the fractional lots adjoining the west boundary of the township. The surplus or deficiency in area that results from the discrepancy in the meridional measurements between the exterior boundaries and the subdivision lines is placed normally in the fractional lots adjoining the north boundary of the township.

Fig. 101 illustrates the adjustments and areas of lots next to the westerly boundary of the township. In computing the areas of lots 1 to 4, Sec. 30, the latitudinal dimensions are calculated by assuming these dimensions to vary proportionally from 18.35

on the south of lot 4 to 18.21 on the north of lot 1. Each meridional dimension is taken as 20 chains.

Fig. 102 illustrates the adjustments and areas of lots in the northwest section (Sec. 6) of a township. The meridional dimensions of lots 1 to 4 and the latitudinal dimensions of lots 4 to 7 (not shown in the plat) are found by proportion.

Fractional areas abutting on irregular boundaries, such as bodies of water, are computed by the usual methods for closed traverses (Chap. VII).

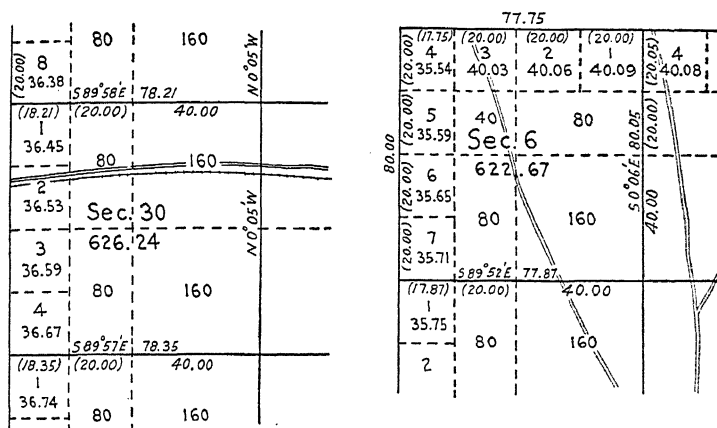


FIG. 102.

PORTIONS OF PLATS SHOWING AREAS OF LOTS.

184. DESCRIPTIONS AND RECORDS. The description of parcels of land should conform to the accepted nomenclature in use by the General Land Office; it should include the name of the proper reference meridian, the appropriate township and range numbers, and the established subdivisions of the township.

Example:

... township 9 north, range 12 east of the Black Hills meridian.

Each reference meridian has its own base-line, and therefore the words "and base-line" are usually omitted. For example,

"T. 5 S., R. 11 E. of the Willamette meridian" means "township 5 south of the associated base-line, in range 11 east of the Willamette meridian."

Parts of the township are referred to by section number according to the official plat. Parts of the section are described by half-section or quarter-section ($N\frac{1}{2}$; $E\frac{1}{2}$; $SW\frac{1}{4}$; etc.), and half-quarter or quarter-quarter-section ($N\frac{1}{2}NE\frac{1}{4}$ sec. 18, meaning "the $N\frac{1}{2}$ of the $NE\frac{1}{4}$ of section 18"). These terms give the so-called aliquot parts of 640 acres as contained in a regular section.

Lot numbers are employed to designate fractional units of the section — that is, other than quarter-quarter units of 40 acres each; but occasionally a subdivision smaller than a quarter-quarter-section is used, such as $E\frac{1}{2}NW\frac{1}{4}SW\frac{1}{4}$. Where terms for aliquot parts are spelled out, "half" and "quarter" are used, not "one-half" and "one-quarter."

The description of lots, other than aliquot parts where the numbers are omitted, is by fractional legal parts of the section, i.e., fractional $W\frac{1}{2}NE\frac{1}{4}$ sec. 5, containing 49.52 acres; fractional $NW\frac{1}{4}SW\frac{1}{4}$ sec. 6, containing 44.73 acres.

In some of the very old surveys the township subdivisions and the units within the sections do not conform to the examples shown above, and there may be other complications that must receive special consideration. Under such conditions it is advisable to obtain a statement from the General Land Office in reference to the appropriate terms to be employed.

In those states where the original surveys have been completed, application may be made to the proper offices for permission to examine the records and to make copies of the same. The list of these offices follows:

- Alabama: Secretary of State, Montgomery.
- Arkansas: Commissioner of State Lands, Little Rock.
- Florida: Commissioner of Agriculture, Tallahassee.
- Illinois: Auditor of State, Springfield.
- Indiana: Auditor of State, Indianapolis.
- Iowa: Secretary of State, Des Moines.
- Kansas: Auditor of State and Register of State Lands, Topeka.
- Louisiana: Register of State Lands, Baton Rouge.
- Michigan: Director, Department of Conservation, Lansing.
- Minnesota: Secretary of State, St. Paul.
- Mississippi: Commissioner of State Lands, Jackson.

Missouri: Secretary of State, Jefferson City.

Nebraska: Commissioner of Public Lands and Buildings, Lincoln.

North Dakota: State Engineer, Bismarck.

Ohio: Auditor of State, Columbus.

Oklahoma: Commissioner of the General Land Office, Washington, D. C.

South Dakota: Commissioner of School and Public Lands, Pierre.

Wisconsin: Commissioners of Public Lands, Madison.

185. RETRACEMENT OF SURVEYS. It has been the common experience that many of the monuments and marks originally established on the lines of the Public Land Surveys become lost or obliterated by the time the country has been settled for a generation or two. Witness and line trees are cut down when the land is cleared, the pits and mounds marking the corners on the prairie are quickly destroyed when the sod is broken up, posts rot away, and no one takes the trouble to see that new and more durable marks are set to perpetuate the location of the points. Largely owing to the fact that in the areas covered by the Public Land Surveys the public roads are usually located along the section lines, even substantial stone monuments oftentimes are carelessly knocked out of place and eventually are thrown into the ditch or the fence corner. The responsibility for the maintenance of the monuments and marks, rebuilding old ones and setting new reference marks, in the interest of the landowners, rests primarily with the county surveyors, who are authorized by law to perform such work, and to file the record.

In retracing old surveys certain general rules* must be observed, which are controlling upon the location of all lands that have been granted or patented. These rules are summarized in the following paragraphs:

First. That the boundaries of the public lands when approved and accepted are unchangeable.

Second. That the original township, section, and quarter-section corners must stand as the true corners that they were intended to represent, whether in the places shown by the field-notes or not.

* Restoration of Lost or Obliterated Corners, and Subdivision of Sections, March 13, 1883, 1 L. D. 339; 2d Edition 1 L. D. 671; revised October 16, 1896, 23 L. D. 361; revised June 1, 1909, 38 L. D. 1; reprinted in 1916; revised April 5, 1939: Superintendent of Documents, Washington, D. C.

Third. That quarter-quarter-section corners not established in the original survey shall be placed on the line connecting the section and quarter-section corners, and midway between them, except on the last half mile of section lines closing on the north and west boundaries of the township, or on the lines between fractional or irregular sections.

Fourth. That the center lines of a section are to be straight, running from the quarter-section corner on one boundary to the corresponding corner on the opposite boundary.

Fifth. That in a fractional section where no opposite corresponding quarter-section corner has been or can be established, the center line must be run from the proper quarter-section corner as nearly in a cardinal direction to the meander line, reservation, or other boundary of such fractional section, as due parallelism with the section boundaries will permit.

Sixth. That lost or obliterated corners are to be restored to their original locations whenever it is possible to do so.

The surveyor has no right to alter the position of an established and recorded corner even though his resurvey may show its position to be faulty. The General Land Office possesses no authority to change the boundaries of the lands that have been disposed of by the Federal government. Resurveys are made in certain cases, when found necessary; the law requires that no such resurvey or retracement shall be so executed as to impair the bona fide rights or claims of any claimant, entryman, or owner of lands affected by such resurvey or retracement.

Whatever the purpose of the retracement may be — whether it calls for the ascertainment of the true lines of the original survey, or for the running of the subdivisional lines of a section, the rules as outlined require some or all of the definite steps which follow:

- a. Secure a copy of the original plat and field-notes;
- b. Secure all available data regarding subsequent surveys;
- c. Secure the names and contact the owners of the property adjacent to the lines that are involved in the retracement;
- d. Find the corners that may be required

First: By the remaining physical evidence;

Second: By collateral evidence, supplemental survey records, or testimony, if the original monument is to be regarded as obliterated, but not lost; or,

- Third: If lost, by the application of the rules for proportionate measurement from other available corners;
- e. Reconstruct the monuments as required, including the placing of reference markers where improvements of any kind might later interfere with the monuments or if the site is such as to suggest the need for supplemental monumentation;
 - f. Note the rules for the subdivision of sections where these lines are to be run; and
 - g. Prepare and file* a suitable record of what was found, the supplemental data that were employed, a description of the methods, the direction and length of lines, the new markers, and any other facts regarded as important.

186. Restoration of Lost or Obliterated Corners. The General Land Office defines the condition of previously established corners as follows:

An **existent corner** is one the position of which can be identified by verifying the evidence of the monument or its accessories by reference to the description that is contained in the field-notes; or one at which the point can be located by an acceptable supplemental survey record, some physical evidence, or testimony.

Even though its physical evidence may have entirely disappeared, a corner will not be regarded as lost if its position can be recovered through the testimony of one or more witnesses who have a dependable knowledge of the original location.

An **obliterated corner** is a point at which there are no remaining traces of the monument or its accessories, but at which the location has been perpetuated or may be recovered beyond reasonable doubt by the acts and testimony of the interested landowners, competent surveyors, or other qualified local authorities, by witnesses, or by some acceptable recorded evidence.

A position that depends upon the use of collateral evidence can be accepted only as duly supported, generally through its proper relation to known corners, and its agreement with the field-notes regarding distances to natural objects, stream crossings, line trees, and off-line tree blazes, etc., or through unquestionable testimony.

* In many of the states there is a well-established practice of filing of field-notes and plats of surveys, usually in one of the county offices; otherwise the record is the property of whoever pays the cost of the survey, and ordinarily it would be filed as an exhibit with a deed, or agreement, or court decree, etc.

A **lost corner** is a point of a survey the position of which cannot be determined beyond reasonable doubt, either from traces of the original marks or from acceptable evidence or testimony that bears upon the original position, and the location of which can be restored only by rerunning of lines from one or more interdependent corners.

If there is some acceptable evidence of the original location, that position will be employed in preference to the rule that applies to a lost corner.

No decision should be made in regard to the restoration of a corner until every means has been exercised that might aid in identifying its true original position.

Existing original corners cannot be disturbed; consequently, discrepancies between the new and the recorded measurements will not in any manner affect the measurements beyond the identified corners, but the differences will be distributed proportionately within the several intervals along the line between the corners, i.e., the new measurement will be distributed among the several parts of the line in the same proportion as the originally recorded measurement was distributed.

The term "single proportionate measurement" is applied to a new measurement made on a single line to determine one or more positions on that line.

The term "double proportionate measurement" is applied to a new measurement made to determine the position of a point with respect to four corners: two on a meridional line and two on a latitudinal line.

In determining positions, those derived from points on standard parallels will be given precedence over other township exteriors, and ordinarily the latter will be given precedence over subdivisional lines; section corners will be relocated before the positions of lost quarter-section corners can be determined.

A lost corner of four townships (or sections) is restored by double proportional measurement, as in Fig. 103 where *A*, *B*, *C* and *D* are four original corners from which the lost corner *X* is to be restored. First *E* is located where *X* should be by proportional measurement from *A* and *B*; similarly *F* is located from *C* and *D*. The intersection of a latitudinal line through

E and a meridional line through F is adopted as the restored position of X . The point X satisfies the proportional measurements for both latitude and departure, but does not lie exactly on either AB or CD .

In many of the surveys the field-notes and plats indicate two sets of corners along township boundaries, and frequently along section lines where parts of the township were subdivided at different dates. In these cases there are usually corners of two sections at regular intervals, and closing section corners that are placed upon the same line, but which were established at a later

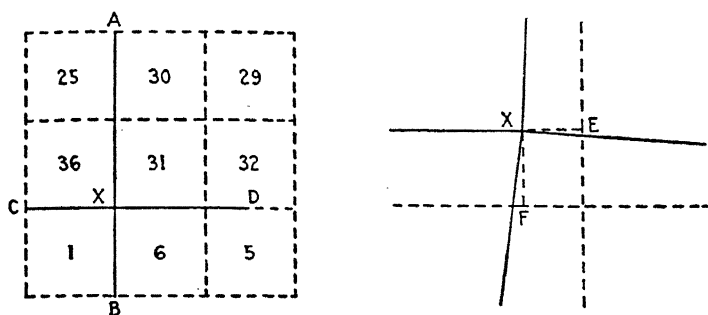


FIG. 103. REPLACING A LOST CORNER BY PROPORTIONAL MEASUREMENT.

date. The quarter-section corners on such lines are usually controlling for one side only in the older practice.

In the more recent surveys where the record calls for two sets of corners, those that are regarded as the corners of the two sections first established and the quarter-section corners relating to the same sections will be employed for the retracement, and will govern both the alignment and the proportional measurement along that line. The closing section corners, set at the intersections, will be employed to govern the *direction* of the closing lines.

In order to restore a lost corner on a township line by single proportional measurement, a retracement will be made connecting the nearest identified regular corners on the line; a temporary stake (or stakes) will be set on the trial line at the original record distance (or distances); the total distance and the falling at the objective corner will be measured.

On meridional township lines an adjustment will be made at each temporary stake for the proportional distance along the line, and it will then be set over to the east or to the west for falling, counting its proportional part from the point of beginning.

On east-and-west township lines and on standard parallels the proper adjustment should be made at each temporary stake for the proportional distance along the line, for the falling, and in order to secure the latitudinal curve*; i.e., the temporary stake will be either advanced or set back for the proportional part of the difference between the record distance and the new measurement, then set over for the curvature of the line, and last corrected for the proportional part of the true falling.

Lost standard corners will be restored to their original positions on a base-line, standard parallel or correction line by single proportionate measurement on the true line connecting the nearest identified standard corners on opposite sides of the missing corner or corners, as the case may be.

Corners on base-lines are to be regarded the same as those on standard parallels. The corners that were set first in the running of a correction line will be treated as original standard corners; those that were set afterward at the intersection of a meridional line will be regarded as closing corners.

All lost section and quarter-section corners on the township boundary lines will be restored by single proportional measurement between the nearest identified corners on opposite sides of the missing corner, north and south on a meridional line, or east and west on a latitudinal line, after the township corners have been identified or relocated.

All lost quarter-section corners on the section boundaries within the township will be restored by single proportional measurement between the adjoining section corners, after the section corners have been identified or relocated.

Lost meander corners, originally established on a line projected across the meanderable body of water and marked upon

* The term latitudinal curve as here employed denotes an easterly and westerly line, properly adjusted to the same mean bearing from each monument to the next one in regular order, as distinguished from the long chord or great circle that would connect the initial and terminal points.

both sides, will be relocated by single proportional measurement after the section or quarter-section corners upon the opposite sides of the missing meander corner have been duly identified or relocated.

A lost closing corner will be reestablished on the true line that was closed upon, and at the proper proportional interval between the nearest regular corners, to the right and left.

In order to reestablish a lost closing corner on a standard parallel or other controlling boundary, the line that was closed upon will be retraced, beginning at the corner from which the connecting measurement was originally made; a temporary stake will be set at the record connecting distance, and the total distance and falling will be noted at the next regular corner on that line on the opposite side of the missing closing corner; the stake will then be adjusted as in single proportionate measurement.

A closing corner not actually located on the line that was closed upon will determine the direction of the closing line, but not its legal terminus; the correct position is at the true point of intersection of the two lines.

187. SUBDIVISION OF SECTIONS. The sections are not usually subdivided in the field by the United States surveyors, but certain of the subdivision-of-section lines are "protracted" (i.e., drawn in) upon the township plats to indicate how the lottings are prescribed by law; the boundaries of the quarter-sections also are generally shown.

Preliminary to subdivision it is essential to identify the boundaries of the section, since it cannot be subdivided until the section corners and quarter-section corners have been found, or restored by proper methods, and until the resulting courses and distances have been determined by survey.

The order of procedure is: First, identify or reestablish the section boundary corners; next, fix the lines of quarter-sections; then, form the smaller tracts by proportional division.

In subdividing a section into quarter-sections straight lines are run between opposite quarter-section corners previously established or reestablished on the boundaries of the section. The point of intersection of these two lines is the common corner of the four quarter-sections and the legal center of the section.

Quarter-section corners on lines closing on the north and west boundaries of a regular township were established originally at 40 chains to the north or west of the last interior section corner, the excess or deficiency in measurement being thrown into the half-mile next to the township or range line. If such quarter-section corners are lost, they are reestablished by proportional measurement based upon the original record.

When the plat calls for double sets of section corners on township and range lines, the quarter-section corners for the sections south of the township line and east of the range line were not usually established in the original surveys. In subdividing such sections, new quarter-section corners should be so placed as to satisfy their calculated areas, as expressed on the official plat. Proportional measurements should be adopted for the sides of the quarter-sections where the new measurements of the north and west boundaries of the section differ from the recorded distances.

Sections that are invaded by meanderable bodies of water, or by approved claims not conforming to the regular legal subdivisions, are subdivided by protraction into regular and fractional parts, as shown by the dashed lines in Figs. 104 and 105.

Fig. 104 represents a regular section except for the parts that are set aside for mineral claims; two of the quarter-sections and two of the half-quarter-sections are normal. Fig. 105 shows a section that is made fractional by the lake; it is subdivided into aliquot parts in the usual manner as far as possible. The descriptions that follow illustrate the principles covered by the general rules in Art. 185.

To subdivide the section shown in Fig. 104, both center lines *ab* and *cd* are run to connect the opposite quarter-section corners; corner *b* on the north boundary, within the mineral claim, will be in place; the intersection at point *e* is the center. Points *f*, *g*, *h*, *i*, *j* and *k* for the sixteenth-section corners are set at mid-points along the boundaries of the two northern quarter-sections, and a stake will be needed at mid-point between *b* and *e*. The center lines of these quarter-sections are then run, thus establishing the boundaries of lots 1, 2, 3 and 4. Intersection of these lines with the boundaries of the mineral claims are established and marked.

In Fig. 105 the boundaries of the section cannot be completed because of the lake. To subdivide this section, the north and south center line lm is run on the mean of the bearings of the east and west boundaries of the section, completing the line across the water by triangulation or by a meander traverse. The east and west center line op is run, connecting the opposite quarter-section corners, and completing the line across the water by triangulation or meander traverse. The theoretical center of the section will be fixed at the calculated intersection of the center lines (in the lake). The points q, r, s, t and u for the sixteenth-section corners along the boundaries of the two southern

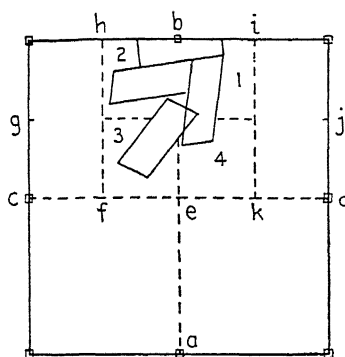


FIG. 104.

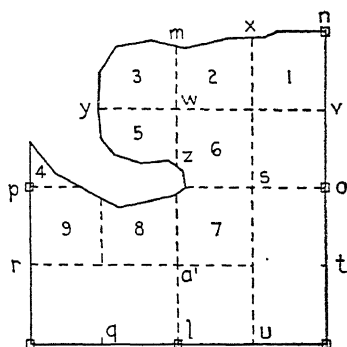


FIG. 105.

EXAMPLES OF SUBDIVISION OF FRACTIONAL SECTIONS.

quarter-sections are set, all at mid-point. The point a' is set at mid-point on the line between l and the theoretical center of the section. The two other sixteenth-section corners v and w for the northeast quarter-section will be placed at the calculated proportional distances between o and n , and between the theoretical center of the section and m respectively, to agree with the intent of the representations on the official township plat. The north and south center line of the northeast quarter-section sx will be run on the mean of the bearings of its east and west boundaries. The line wy between lots 3 and 5 will be run parallel to op , the east and west center line of the section.

CHAPTER IX

CONTOURS

188. CONTOURS. The earth's surface is made up of hills, valleys, and ridges. Because the solution of many engineering problems is dependent upon a knowledge of the form of its surface, several ways have been devised for showing the shape of the ground (relief) on maps. The *contour* method is most commonly used; it is the most satisfactory from the standpoint of the engineer.

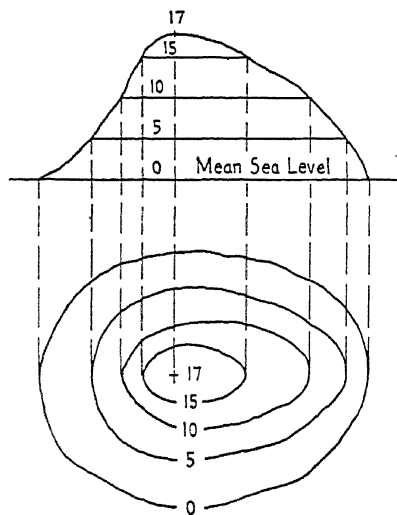


FIG. 106. CONTOUR LINES OF AN ISLAND.

A *contour* is an imaginary line on the surface of the earth connecting all points that are of equal height above some reference plane or level datum surface, usually mean sea-level. The height above the datum plane is called the *elevation*; it is the vertical distance from the chosen datum to the contour.

Contour lines on the map are similar in shape to the contours on the ground but are drawn to the scale of the map. A conception of the contour method may be obtained from Fig. 106, which represents an island. The shore-line is a contour since all points on this line have an elevation zero; it is called the zero (mean sea-level) contour. Now assume that the water rises 5 feet. The new shore-line will then define another contour which will be called the 5-foot contour, because all points in this new line are 5 ft. vertically above the zero contour. Imagine that

the water again rises 5 ft., forming a new shore-line and contour. This contour will be the 10-foot contour, since all points in it are 10 ft. above the zero contour. Similarly, the next contour will be the 15-foot contour. The highest point on the island is 17 ft. above the zero contour (mean sea-level). If these contours are plotted, the result will be as indicated in the lower part of Fig. 106, a contour map of the island. This contour map represents the different shore-lines described above as they would appear if one were looking directly downward upon the terrain.

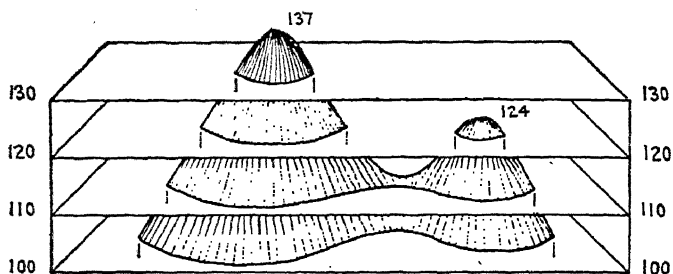
Another illustration of contours is shown in Fig. 107. The upper part of the figure shows two summits intersected by level surfaces (contour planes), spaced uniformly apart vertically. The middle portion of Fig. 107 shows the areas cut from the solids by these parallel (contour) planes. The outlines of these areas are contour lines; they are assembled in a contour map in (c), the bottom part of Fig. 107.

189. Contour Interval. Any number of contours may be drawn between two planes of different elevation, but it is obvious that if too many are drawn it will be impossible to distinguish one contour from another. In practice a definite contour interval (vertical distance between contours) is chosen for each map, so that each contour may be easily distinguished. The interval will depend upon the scale of the map and upon the purpose for which the map is being prepared. The intervals commonly used are 1, 2, 5, 10, 20, 50, and 100 ft.; generally speaking, the larger the scale of the map, the smaller the contour interval.* All contours shown will be multiples of the contour interval. The elevation of any point on the map between contours may be determined by interpolation.

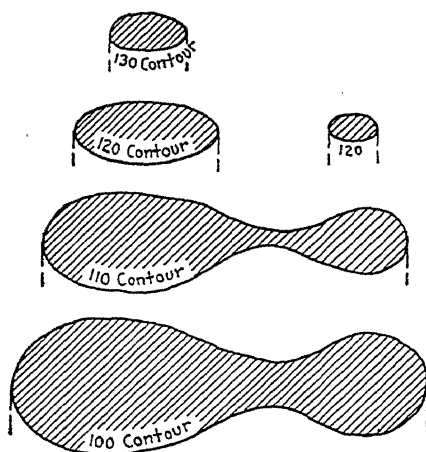
The contour lines also indicate the shapes of the various land forms that make up the earth's surface. With a little study one may visualize the terrain from a well-drawn contour map as accurately as if he were on the site.

190. Ground Slopes. The lower part of Fig. 108 shows the contours of the three geometric solids shown in the upper part of the sketch. The contours of Fig. 108(a) are concentric circles

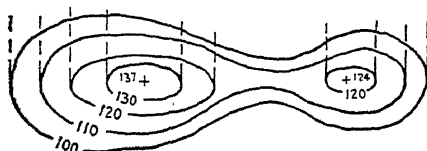
* A scale of 1 in. = 10 ft. would be termed a large-scale map; whereas a map on a scale of 1 in. = 500 ft., or $\frac{1}{500}$ in., would be called a small-scale map.



(a) Contour Planes Intersecting Terrain.



(b) Sections Cut by Contour Planes in (a).



(c) Contour Map of Terrain in (a).

FIG. 107. CONTOUR PLANES.

and are *uniformly* spaced; they indicate a uniform slope on the sides of the cone. When contours are uniformly spaced on a map, we know at once that they represent a **uniform** slope. The contours of Fig. 108(b) are closer together at the higher than at the lower elevations and represent a **concave** surface; the con-

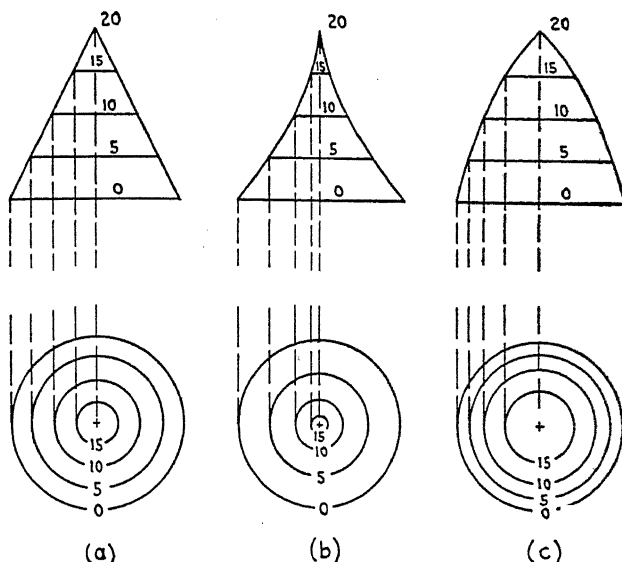


FIG. 108. CONTOUR MAP OF SIMPLE SOLIDS.

tours of Fig. 108(c) are closer together at the lower than at the higher elevations, indicating a **convex** slope. The steepness of the slope is shown also by the spacing of the contours; the steeper the slope is, the closer the contours are.

191. CHARACTERISTICS OF CONTOURS. Fig. 109 is a contour map showing a divide along *ABCDE* between the drainage systems to the north and to the south. Branching from both sides of the divide, and approximately at right angles to it, are ridges or spurs which separate the river valleys. The principal ridges on the south side of the divide lie along the directions *A* to *F*, *D* to *G*, and *E* to *H*.

The following characteristics of contours are illustrated in Fig. 109.

(1) All points on any one contour have the same elevation, as at *Y*.

(2) Every contour closes on itself either within the limits of the map, as at *A*, *B*, *C*, or beyond the limits of the map. If the contour does not close within the limits of the map, it will run to the edge of the map, as at *Z*.

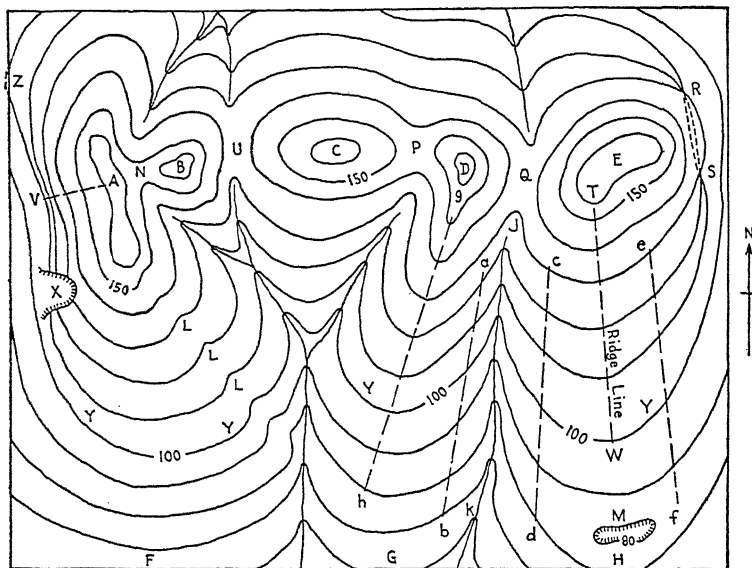


FIG. 109. CONTOUR CHARACTERISTICS.

(3) Summits are indicated by closed contours, as at *A*, *B*, *C*. Closed contours also represent depressions or hollows in the ground. Depression contours are marked, as at *M*, to distinguish them from summit contours.

(4) Contours never cross each other except in the case of an overhanging cliff; there must then be two intersections, as at *R* and *S*. Contours never split or branch. Such a condition would be represented by a knife-edged ridge or valley at exactly a contour elevation; this is not found in nature. Contours may appear to branch at the edges of a vertical cliff, but in reality they

do not branch because they are a contour interval apart along the face of the cliff.

(5) On a plane surface the contours are straight parallel lines. There is no such surface on Fig. 109.

(6) On uniform slopes the contours are equally spaced, as along the line *gh*. The slope along *TW* is concave, and along *AV* it is convex. The closer the contours are, the steeper the slope is.

(7) Between the dash lines *ab* and *cd* is a valley represented by contours. It will be noted that in a valley the contours run up one side of the valley, cross the stream at right angles and run down the other side. Since contours are always at right angles to the lines of steepest slopes, they are at right angles to the thread of the stream at the point of crossing. In general, valley contours are convex toward the stream. Small valleys are represented by indentations in the contours, as at *L*.

(8) Between *cd* and *ef* is a ridge represented by contours. The ridge contours cross the ridge line (watershed) at right angles, as at *W*.

(9) The depressions between summits are called saddles. They are shown at *N*, *U*, *P* and *Q*, and their shapes are indicated by contours between *A* and *B*, *B* and *C*, etc.

(10) At very steep slopes such as eroded banks of gravel pits, where one contour in plan would lie practically on top of the next, it is customary to use the symbol shown for the gravel pit at *X*.

192. RELATION BETWEEN CONTOUR MAP AND PROFILE.

A profile of the surface of the ground may be constructed along any line drawn on a contour map, since the elevations at which the line crosses the contours are known and the horizontal distances between the contours may be scaled or projected from the map.

For example, if a profile along the line *AB* in Fig. 110(a) is required, a series of equally spaced lines parallel to *AB* are first drawn, as in Fig. 110(b). The distances between the lines represent the contour intervals. The lines are numbered to correspond to the elevations of the contours on the map. The points at which the line *AB* intersects the contours on the map are projected perpendicularly from the line *AB* (on the plan) to the

line of corresponding elevation on the profile in Fig. 110(b). Thus point M , the intersection of line AB and the 460-ft. contour on the map, is projected to m on the 460-ft. line in the profile grid. Point m is then a point on the profile. Similarly the other points of intersection of the contours and line AB are projected. If the points are then connected as shown in Fig. 110(b), the resulting line is a profile along the line AB .

In constructing a profile along a broken line CD , a vertical section grid is first drawn, as shown in Fig. 110(c). Along the bottom line, or base, of this grid are laid off in order the horizontal distances between the contours on the map, beginning at C and ending at D . The total length of the profile (Fig. 110(c)) will then be equal to the total length of the broken line CD on the map. The "contour" points along the base are then projected upward from the base to the horizontal line in the grid which corresponds to the elevation of the contour point on the map. By connecting these points, as shown in Fig. 110(c), a profile is obtained along the broken line CD .

In Fig. 110(b) the distance between the parallel lines is plotted as $\frac{1}{10}$ in., while in Fig. 110(c) the distance between the lines is $\frac{1}{20}$ in., each space corresponding to a contour interval of 20 ft. Different vertical scales were purposely chosen to illustrate different degrees of accentuation in the profile, i.e., magnification of the vertical scale.

193. Determination of Ground Slopes from Contour Maps. The slope of the ground along any line drawn on the map is the interval between two adjacent contours divided by the distance between them as scaled from the map. This fraction is usually expressed as a per cent, in which case it is usually called the *rate of grade*. Thus, if the maximum grade along the road AB (Fig. 111) is desired, it will be found at that place where the contours crossing AB are the closest, i.e., between X and Y . The map shown in Fig. 111 has a scale of $\frac{1}{25000}$, which means that all distances on the actual ground are 25000 times the actual scaled distances on the map. To determine the rate of this grade, scale

$$XY = 0.15 \text{ in.} = \frac{25000 \times 0.15}{12} = 312 \text{ ft.} \text{ Since the contour}$$

interval 20 ft. the grade at XY is $\frac{20 \times 100}{312} = 6.42\%$.

194. Laying Out a Grade Line on a Contour Map. In Fig. 111 it is desired that a line be drawn on the map from C to M which will be on a $+2\%$ grade. The first step is to compute the length of 2% grade that will produce a rise in elevation corresponding to the contour interval. Since a 2% grade means a rise or fall of 2 ft. vertically in a horizontal distance of 100 ft., the horizontal

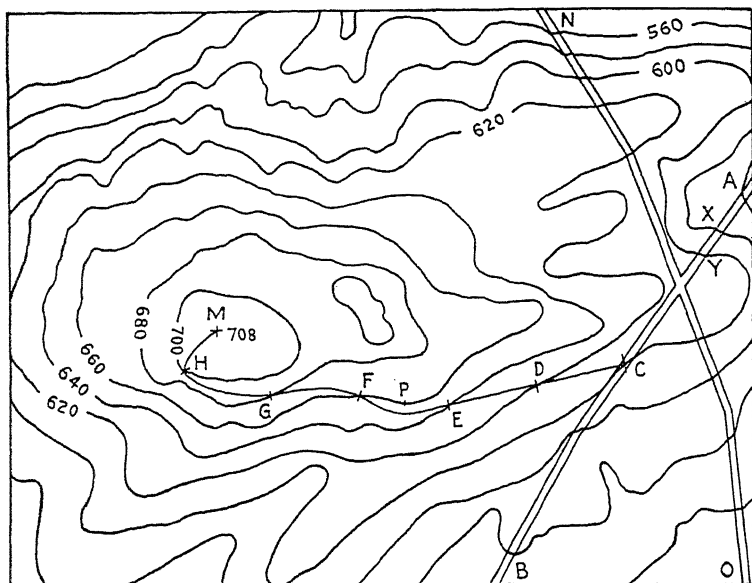


FIG. 111. LAYOUT OF ROAD ON CONTOUR MAP.

distance for a contour interval of 20 ft. is $\frac{20}{2} \times 100 = 1000$ ft. CD , DE , etc., are laid out to represent 1000-ft. distances. As the scale of the map is $\frac{1}{25000}$, these distances are actually $\frac{1000 \times 12}{25000} = 0.48$ in. Point D on the 620 contour is there-

fore plotted 0.48 in. from C , point E on the 640 contour is plotted 0.48 in. from D , and so on for the points F , G and H . The lines connecting these points are not necessarily straight

lines; they should be drawn to follow the shape of the ground. For example, if a line had been drawn straight between *E* and *F*, it would have intersected the 660-ft. contour at the point *P*, about half-way from *E* to *F*. The grade from *E* to *P* along *EP* would then be considerably greater than 2%, while the grade from *P* to *F* along *PF* would be less than 2%. To keep the grade from *E* to *F* uniformly 2%, it is necessary to draw *EF* so that the middle point of *EF* will be midway between the 640 and 660 contours, i.e., at an elevation of 650 ft. This may be done by sketching the 650 contour by interpolation and laying off $\frac{0.48}{2} =$

0.24 in. from *E* to the 650 contour, and another interval of 0.24 in. from that contour to the 660 contour. In some cases it may be necessary to interpolate more than one intermediate contour.

195. SCALE OF CONTOUR MAP. The amount of detail shown in a contour map will relate to the scale of the map as well as to the contour interval. A map is said to be a small-scale map if a large extent of the terrain is represented on a small map area. For example, a scale of 1 in. on the map to 1 mile on the ground, or 2 in. to the mile, is usually referred to as a small-scale map, whereas a scale of 10 ft., 20 ft., or 40 ft. to the inch, is referred to as a large-scale map. Intermediate scales would be, for example, 200 ft., 400 ft., or 800 ft. to the inch.

Fig. 112 shows a contour map on two different scales. It will be noticed that the same contours are represented in each drawing, but the small-scale map omits many of the details in the shape of the contours, whereas the large-scale map is full of detail which it was impossible to show on the small-scale map. Both of these maps represent exactly the same terrain. The surveyor must learn to interpret the ground shapes from the plans with due regard to the scale of the maps.

196. LOCATING AND PLOTTING CONTOURS. Three methods for locating contours are in common use. In one method the contour is traced out on the ground by locating points **at the contour elevation**. These points are then plotted on the map and a smooth curve drawn through them represents a contour line.

In the second method, **controlling points** in the topography are located. Controlling points are points situated along ridge

and valley lines and at places where the slope changes. They are first plotted on the map, and the contours are then interpolated between these points of known elevation.

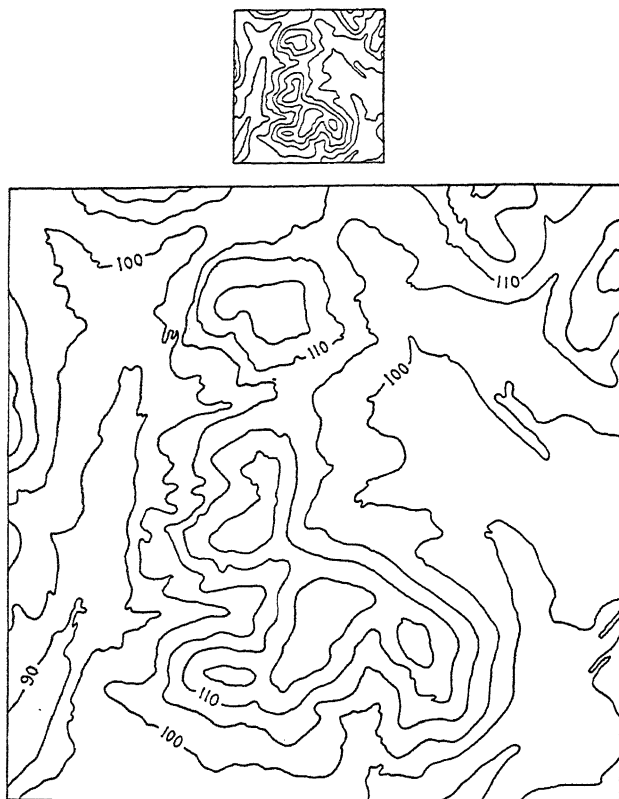


FIG. 112. CONTOURS OF SAME AREA SHOWN ON DIFFERENT SCALES.

In the third method, a grid (cross-section) system is laid out on the ground, and elevations are determined at the intersections of all cross-section lines and at points between the grid points wherever the ground slope changes. The grid system is plotted, and the elevations of all corner and intermediate points are recorded. Contours are then drawn on the map by interpolation.

Field practice in each of the three methods are now described.

197. LOCATING POINTS ON A CONTOUR. This method of locating contours is especially applicable to small areas where the scale of the map is large (such as 10 ft. or 20 ft. to an inch) and the contour interval small (such as 1 or 2 ft.), and in areas where it is desired to locate the contour lines accurately. Either a transit or the plane table may be used for locating points on contours; or a level may be used in combination with these instruments. When only the transit is used the points are located horizontally by the stadia method, and the elevations are determined by using the instrument as a level (Chap. X). The procedure of locating a point on a contour is as follows. The transit is set up at a station, and oriented by taking a sight on a traverse point with the horizontal circle reading 0° . The telescope is then leveled, and a reading taken on a rod held on a B.M. to determine the H.I. of the instrument. Assume the elevation of the B.M. to be 162.75 ft. and the rod-reading on the B.M. to be 6.48 ft., then the H.I. = 169.23 ft. The 160 contour is 9.23 ft. below the H.I. and the 165 contour 4.23 ft. below the H.I. If it is first desired to locate points on the 160 contour, the instrumentman will direct the rodman to move the rod up or down the slope until the level telescope reading on the rod is 9.2 ft., since rod-readings on the ground need only be taken to the nearest 0.1 ft. The bottom of the rod is then on the 160 contour, and its horizontal position is located by an angle from a known line and a stadia distance. Other points on the contour are located in the same manner. The above procedure likewise applies when the plane table is used. The stadia method and the use of the plane table are explained in Chap. X.

When a level instrument is used with the transit, or the plane table, the former is used to determine the position where the rod should be held on the ground to be on a contour, and the latter to locate the position of the rod when it is on the contour.

198. LOCATING CONTROL POINTS IN TOPOGRAPHY. This method is illustrated in Fig. 113. It is applicable to smaller scale maps, to larger contour intervals and larger areas than the preceding method. The points controlling the topography are

located by means of the transit, using the stadia method; or by the plane table. Good judgment is required in selecting a sufficient number of points to determine adequately the shape of the ground. The plotting is usually done in the office by a draftsman who has no knowledge of the ground and must rely on the field data alone in drawing contours. If the plotting is done in the field, as in the plane-table method, a more accurate representation of the ground will be obtained.*

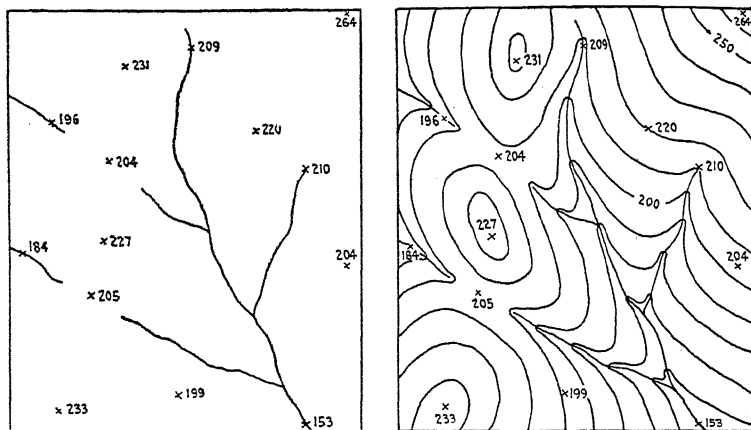


FIG. 113. LOCATING CONTOURS FROM CONTROL ELEVATIONS.

199. LOCATING CONTOURS BY CROSS-SECTION METHOD.

This method is best adapted to small areas, such as house lots. The surface to be contoured is divided into a series of squares, Fig. 114, using the transit and tape, and the corners are marked with temporary stakes. Elevations of the ground are taken to tenths of a foot at each corner of the squares and at as many intermediate points as are necessary to determine the changes in the slopes. The size of the grid squares varies from 10 ft. to 100 ft. on a side, depending on character of ground and purpose of survey. The size of the grid squares chosen should be such that the slopes between the corners of the squares can be con-

* A treatise on the relation of geology to topography will be found in Higher Surveying by Breed and Hosmer, Vol. II, Chap. VI.

sidered uniform. Where this is not the case additional points should be chosen, their positions located, and the elevation of the ground at the foot of stakes determined.

The stakes are designated by a system of rectangular coordinates, one set of parallel lines commonly being marked by letters and the other by numbers, as in Fig. 114. For example, the point

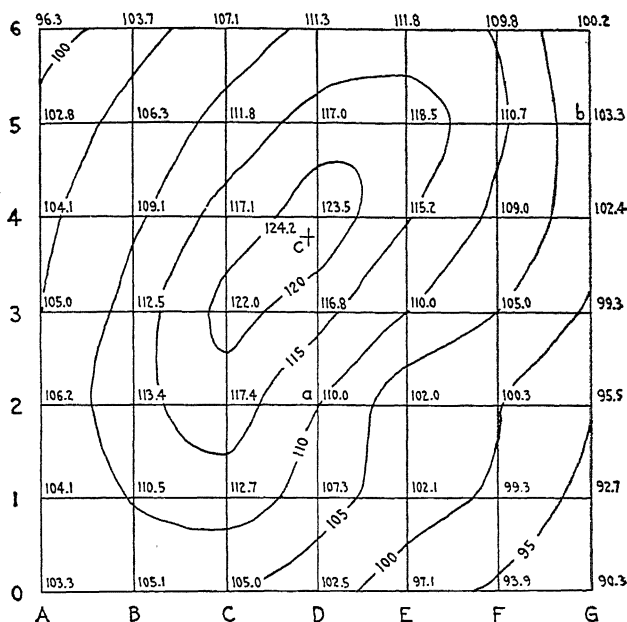


FIG. 114. LOCATING CONTOURS BY CROSS-SECTION METHOD.

a would be recorded (D₂); the point *b* as (G₅); the point *c* as (C + 90, 3 + 79). The notes are kept as in profile leveling.

When the fieldwork has been completed, the grid (squares) is plotted to the desired scale, and the elevation of each corner is marked on the plan, as shown in Fig. 114. The points at which the contours intersect the sides of the squares are then determined by interpolation. In the upper left-hand square the point at which the 100-ft. contour crosses the line A6-B6 is found as follows: the difference in elevation of points A6 and B6 is

$103.7 - 96.3 = 7.4$ ft. The difference in elevation of the 100 contour and point A6 is $100.0 - 96.3 = 3.7$ ft. Therefore the point where the 100-ft. contour intersects the line A6-B6 is $\frac{3.7}{7.4} = \frac{37}{74}$, or $\frac{1}{2}$ the distance from A6 to B6. An experienced topographer will perform these calculations mentally and sketch the contours by eye.

PROBLEMS

1. From the map shown in Fig. 110(a), draw a profile along a line parallel to the right-hand border line and passing through the point on line AB marked 662. Use a vertical scale of 1 in. = 200 ft.

2. Draw a profile along the diagonal line connecting the upper right and lower left corners of the map in Fig. 110(a). Use a vertical scale of 1 in. = 200 ft.

3. (a) Plot a profile of the road NO in Fig. 111, using a vertical scale of 1 in. = 100 ft.

(b) Compute the maximum per cent of grade on road NO.

4. The instructor should prepare a plot similar in form to that in Fig. 113 (left), showing the location and elevations of points controlling the topography of a portion of terrain, and should require the student to draw in the contours. The scale of the plot should be about twice that in Fig. 113.

5. A cross-section grid with squares 50 ft. on a side has been laid out for locating topography within a rectangular area 200 by 250 ft. The grid lines running north and south are lettered A to E from the west, and those running east and west are numbered 1 to 6 incl. from the south.

Elevations were taken as follows:

POINT	ELEV.	POINT	ELEV.	POINT	ELEV.	POINT	ELEV.
A, 1	92.1	B, 2	97.1	C, 5	90.0	D, 6	124.8
A, 2	108.2	B, 3	119.2	C, 5 + 8	87.1	D + 25, 1	106.2
A, 3	126.9	B, 3 + 12	129.0	C, 6	105.1	D + 25, 2	120.0
A, 3 + 15	130.0	B, 4	117.1	C + 25, 1	92.2	D + 15, 3	130.0
A, 3 + 33	130.0	B, 5	97.2	C + 26, 2	101.1	D + 32, 3	130.0
A, 4	113.2	B, 6	83.3	C + 40, 4	97.2	E, 1	100.0
A, 5	94.3	B + 39, 3	130.0	D, 1	103.2	E, 2	112.1
A, 6	71.1	C, 1	101.1	D, 2	108.1	E, 3	125.2
A + 15, 6	70.0	C, 2	120.2	D, 3	112.8	E, 4	133.8
A + 25, 1	83.2	C, 3	123.1	D, 4	107.1	E, 5	124.5
B, 1	91.8	C, 4	106.2	D, 5	102.9	E, 6	151.4

There is a stream in the northwesterly part of the area which rises at C + 45, 3 + 35 and flows through C + 40, 4; C + 19, 5; C, 5 + 8; B, 5 + 14; and A + 15, 6. This stream has a short branch starting at B + 47, 4 + 32 and joining the main stream at B + 30, 5 + 7.

In the southeasterly portion of the area is another stream which arises at C + 40, 2 + 43 and flows through C + 26, 2 and C + 25, 1.

Plot the grid to a scale of 1" = 40', and record elevations beside the points where they were taken. Draw in 5-ft. contours using the method of Art. 199.

CHAPTER X

STADIA SURVEYING WITH TRANSIT AND PLANE TABLE

200. STADIA SURVEYING. In this method a transit and rod are used, and distances from the transit to the rod are computed from the interval observed on the rod between two horizontal cross-wires in the telescope. It is a useful and economical method of surveying in work where a precision of 1 in 500 or less is sufficient. It is especially suitable for obtaining details for maps and for obtaining data for plotting contours. Its chief advantage over the use of the transit and tape is the rapidity and ease with which distances can be measured, irrespective of the intervening terrain.

201. Stadia Rods. In stadia surveying a graduated rod about 12 ft. long is used. These rods are about 4 in. wide and $\frac{3}{4}$ in. thick. (See Fig. 115.) They are made in a number of different patterns. The purpose of the design is to obtain a rod that can be easily read at distances up to about 1000 ft., and one which can be easily distinguished in varying topography, particularly in wooded terrain. Types a and e, Fig. 115, read directly to tenths of feet. Types b, c and d read to half-tenths. The manner of reading the rods to hundredths may be obtained by studying the designs and the typical readings shown in Fig. 115.

202. PRINCIPLES OF STADIA — HORIZONTAL SIGHTS. In addition to the horizontal and vertical cross-hairs found in all transits, most engineer's transits contain two horizontal hairs, called stadia hairs, spaced at equal distances on each side of the middle horizontal hair. These stadia hairs are used for obtaining the distance from the instrument to any point at which the graduated rod is held. To do this the rod is held **vertically** on the point and is sighted with the instrument. One of the stadia hairs is set on a full foot-mark, and the rod intercept to the other stadia hair is read to the nearest $\frac{1}{100}$ ft., as shown in Figs. 115 (c and d).

The underlying principle upon which the stadia method depends is the geometric proposition that in two similar triangles homologous sides are proportional. Referring to Fig. 116, $f_2 : f_1 = AB : ab$. If AB is called s and ab is i , then

$$f_2 : f_1 = s : i \quad [1]$$

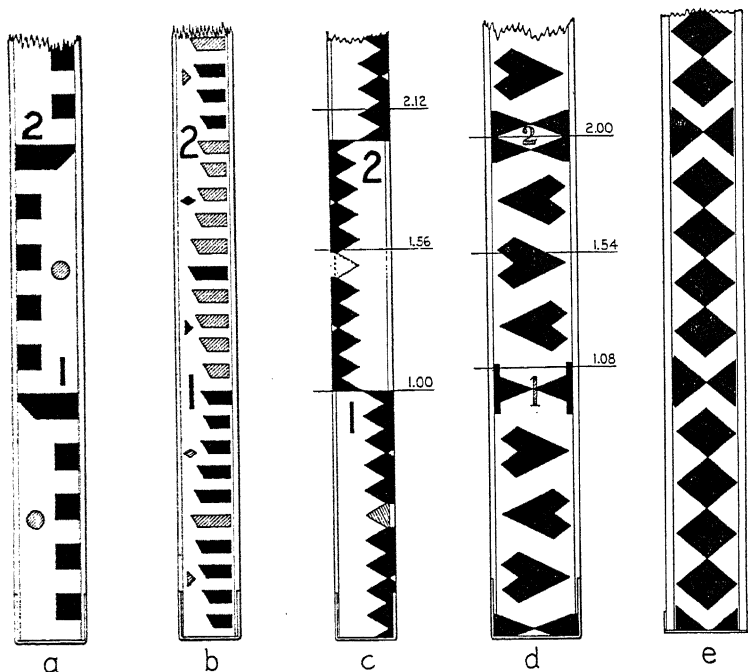


FIG. 115. STADIA RODS.

(Cross-hatched portions are painted red.)

The formula for a horizontal stadia distance may be derived through the application of the "Law of Lenses." This law is

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} \quad [2]$$

where f_1 and f_2 are conjugate foci and f is the principal focal distance.

Solving equations [1] and [2] simultaneously gives

$$f_2 = \frac{f}{i} s + f \quad [3]$$

Obviously, distance D , from the rod to the center of the instrument, equals

$$D = \frac{f}{i} s + (f + c) \quad [4]$$

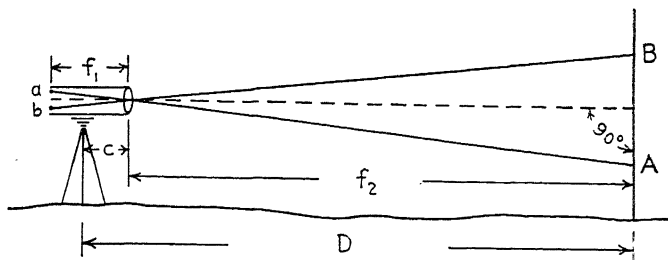


FIG. 116. STADIA PRINCIPLE, HORIZONTAL SIGHTS.

In most transits the ratio $\frac{f}{i} = 100$. Therefore the formula for horizontal distances becomes

$$D = 100 s + (f + c)$$

The term $(f + c)$ is known as the stadia constant and varies for transits from 0.75 ft. to 1.40 ft. The value f is obtained by measuring with a rule from the center of the lens, when focused at a distant object, along the telescope to the capstan-headed screws holding the cross-hair ring, and c is the distance measured along the telescope from the center of the lens to the plumb-line. In stadia practice, for transit telescopes, $(f + c)$ is taken as 1 ft.

If the telescope is the internal focusing type there is no constant to be added. The distance D will be $100 s$.

Sometimes the stadia hairs are not set in the instrument at the exact interval for a multiplying factor of 100. It is good practice therefore to check this factor by laying out a level line about 400 ft. in length with stakes every 100 ft. along the line, and by

taking rod-readings at points so marked. The rod interval readings (for the external focusing transit) should be .99, 1.99, 2.99, etc. If the readings differ from these amounts, the indicated ratios should be used in computing distances.

Stadia rods are usually about 12 ft. long. If the rod is 1400 ft. from the instrument it is evident that the rod is not long enough to permit the full stadia interval to be read. When the entire interval cannot be read, the interval between one of the stadia hairs and the middle horizontal hair is read. If the stadia hairs are equally spaced on each side of the middle hair, the half intercept is multiplied by two to obtain the full intercept; it is better practice to read both half intercepts and add them to obtain the full interval.

203. PRINCIPLES OF STADIA — INCLINED SIGHTS. In practice all rod-readings are taken when the rod is vertical; but the line of sight may be inclined to the rod, as shown in Fig. 117. In this case horizontal distances are obtained from inclined sights by the following procedure:

First measure the height of instrument (H.I.). This is the vertical distance MO and is measured with a tape from the top

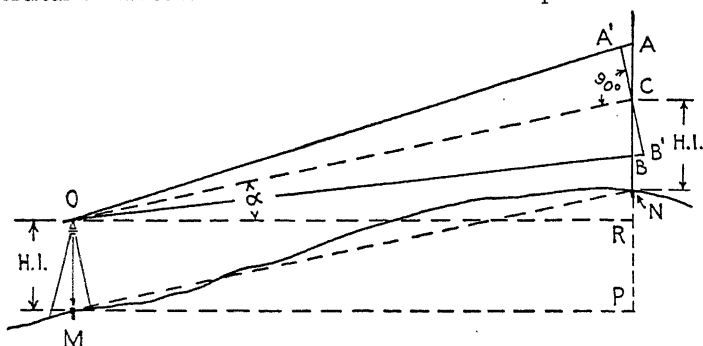


FIG. 117. STADIA PRINCIPLE, INCLINED SIGHTS.

of the stake M to the center of the horizontal axis of the transit. This definition of the H.I. should not be confused with the H.I. used in leveling. A narrow band of red cloth is tied around the rod at the height ($MO = \text{H.I.}$) measured from the bottom of the rod. With the rod held vertically at N , direct the telescope to the rod so that the middle horizontal cross-hair is near the red

band C . Slowly move the telescope with the vertical tangent screw until one of the stadia hairs bisects the nearest full foot-mark; then read the stadia interval. Next, move the telescope (using vertical slow-motion screw) until the red band C on the rod is bisected by the middle horizontal hair, and then read the vertical angle α . From the rod-reading AB and the vertical angle α , the horizontal distance MP and the vertical distance PN may be computed. Note that $PN = RC$.

The difference in elevation between the center of the instrument O and the point C on the rod is derived as follows:

$$\begin{aligned}
 RC &= CO \sin \alpha = \left\{ \frac{f}{i} A'B' + (f + c) \right\} \sin \alpha \\
 A'B' &= AB \cos \alpha \text{ (closely approximate)} \\
 RC &= \frac{f}{i} AB \sin \alpha \cos \alpha + (f + c) \sin \alpha \\
 &= \frac{f}{i} AB \frac{1}{2} \sin 2\alpha + (f + c) \sin \alpha \quad [5]
 \end{aligned}$$

For the horizontal distance from the transit point M to the rod

$$\begin{aligned}
 OR &= CO \cos \alpha = \left\{ \frac{f}{i} A'B' + (f + c) \right\} \cos \alpha \\
 &= \frac{f}{i} AB \cos^2 \alpha + (f + c) \cos \alpha \quad [6]
 \end{aligned}$$

Since α is usually a small angle, $\frac{1}{2} \sin 2\alpha$ is nearly the same as $\sin \alpha$, and $\cos^2 \alpha$ is nearly the same as $\cos \alpha$; and since $(f + c)$ is small in comparison with $\frac{f}{i} AB$, it is customary to combine $(f + c)$ with $\frac{f}{i} AB$ resulting in the following formulas when $\frac{f}{i} = 100$:

$$\begin{aligned}
 \text{Diff. Elev.} &= (100 AB + \text{constant}) \frac{1}{2} \sin 2\alpha \\
 \text{Hor. Dist.} &= (100 AB + \text{constant}) \cos^2 \alpha
 \end{aligned}$$

*These formulas apply with an external focusing telescope; with an interior focusing telescope, omit the constant.

* For more detailed derivation of formulas see *Elementary Surveying* by Breed and Hosmer, Vol. I, Arts. 219-20, or *Higher Surveying* by Breed and Hosmer, Vol. II, Chap. IV.

The work of making stadia reductions is greatly simplified by the use of stadia tables, stadia diagrams, or stadia slide rules.

204. Stadia Tables. Table VIII, based on the stadia reduction formulas, is in two parts. The part headed "Vertical Heights" gives the values for the vertical distance (difference of elevation) between two points for a rod-reading of 1.00 (distance 100 ft.) and for any vertical angle between 0° and 40° . The part of the table headed "Horizontal Corrections" gives the values to be subtracted from the distance read from the rod for distances up to 1000 ft. and for vertical angles up to $39^\circ 30'$. To illustrate the use of the table, assume a stadia intercept on the rod to be 5.09 ft. and the vertical angle $+5^\circ 29'$ (external focusing telescope).

Enter table of Horizontal Corrections. Under $5^\circ 29'$ and opposite 100 (5.09) $+ 1 = 510$, find by interpolation 5 ft. as the horizontal correction to the nearest foot. Subtract this from 510 and obtain horizontal distance = 505 ft.

Enter table of Vertical Heights. Under $5^\circ 29'$, find 9.51 ft. by interpolation. This is the vertical distance for an inclined distance reading of 100 ft. and vertical angle of $5^\circ 29'$. Hence, $9.51 \times 5.10 = 48.5$ ft. is the vertical distance (diff. elev.) for a rod-reading of 5.09 ft. and a vertical angle of $5^\circ 29'$. If the elevation of $M = 263.25$ ft., then the elevation of $N = 263.25 + 48.5 = 311.8$ ft. (See Fig. 117).

205. Stadia Diagrams. Diagrams are sometimes used in the office and occasionally in the field for reducing inclined stadia readings to the corresponding horizontal and vertical distances. These diagrams are usually plotted on large sheets of cross-section paper varying from 20 to 30 inches square, and the reduced values are obtained directly by inspection.

There are several kinds of stadia diagrams in use. One of the simpler forms is shown in Fig. 118. This is not a completed diagram, but is sufficient to show the principle of construction and the method of using it. In the portion of Fig. 118 that is marked "Horizontal Correction" is found the correction (corresponding to an observed inclined stadia distance and vertical angle) which is to be subtracted from the observed distance to obtain the horizontal distance; and in that part of the diagram

marked "Difference in Elevation" is found the vertical distance (corresponding to an inclined stadia distance and vertical angle) between the horizontal axis of the transit and the point on the rod intersected by the middle horizontal cross-hair; that is, the point on the rod at which the vertical angle was taken. If, in obtaining the vertical angle, the middle horizontal hair is set on

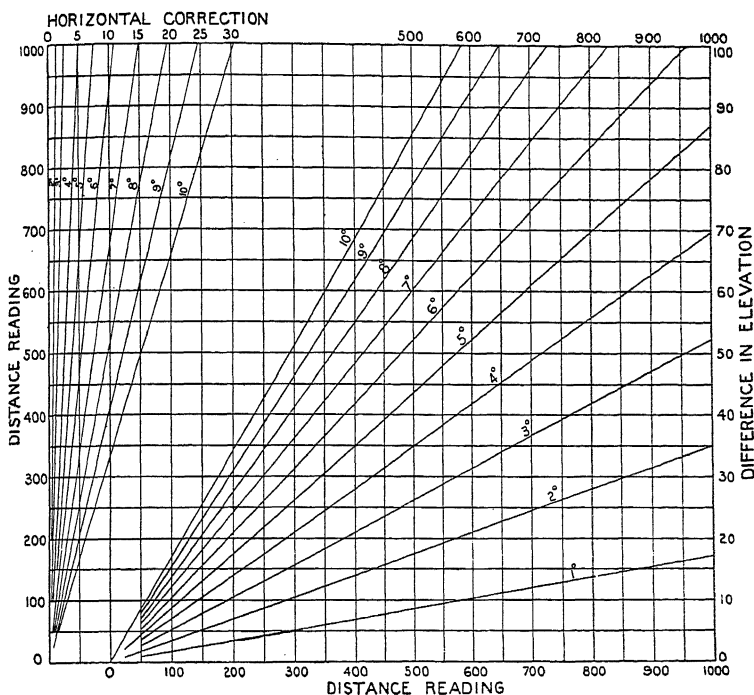


FIG. 118. STADIA REDUCTION DIAGRAM.

the rod at a point equal to the H.I. of the transit, the "difference in elevation" obtained from the diagram is the true difference in elevation of the point under the transit and the foot of the rod.

To illustrate the use of the diagram, assume an observed inclined stadia distance to be 650 ft., ($f + c$) constant included, and the vertical angle 4° . For the "difference in elevation," along "distance reading" find the vertical line corresponding to 650

along the bottom of the diagram. Follow this line vertically until it intersects the inclined line marked 4° . Project this point horizontally to the right-hand line of the diagram and read 45.5 ft.; this is the difference in elevation (vertical distance). In a similar manner the horizontal correction is found to be 3 ft.

The above diagram can be readily plotted from the values in the stadia tables. The inclined lines will be straight lines because an examination of the formulas in Art. 203 shows that both the horizontal correction and vertical distances (differences in elevation) vary directly as the distances (or the rod intervals). To be of practical use the vertical angle should be drawn for every 5 or 10 minutes on the diagram for differences in elevation, and every 30 minutes for horizontal corrections; in this way the observed values may be readily interpolated.

206. Stadia Slide Rule. The most rapid means of reducing stadia readings is by the use of a slide rule which has, in addition to the ordinary scale of numbers (logarithms of the distances), two scales especially constructed for stadia work; one consisting of values of $\log \cos^2 \alpha$, and the other of $\log \frac{1}{2} \sin 2\alpha$ for different values of α . (See Art. 203.) On some rules the values of α range from $0^\circ 34'$ to 45° , on others from $0^\circ 03'$ to 45° . In some forms the horizontal distance is read directly; in others the horizontal correction $1 - \cos^2 \alpha$, or $\sin^2 \alpha$, is given. A 10-in. slide rule gives results sufficiently accurate for all ordinary purposes.

One type of stadia slide rule is shown in simplified form in Fig. 119; certain of the finer divisions that appear on the rule have been omitted. The stadia distance for which reductions are desired is set by moving the **slide** (middle portion of the rule) to the left until the zero on the right-hand end of the slide (H scale) coincides with the stadia distance reading on the upper scale of the rule (R scale); the setting in Fig. 119 is at 650 ft.

For a vertical angle of 4° (as in Art. 205), the correct horizontal distance is read on the upper scale of the rule opposite 4° on the upper right-hand scale of the slide. It will be observed that there are two angle scales on the upper edge of the slide; one for horizontal distances running from zero to 45° from right to left, and the other for difference of elevation running from $35'$ to 45° from left to right. At the right-hand end of the H scale the

first division mark close to zero is 5° , the second 10° , the third 15° , and the fourth 20° , as marked. The distance reading for 4° of vertical angle is taken a shade to the right of the first (5°) division and shows the horizontal distance to be 647 ft.

For small angles such as are commonly encountered in stadia work the horizontal distance is difficult to read to three significant figures on the stadia slide rule, since the divisions indicating horizontal distance are very close together especially near the zero end of the slide.

It is much **easier to read the horizontal correction** than the horizontal distance. In most cases the former is only required to one or two significant figures. In Fig. 119 a scale of horizontal corrections (H.C.) is engraved on the middle of the slide. To obtain the horizontal correction for a distance of 650 ft., the rider is moved to the left until the wire coincides with 4° on the middle scale. This reading is indicated in the figure by the dash-line position of the rider. The horizontal correction is read on the lower (A) scale; it is 3.15, or 3 ft. to the nearest foot, which agrees with the horizontal correction of 3 ft. found for the same set of conditions on the stadia diagram in Art. 205.

The difference in elevation for a vertical angle of 4° and stadia reading of 650 ft. is found on the upper scale of the rule, opposite this angle on the upper left-hand scale of the slide; the reading is 45.3 ft. In Fig. 119 the rider (shown in full lines) is set at this reading.

If the vertical angle were small, such as $0^\circ 45'$, no discernible difference from 650 could be read for the horizontal distance. The

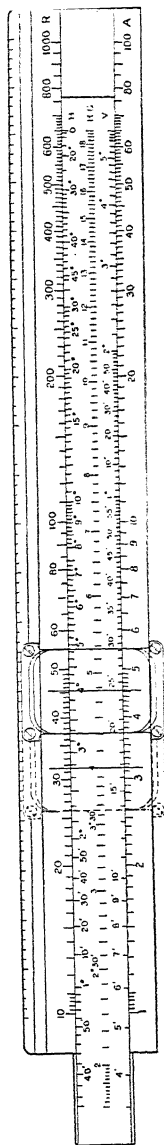


FIG. 119. STADIA SLIDE RULE.

angle of $0^{\circ} 45'$ on the upper scale of the slide falls off (outside of) the upper scale of the rule. Therefore the lower scale "A" of the slide must be used. Looking near the middle of the rule on the bottom pair of scales, it will be noted that $0^{\circ} 45'$ falls opposite 8.5 ft., which is the difference in elevation for a distance of 650 ft. and vertical angle of $0^{\circ} 45'$.

A condensed table of horizontal corrections can be easily prepared which may be carried in the field. The correction to apply to stadia distances to obtain the equivalent horizontal distances may be read from this table with ease and rapidity, the slide rule being used only to determine differences of elevation.

207. Beaman's Stadia Arc. The Beaman stadia arc is an attachment to the vertical arc of an engineer's transit, devised for the purpose of simplifying the reduction of inclined stadia observations to horizontal and to vertical distances. When the line of sight is sloping its inclination to the horizontal may be expressed as usual, in degrees and minutes; or it may be expressed in per cent of the inclined distance.

The vertical angle for which the difference in elevation is 1 per cent of the (rod interval \times 100) is $0^{\circ} 34'$; 2 per cent is $1^{\circ} 09'$; 5 per cent $2^{\circ} 52'$; 10 per cent $5^{\circ} 46'$.

The Beaman arc is marked in per cents, using these equivalents. If therefore the line of sight is so inclined that the vertical angle reads $2^{\circ} 52'$, the attached Beaman arc will read 5 per cent. This will be true if for 0° (a level line) the Beaman arc reads 0; the arc is arbitrarily marked to read 50 per cent when the line of sight is level. Hence, if the line is inclined upward 5 per cent it will read 55 per cent, and if it is inclined 5 per cent downward it will read 45. If 45 is the reading of the Beaman arc, the rod interval is 2.71, instrument constant 1 ft., middle hair rod-reading on 6.2 ft., elevation of center of horizontal axis of transit 249.3 ft.; the elevation of the ground at the foot of the rod is $249.3 - 5 \times 2.72 - 6.2 = 229.5$ ft. The effect of index correction must be eliminated either by leveling the telescope and then setting the index at 50, or else by using the vernier level described in Art. 71.

On the Beaman arc is a horizontal correction scale which gives the per cent the inclined distance must be reduced to equal the horizontal distance.

208. Azimuths. In addition to obtaining the data for determining the horizontal and vertical distances, it is also necessary in order to plot the survey to find the directions from the transit to the points on which the rod is held. These directions, called azimuths, are horizontal angles measured clockwise from some reference line, from 0° to 360° . The reference line may be an established line of a previous survey, a true meridian, or a magnetic meridian. In the case of a meridian, the south point (in surveying) is taken as 0° azimuth. For example, in Fig. 120 the azimuth of OA is 225° .

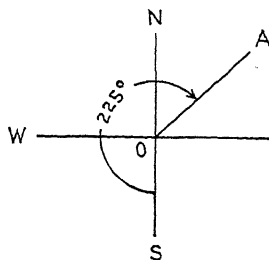


FIG. 120. AZIMUTH FROM SOUTH DIRECTION.

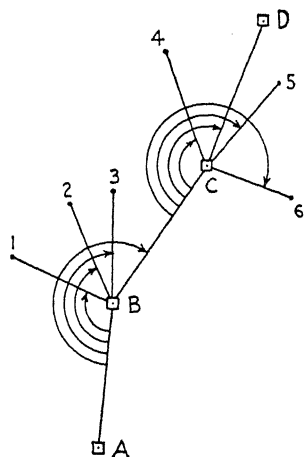


FIG. 121. AZIMUTH FROM LAST TRAVERSE LINE.

If the 0° azimuth is taken as a backsight on the last traverse point, the azimuths will be measured as illustrated in Fig. 121.

209. STADIA SURVEYS. Stadia surveys usually consist of two parts which are carried on simultaneously in the field. The first part is a traverse which controls the second part, namely, locating of details. This second part is called "*taking the side shots.*" Obviously the traverse part of the survey should be done carefully and checked. Where it is possible the traverse should form a closed figure or should be tied in at each end to established

survey points. Traverse azimuths are customarily read to the nearest minute, and side-shot azimuths to the nearest 5 minutes. The side shots are usually plotted on the plan with

a protractor, which cannot be read closer than 5 or 10 minutes of angle; hence greater precision than this in the field is unnecessary for these angles. In some cases the traverse is a transit and tape traverse, stadia being used only for locating details.

210. Orienting the Transit at the Stadia Stations. In stadia surveying a transit is said to be oriented when the reading on the horizontal circle is the azimuth of the line along which the instrument is sighting. Thus, the instrument is oriented with reference to the magnetic meridian if the circle reads 0° when the telescope points magnetic south.

The reference line may be the true south meridian, the south magnetic meridian, or the line to the previous station, as described in Art. 208. If the meridian is used, its direction is determined by sighting along the needle at the beginning of the survey. Once that south direction is determined it is carried forward from station to station by means of angles measured with the transit. If, for example, magnetic south is used to orient the instrument at the first station, this direction, no matter how inaccurate, is carried forward at each transit station. Obviously, if magnetic south were used to orient the instrument at each station, the survey would become a compass survey, with readings only good to about $\frac{1}{4}$ degree and with the possibility of even greater errors due to local attraction. The use of the magnetic south for orienting the instrument at the first stadia station is a purely arbitrary procedure.

In Fig. 122 the instrument is oriented at *A* by pointing the telescope toward magnetic south as closely as the needle will permit, the horizontal circle reading 0° , and the lower clamp tightened. Angles are then measured using the upper motion, and after all the fieldwork is completed at *A* and before moving the instrument to *B*, a sight is taken on *B* from *A*, and the azimuth of *AB* read as $201^\circ 12'$ (Fig. 122). Keeping this azimuth reading $201^\circ 12'$ set on the circle, the instrument is moved to *B* and set up over that station. The telescope is inverted and sighted on *A* and the lower motion clamped. The azimuth $201^\circ 12'$ should be read to see that the circle reading has not changed while the transit was carried from *A* to *B*. After sighting *A* with the telescope inverted, the telescope is turned on

its horizontal axis (plunged) until it is in its normal position (right side up). The telescope is now pointing in the direction AB , the azimuth of which we found when at A to be $201^{\circ} 12'$. The instrument is now oriented, for if the circle were brought back to 0° , the telescope would then be pointing exactly in the same direction that was accepted as the south direction when the transit was originally oriented at A .

When the azimuth is carried from one station to the next the transit must be handled with great care. There is always a chance of jarring it and changing the angle reading, which is one of the defects in this method.

211. Another method of orienting the instrument at B is as follows: from Fig. 122 it will be seen that if the azimuth of $AB = 201^{\circ} 12'$ the azimuth of $BA = 201^{\circ} 12' - 180^{\circ} = 21^{\circ} 12'$. Hence, when the transit is set up at B , the circle should be set to read $21^{\circ} 12'$, and A sighted with telescope **direct**. The instrument is now oriented. This method avoids the danger of the circle setting becoming shifted as the transit is carried from A to B ; it also eliminates any error that may be due to the fact that the line of sight may not be exactly perpendicular to the horizontal axis, because in this method the telescope is not plunged. There is always the possibility, however, of computing the back azimuth BA incorrectly.

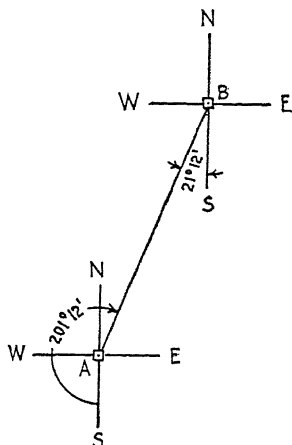


FIG. 122. CARRYING AZIMUTHS.

Stadia surveys are frequently tied in to already defined traverse lines, the bearings of which (true or magnetic) have been previously established, and this fact may be used for orienting the instrument. Suppose the bearing of an established line EF is $S 30^{\circ} 18' E$, and that the instrument is set at F . To orient, sight E with the circle reading $149^{\circ} 42'$. If the upper motion is now loosened and the telescope is turned until the circle reads 0° , the telescope will be pointing south.

Many surveyors measure and record the azimuth to the stadia station ahead as the **first** sight taken with the instrument after it is oriented at a new set-up (see Fig. 126); this is done to eliminate the possibility of errors in the traverse angles, resulting from the lower plate being moved when the many side-shot azimuths were being measured. In this case, however, the transit is again sighted to the station ahead, and the circle reading is again taken (but not recorded) as a check just before the transit is moved ahead to the next traverse station.

212. Some surveyors prefer to orient the instrument by backsighting on the preceding survey point **with the circle reading 0°** , and to measure all angles at that traverse point clockwise from that direction. In this method no azimuths are carried from one traverse point to another, since the angles at each point are referred to the direction of the back traverse line. This method avoids the danger of the circle setting becoming shifted when being moved from one stadia point to the next. It has this disadvantage, however: should the note-keeper carelessly forget to state in his field-notes the backsight used for 0° at any set-up, the field-notes taken from that point cannot be plotted.

The advantage in using the meridian as 0° direction at each set-up lies in the fact that all points may be plotted from one direction (the meridian), which can be transferred with the triangle and straightedge to any point on the plan from which angles (azimuths) are to be plotted with a protractor.

The surveyor must determine from experience which of these methods he prefers. In the notes in Fig. 123 the former method (0° on magnetic south) is employed, whereas in Fig. 126 the latter method (0° on last traverse line) is used.

213. Organization of Stadia Party. An effective stadia survey party should consist of three men; a transitman, a note-keeper, and a rodman. If the distances between shots are long, two rodmen may be used to advantage. If the survey is being done in a wooded area, two axemen may be necessary. The speed of the party is usually limited by the note-keeper. In addition to recording the data given by the transitman, his notes must show neat sketches and must give other information which can be readily and correctly interpreted by the draftsman.

214. Stadia Surveys of Shore-Line. Fig. 123 shows stadia survey notes of a portion of a lake shore. Field-notes are recorded on the left-hand page. Traverse stations are indicated thus \square . The stadia stations at which the instrument was set up are indicated, $\overline{\square}$ at $\square A$, $\overline{\square}$ at $\square B$, etc. Under each set-up are shown the data taken at that station. The first column headed "Pt." indicates the point at which the rod was held when sighted from the station recorded at the top of the column of side shots. The next two columns give the distances and azimuths to the points. The right-hand page shows a sketch of the shore-line, indicating the relative positions of the stations, and the points at which the rod was held. Note that the constant (1 ft.) is not included in the value recorded in the column headed Distance.

In making this survey, $\square A$ was chosen so that a reasonable portion of the shore-line could be seen. The instrument was set over $\square A$, the circle set to read 0° , and the telescope oriented by pointing it to magnetic south as indicated by the magnetic needle. The lower motion was then clamped, and the azimuth angles measured using the upper motion. From $\square A$ the instrumentman observed the distance and the azimuth when the rod was held at Pt. 1; then at Pt. 2, etc. These side-shot points were selected at such intervals as would properly represent the shape of the shore-line when plotted to the scale chosen for the map. Between points 1 and 2 the shore-line was straight, hence these points were chosen a considerable distance apart. Near $\square A$ there was a bend in the shore, hence points 2, 3 and 4 were taken at closer intervals to define this bend. After point 7 was observed, it was decided to choose a new stadia station. The point $\square B$ was chosen because it could be seen from $\square A$, and at the same time it would permit a view of a reasonable portion of the shore-line ahead.

In sighting the side-shot points, the face of the rod was held in the direction of the instrument for both distance and azimuth readings. When sighting $\square B$ from $\square A$, however, the rod was held on $\square B$ with its edge toward $\square A$, and the azimuth was read to the nearest minute or closer. The face of the rod was then held toward $\square A$ in order to obtain the distance. Observ-

ing $\square B$ completed the instrument work at $\square A$. The transit was then moved to $\square B$ and set up. With the telescope inverted, and the vernier checked to make certain the circle reading was the azimuth of the line AB , the edge of the rod, held on $\square A$, was sighted, the lower motion clamped, and the telescope brought erect. The distance BA was then read on the rod and compared with the previous reading AB for a check.

Stadia Survey of Shore Line Bog Lake, Watertown, Ohio					Jones π Ray	April 30, 1941
Pt.	Dist.	Az. L			Description	
	π at $\square A$	$\square A$			$f + c = 1.1$ ft. not incl. in recorded distances. π at A oriented 0° on Mag. S.	
1	249	207 10'				
2	62	170 55				
3	41	91 05				
4	74	359 20				
5	281	337 15				
6	407	348 30				
7	515	359 45				
$\square B$	692	8 52				
	π at $\square B$	$\square B$				
$\square A$	691					
8	53	189 00				
9	29	91 10				
10	60	315 10				
11	188	312 55				
12	314	314 55				
13	419	330 05				
14	524	332 20				
$\square C$	525	330 15				
	π at $\square C$	$\square C$				
$\square B$	525					
Etc.						

FIG. 123. STADIA NOTES OF SHORE-LINE.

It should be particularly noted that no vertical angles are shown in the notes. This is because no elevations were required, the transit stations being practically at the shore-line, thereby virtually eliminating any inclined sights which would have affected the horizontal distances. Had the elevation of the stadia stations been such that greatly inclined sights had to be taken, vertical angles should have been read and correction

applied for reducing the observed distances to the horizontal distances, especially on the traverse portion of the survey.

215. STADIA SURVEY REQUIRING ELEVATIONS. Fig. 124 shows stadia notes of a simple topographical survey. The sketch in the note-book shows the stations where the transit was set up and the points at which the rod was held. It will be noted that the property line on the south side of Harvard St. is defined by two concrete monuments, numbered 150 and 151. These monuments were established during a previous survey and were used for orienting the instrument in the present survey, since the true bearing of the line from Mon. 150 to Mon. 151 was known. The note-book shows that the true bearing was $N\ 40^{\circ}\ 30'\ W$, the equivalent azimuth being $139^{\circ}\ 30'$.

In making the survey, the instrument was set up over Mon. 150. The H.I. = 4.61 ft. was measured and a red cloth band fastened at this reading on the rod. By direct leveling the elevation of Mon. 150 was determined from the nearby B.M. as follows: the rod was held on the B.M., elevation 152.75 ft.; the telescope on the transit was brought to a horizontal position by means of the telescope bubble, and a rod-reading of 5.18 ft. was obtained. The elevation of the telescope would then be $152.75 + 5.18 = 157.93$ ft. From this value the H.I. = 4.61 ft. is subtracted. The result 153.32 ft. is the elevation of the Mon. 150. To orient the instrument, an angle equal to the azimuth was set off on the horizontal circle. The rod was held on Mon. 151, edge toward the instrument, and bisected by the vertical hair. The azimuth was again read for a check. Next, the face of the rod was turned toward the transit, and the middle horizontal hair was set near the red band. One of the stadia hairs was then set on a full foot-mark, and the interval read to the other stadia hair. One hundred times this interval (250) was recorded in the note-book. The center horizontal hair was next adjusted to bisect the red band, and the vertical angle $-0^{\circ}\ 16'$ was read and recorded. Note that some of the vertical angles are recorded as + or positive, and some as - or negative. A plus angle is measured when the telescope is inclined upward, and a negative angle is measured when the telescope is inclined downward from the horizontal. These signs designate whether

Topographical Survey of Property Belonging to John Brown Glen Corner, Winfield, Mass.					
Sta.	Dist.	Az. Ang.	Vert. Ang.	Diff. El.	Elev.
	π at Mon. 150	H.I. = 4.61			153.32
BM A98	79	$103^{\circ} 00'$	$0^{\circ} 00'$ on 5.18		152.75
Mon. 151	250	$139^{\circ} 30'$	$-0^{\circ} 16'$	- 1.2	152.1
1	251	$147^{\circ} 00'$	$-0^{\circ} 14'$	- 1.0	152.3
2	171	$150^{\circ} 15'$	$-0^{\circ} 30'$	- 1.5	151.8
3	64	$169^{\circ} 20'$	$-0^{\circ} 41'$	- 0.8	152.5
4	62	$176^{\circ} 55'$	$-0^{\circ} 21'$	- 0.4	152.9
5	68	$180^{\circ} 05'$	$-0^{\circ} 15'$	- 0.3	153.0
6	76	$204^{\circ} 55'$	$-0^{\circ} 18'$	- 0.4	152.9
7	43	$227^{\circ} 05'$	$-0^{\circ} 20'$	- 0.3	153.0
8	52	$322^{\circ} 10'$	$+0^{\circ} 53'$	+ 0.8	154.1
9	42	$0^{\circ} 05'$	$+0^{\circ} 49'$	+ 0.6	153.9
A	243^{-2}	$162^{\circ} 59'$	$+5^{\circ} 25'$	+ 22.9	176.2
	π at A	H.I. = 4.75			176.2
Mon. 150	242^{-2}		$-5^{\circ} 26'$	- 22.9	153.3
10	133^{-2}	$340^{\circ} 05'$	$-6^{\circ} 50'$	- 15.8	160.4
11	58^{-2}	$359^{\circ} 55'$	$-10^{\circ} 18'$	- 10.4	165.8
12	45^{-4}	$88^{\circ} 50'$	$-16^{\circ} 58'$	- 12.8	163.4
13	75	$151^{\circ} 45'$	$+0^{\circ} 32'$	+ 0.7	176.9
14	91	$179^{\circ} 45'$	$-4^{\circ} 07'$	- 6.6	169.6
15	117^{-1}	$197^{\circ} 30'$	$-5^{\circ} 31'$ on 8.75	- 15.3	160.9
16	98^{-2}	$206^{\circ} 55'$	$-8^{\circ} 50'$	- 15.0	161.2
Etc.					

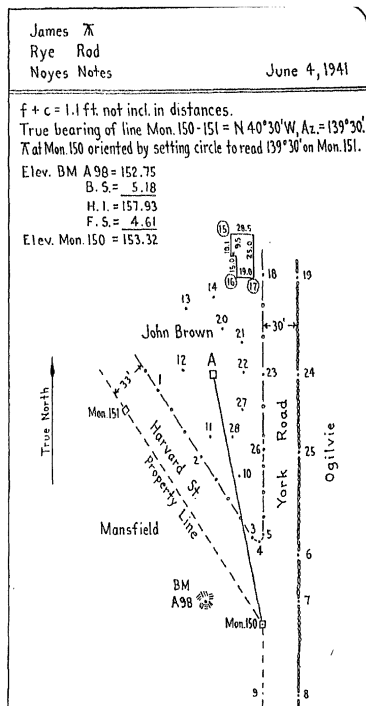


FIG. 124. STADIA FIELD-NOTES.

the "difference in elevation" should be added to or subtracted from the elevation of that point over which the instrument was set in the process of obtaining the elevation of the point at which the rod was held.

The survey includes so many inclined sights that the only practical way of computing the reductions to the horizontal distances and differences in elevation is by the use of the stadia slide rule, stadia diagrams, or stadia tables. Use of the tables is illustrated in the following examples for Mon. 151 and Pt. 1.

Mon. 151. For an observed distance of $250 + 1$ ft. and a vertical angle of $-0^{\circ} 16'$; the Hor. Corr. = 0; Diff. Elev. = $2.51 \times 0.47 = -1.2$.

The horizontal correction, if any, is recorded in the notes by a small figure in the distance column which indicates the amount to be subtracted from the recorded distance to obtain the horizontal distance. Note that in Fig. 124 the instrument constant has not been included in the "Dist." column; consequently at side-shot 10, the horizontal distance is $133 + 1 - 2 = 132$. Sometimes the horizontal distances are recorded in a special column. The difference in elevation is recorded in the column headed "Diff. El." with the proper $+$ or $-$ sign. The elevation of the Mon. 151 is found by applying this value to the elevation of the stadia station. Thus $153.3 - 1.2 = 152.1$ ft. For Pt. 1, the reductions are as follows:

$$\text{Hor. Corr.} = 0$$

$$\text{Diff. Elev.} = 2.52 \times 0.41 = -1.0$$

$$\text{Elev.} = 153.3 - 1.0 = 152.3 \text{ ft.}$$

Opposite Pt. 15 and in the vertical angle column, the notation " $-5^{\circ} 31'$ on 8.75" is found. This means that for some reason the red band on the rod could not be seen from the instrument; possibly a bush was on line (Fig. 125). Hence for obtaining the vertical angle, the middle horizontal hair was set on the rod at a point 8.75 ft. from the bottom, or just 4 ft. above the red band. The corresponding vertical angle was $-5^{\circ} 31'$. The horizontal correction is found by proceeding as before. Interpolating the table, we find the horizontal distance correction to be 1 ft.; the

actual horizontal distance is $117 + 1 - 1 = 117$. The difference in elevation is computed by proceeding as before, and then applying to this value the difference between the point sighted on the rod and the H.I. This process is indicated in Fig. 125, as follows:

$$\text{Diff. Elev.} = 1.18 \times 9.57 + 4.0 = 11.3 + 4.0 = 15.3 \text{ ft.}$$

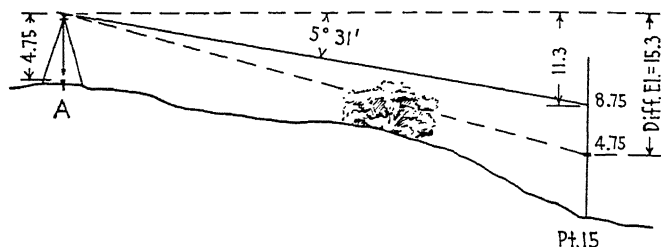


FIG. 125. VERTICAL ANGLE TAKEN TO OTHER THAN H.I. POINT.

It will be noticed that in Fig. 124 several of the vertical angles are small. In such cases the rod-readings might have been taken by the direct leveling process; in which case the elevations of the points on which the rod was held would be found by subtracting the rod-readings from the elevation of the horizontal axis of the telescope.

This direct leveling procedure is particularly applicable when a contour is being traced out on the ground. If, for example, the 140-ft. contour is to be traced out, the instrumentman directs the rodman to the place on the ground where the rod-reading is (elevation of axis of telescope - 140.0). This point is then located by stadia distance and azimuth. Such work can be expedited if a level party works with the transit party, the former detecting the points on contours, and the transitman locating them.

216. Alternate Method of Stadia Survey. Fig. 126 illustrates another method of making a stadia survey and recording the field-notes. The instrument was oriented by setting up on a previously established transit station $\odot A$, and backsighting $0^{\circ} 00'$ on another $\odot K$. An angle was then turned in a clockwise direction $48^{\circ} 12'$ to $\square B$. In this set of notes there is no fixed

reference line for the azimuths (such as the south point); the instrument was oriented at each set-up by backsighting $0^{\circ} 00'$ on the preceding station.

The elevations for the survey were obtained from the elevation of the top of stake at $\odot A$, which was previously established as 422.91. This elevation was carried forward to stadia stations B , C , D , etc., by the usual stadia method. The side-shot elevations, however, were obtained by a different process from that previously described. At every side shot the horizontal hair of the telescope was **sighted on any convenient full foot-mark** on the rod and the vertical angle was measured, rather than sighted on a particular place on the rod called the "H.I."

Referring to Fig. 126 it will be observed that for side shot 1, the plus vertical angle $2^{\circ} 14'$ was measured to the 3.0-ft. mark on the rod. The table of vertical heights (Table VIII) gives +6.4 as the difference in elevation between the telescope axis and the sight on the rod at the 3.0-ft. mark. That distance (+6.4) is recorded in the notes above the vertical angle to which it applies. Since the reading was taken 3.0 ft. above the ground, the foot of the rod is $6.4 - 3.0 = 3.4$ above the center of the telescope, i.e., it is **3.4 above the elevation of the transit telescope**. This is also recorded in the notes above the rod-reading. Hence, the elevation of the ground at Sta. 1 is the instrument elevation (430.8) plus $3.4 = 434.2$ ft. as recorded in the "Elev." column of the notes.

In this method **all elevation computations refer to the elevation of the horizontal axis of the instrument**. The rod-reading at which the vertical angle is read is **always subtracted** from the vertical height obtained from the stadia tables, diagrams, or slide rule. This is so because the vertical angle, whether + or -, is always measured to some foot-mark above the bottom of the rod; the rod-reading must therefore be subtracted algebraically from the tabular vertical height to obtain the true difference in elevation between the axis of the transit and the ground where the rod is held.

There are several advantages to be found in this method. First, the use of a band around the rod at the H.I. distance above the ground may be avoided. This is important since the band

may be misplaced as the rod is carried from point to point. Second, the vertical angle may be taken in that part of the rod which is clearly visible. Third, the computations are routine,

Stadia Survey of Watershed of Black River, N.J.					
May 15, 1940			Hall, Rand, Toms		
Point	Dist.	Az.	Vert. Ang.	Rod Reading	Elev.
At A, B.S. 0°00'	on K, H.I.	= 4.55			422.91
B	146	48° 12'	+ 3.27 + 1° 17'	4.55	426.18
At B, B.S. 0°00'	on A, H.I.	= 4.65		Elev. A =	430.8
A	146	0° 00'	- 3.27 - 1° 17'	4.65	422.91
C	361	257 01	+ 4.10 + 0° 39'	4.65	430.28
1	163	19 47	+ 6.4 + 2° 14'	+ 3.4 3.0	434.2
2	214	319 00	- 13.8 - 3° 42'	- 18.8 5.0	412.0
3	521	295 20	- 10.9 - 1° 12'	- 16.9 6.0	413.9
4	50	89 05	+ 5.0 + 5° 43'	+ 1.0 4.0	431.8
5	110	69 30	- 0.6 - 0° 18'	- 2.6 2.0	428.2
6	34	150 25	+ 4.0 + 6° 53'	- 6.0 10.0	424.8
7	129	190 20	- 5.8 - 2° 3	- 12.8 7.0	418.0
8	246	226 50	- 14.0 - 3° 16'	- 18.0 4.0	412.8
At C, B.S. 0°00'	on B, H.I.	= 4.60		Elev. B =	434.9
B	362	0° 00'	- 4.11 - 0° 39'	4.60	426.17
D	293	202 44	+ 11.25 + 2° 12'	4.60	441.53
9	57	53 25	+ 1.1 + 1° 06'	- 2.9 4.0	432.0
etc.					
Note: Stadia Constant			included in Distances.		

FIG. 126. STADIA NOTES.

whereas when the sights are taken on the H.I. mark on the rod and referred to the ground elevation at the instrument (as in Art. 215), a change of method is required whenever the sight must of necessity be taken at some other point on the rod.

The distance interval is measured as usual in the same general portion of the rod in which the vertical angle is taken.

It will be noticed in Fig. 126 that the vertical angles taken from $\square B$ to $\odot A$ and $\square C$ were read on the H.I. (4.65). This was done so that the vertical angle from $\square B$ to $\square C$, for example, could be compared with that from $\square C$ to $\square B$. If the readings are taken on the H.I. forward and backward on a traverse line, they should be the same. In this way a field check may be obtained on the traverse station angles which control the elevations of the several stadia stations.

217. STADIA LEVELING. In rough or hilly country, stadia leveling affords a rapid and more economical means of determining elevations than direct leveling does. Leveling by stadia is carried on in a manner similar to direct leveling with the exception that, instead of the difference in elevation of two points being obtained from two rod-readings taken with a level, it is obtained from two stadia distances and two vertical angles taken with a transit.

In general, sights should not be over 500 ft., but in some cases sights of 1000 ft. or more may be warranted. If the line of sight of the instrument is parallel to the axis of the telescope bubble, no error will result from foresights and backsights of different length, but it is good practice to assume that the adjustment of the instrument is not perfect and to make the sum of the foresights and backsights approximately equal, as in ordinary leveling (Art. 101).

Stadia leveling may be used to establish approximate elevations of bench marks (B.Ms.) or turning points (T.Ps.). It may also be employed to obtain the profile of a line laid out on the terrain. In fact it can produce within its limitations the same results as direct leveling.

Examination of the field-notes in Figs. 124 and 126 shows that stadia leveling was there carried on, using the stadia traverse points as turning points. The transit was set up over each turning point after its elevation had been determined by stadia distance and angle from the last turning point.

Determination of elevations of selected points can also be made by stadia leveling, following the same general procedure as

in direct leveling, in which case the transit will occupy any convenient location between turning points. In hilly country direct leveling is a very slow process because many set-ups are required; whereas stadia leveling, although not so precise as direct leveling, is equally rapid in flat or hilly country.

At the start, the transit is set up at a location where a rod held on the bench mark and on the first turning point can be conveniently seen. The rod is held on the bench mark, and a stadia distance is read. The rod-reading on the middle cross-hair is recorded, and the vertical angle is read to the nearest minute. The index correction for the vertical angle is then determined, unless it has been eliminated by means of a vernier level attachment (Art. 71). The difference in elevation is determined by use of the tables, which gives the amount that the place on the rod cut by the middle cross-hair was above, or below, the horizontal axis of the transit, from which the elevation of the horizontal axis may be computed. The rod is next held on a turning point, and the distance and vertical angles are observed. From these data the amount the T.P. is above or below the horizontal axis of the transit can be computed. Fig. 127 illustrates the form of recording data for stadia leveling.

Two precautions should be carefully observed in stadia leveling; i.e., **the rod should be held vertically**, using a rod level if possible, and **the index correction should be applied to all vertical angles** except in the case where a vernier level attachment is used.

There is no check on the above-described process unless it closes on a B.M., the elevation of which is known, or unless a circuit is made returning to the starting point. The same is true, of course, in ordinary leveling. It will be noticed, however, that in stadia leveling where the transit occupies the turning points as in the field-notes, Figs. 124 and 126, two determinations are made of the elevation of each transit station; this provides a check on the work at each set-up, in much the same way as in the so-called "double-rodged" line of levels (Art. 102).

Stadia leveling can be used to advantage for profile leveling, either with the instrument set up at station points and at angle points in the line to be profiled; or with the transit set up at

Sta.	Backsights			Diff. in Elev.	Elev. at Hor. Axis of Transit	Foresights			Diff. in Elev.	Elev. of Station	Remarks
	Stadia Dist.	Vert. Ang.	Tabular Diff. Elev.			Stadia Dist.	Vert. Ang.	Tabular Diff. Elev.			
BM ₁	479	on 6.4 -8° 32'	+70.4	+76.8	1062.5					985.7	Add $f + c = 1$ ft. to Distances.
TP ₁	630	on 5.2 -5° 24'	+59.1	+64.3	1189.9	556	on 6.8 +7° 16'	+69.9	+63.1	1125.6	
TP ₂	540	on 6.7 +2° 10'	-20.4	-13.7	1217.9	659	on 7.3 +4° 16'	+49.0	+41.7	1231.6	
						489	on 5.4 -3° 24'	-28.5	-33.9	1184.0	

FIG. 127. STADIA LEVELING.

It should be observed that in these notes the usual conventions followed in direct leveling do not hold, i.e., backsights are not necessarily plus, nor foresights minus. The sign applied to the difference in elevation depends upon the position of the telescope with respect to the point sighted. For example, when the instrument was sighting on B.M.₁ a negative angle was read which indicated that the axis of the instrument was higher than the point sighted on the rod at B.M.₁, hence the difference in elevation was added to the elevation of the bench mark to obtain the elevation at the instrument.

one side of this line and readings taken at the points along the line where the slope of the ground changes.

THE PLANE TABLE

218. The plane table is one of the most effective surveying instruments for accurately obtaining detail and topography. It consists of a tripod on which is mounted a drawing board (about $24'' \times 31''$) which can be leveled and oriented. Drawing paper is attached to the top of the board, and on this the map is drawn in the field. Sights are taken by means of an instrument

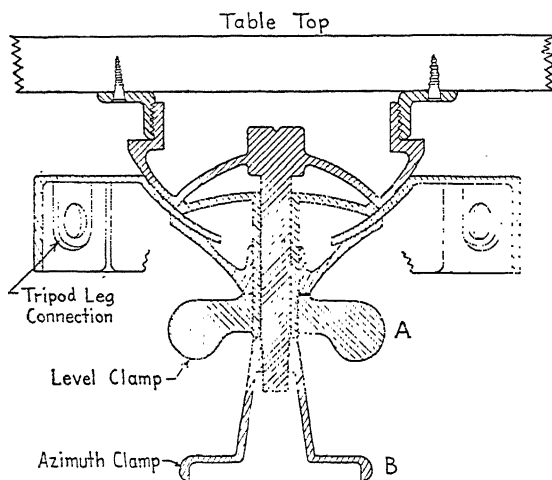


FIG. 128. JOHNSON PLANE-TABLE MOVEMENT.

called the *alidade*, which is placed on top of the table. When using the plane table the topographer can see the form of the ground that he is mapping, and thereby obtain a more accurate and representative interpretation of the terrain than is usually obtained by other surveying methods.

219. **Plane Table.** There are several makes of plane tables, one of the most commonly used is the Johnson type. Fig. 128 shows the details of the tripod head used for leveling and orienting the table. This tripod head consists essentially of several spherical surfaces in contact with each other. When both clamps

A and *B* are loose, the table top is free to tilt up and down and to revolve about the vertical axis. After the table has been leveled (Art. 223), clamp *A* is tightened to hold the board in the horizontal position. The top may then be rotated about the vertical axis, and by tightening clamp *B*, it may be held in any desired direction.

220. Alidades. There are two kinds of alidades: (1) the open sight; (2) the telescope.

The *open sight alidade* (Fig. 129) consists of a flat piece of rectangular metal about 1 in. wide, $\frac{1}{8}$ in. thick, and varies in

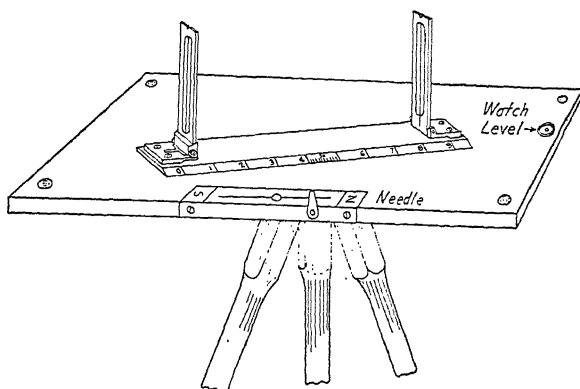


FIG. 129. OPEN SIGHT ALIDADE AND TRAVERSE TABLE.

length from 8 to 18 in. Vertical sight vanes are mounted at both ends in such positions that the sides of the base are parallel to the line defined by the vanes. The open sight alidade is used to advantage in traversing when the scale of the map is small.

221. The telescope alidade (Fig. 130) consists essentially of a telescope mounted on a transverse (horizontal) axis the ends of which are supported in wyres or standards as in a transit. The standards are attached to, or integral with, the top of a post, the bottom of which is rigidly fastened to the center of a rectangular shaped piece of metal forming the base of the alidade. The base is about 3 in. wide, $\frac{1}{8}$ in. thick; the length varies from 12 to 22 in. Both sides of the base are parallel to the plane of motion of the telescope, i.e., parallel to the line of sight so long

as instrument is in perfect adjustment. Attached to the base is a circular level, or two spirit levels at right angles to each other. These levels are used in leveling the plane table. Some alidades have a *declinatoire* (compass-box) attached to the base for use in orienting the table when high precision is not required.

The telescope is provided with stadia hairs and a vertical arc for use in determining stadia distances and differences in elevation. As the vertical motion of the telescope is limited, vertical angles of elevation and depression can only be read to about 30° .

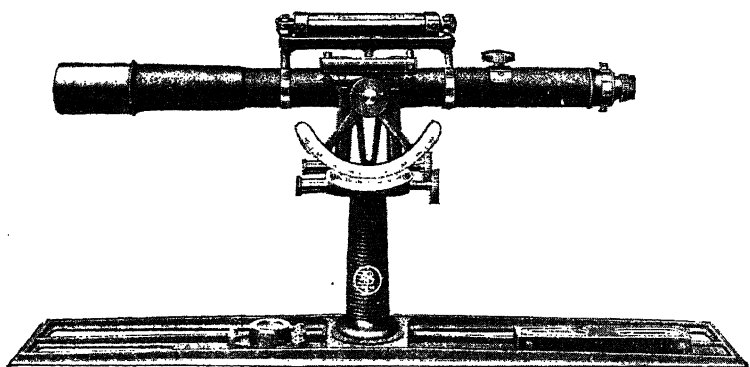


FIG. 130. THE ALIDADE.

An important accessory of all alidades is the *striding level*. This level is more sensitive than the levels which are attached to the base. It may be fastened on top of the telescope tube, and it is used for determining the index correction to the observed vertical angles; this correction being the reading of the vertical arc when the line of sight is truly level. The striding level is also used for leveling the telescope when the alidade is used for direct leveling. (See Art. 230.) Many alidades are provided with a "vernier control level." This level is mounted on an arm with the vernier and may be moved independently of the telescope by means of a tangent screw. Its purpose is to eliminate the index correction before a vertical angle is read. This is accomplished by centering the vernier control bubble just before reading the angle. (See Art. 71.)

The **index correction** is of the utmost importance in **plane-table work**. When a plane table is set up, the board is intended to be perfectly level. This condition rarely exists because of the flexibility of the wooden table top and the weight of the alidade. Moving the alidade into different positions on the plane-table board will cause the table top to deflect and thus cause variable index corrections, which at times may be quite large.

The usual stadia rods for giving sights and obtaining distances are used with the plane table and alidade. (See Art. 201.)

222. Organization of a Plane-Table Party. A plane-table party making a topographical survey should include at least three men: the instrumentman, the note-keeper, and the rodman. A second rodman may here be used to advantage, as in a stadia survey, where the distances between shots are great. The instrumentman operates the plane table and alidade, and does the plotting. The note-keeper records the data given by the instrumentman, i.e., the rod-readings and vertical angles, and reduces these data immediately, usually by means of a stadia slide rule, to the corresponding horizontal distances and elevations of the points at which the rod is held. The rodman should be trained to choose proper control points, to hold the rod on these points, and to record in his own note-book any information that will be helpful in constructing the map; especially observations relating to non-uniformity or unusual shape of ground which is visible to him, but not to the note-keeper or instrumentman.

223. SETTING UP AND ORIENTING THE PLANE TABLE. Theoretically, when a plane table is being set up over a station and oriented, the point on the map corresponding to the station will be vertically above the station, and lines on the map will be parallel to the corresponding lines on the ground, as in Fig. 131. In practice, however, the map point is usually set vertically over the station by estimation, i.e., a is over A . The care which should be taken depends upon the scale of the map. If the scale is an inch to the mile, the map point may be a foot to one side of the ground point without introducing appreciable error; but for large-scale maps the error should not exceed the smallest distance that can be plotted. For example, if the scale is 20 ft.

to an in., and the smallest plottable distance $\frac{1}{100}$ in., the map point should be placed within 0.2 ft. of its exact position over the ground point.

In actually setting up the table it is good practice to approximately orient it by eye, and then move it until the map point is roughly over the station mark. If it is desired that the map point be set precisely over the station mark, this may be accomplished by holding a plumb-line, or by dropping a small stone from a point on the under side of the board which is directly under the map point, and then moving the table until the map point is directly over the station mark.

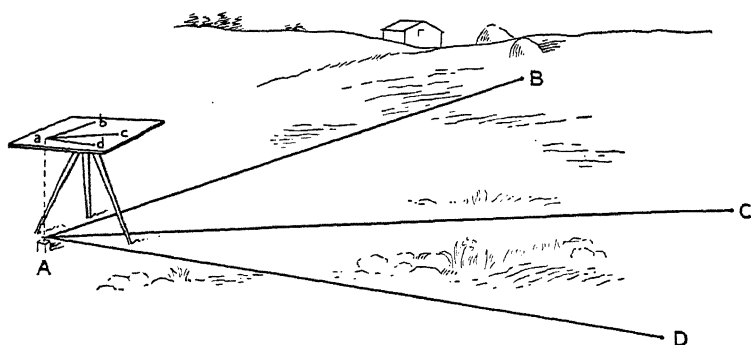


FIG. 131. ORIENTING THE PLANE TABLE.

When finally set up the tripod head should be approximately level, and the tripod legs should be spread well apart and firmly set in the ground.

The table is next leveled by placing the alidade near the center of the table top and tilting the board until the bubbles on the alidade are centered. The top is then clamped in this correct position.

To orient the table (Fig. 131), place one edge of the alidade through the station map point *a* and through any other plotted map point *b* whose position *B* is fixed on the ground. Rotate the table on its vertical axis until *B* is bisected by the vertical hair of the alidade. Clamp in this position. The table is now oriented.

224. Locating Points by Radiation. A survey is required of the property bounded by $ABCDE$, Fig. 132, by the method of radiation. Point O is chosen on the ground from which $ABCD$ and E can be seen. Point o on the paper is next chosen so that the outline of the property may be plotted to the desired scale, within the limits of the map board. The plane table is set up at Sta. o and clamped in the desired position.

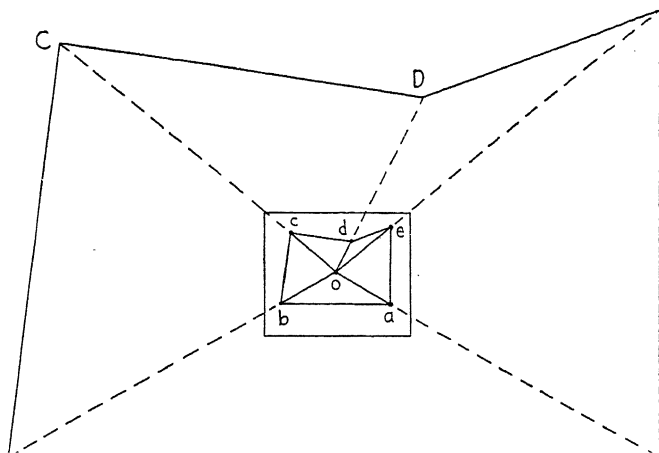


FIG. 132. RADIATION.

With one edge of the alidade through o , sight A and draw a line of indefinite length along the side of the alidade through o . Proceed in the same manner and sight on Stas. B , C , D and E . The distances OA , OB , OC , OD and OE are obtained by stadia or taping, and plotted to scale along the corresponding rays on the map. These points are shown on the map at a , b , c , d and e . A plan of the property is obtained by connecting these points.

225. Traversing with the Plane Table. Fig. 133 illustrates the method of plotting figure $ABCDE$ by traversing with the plane table. Point a is selected and a direction ab is chosen on the paper so that all of the points to be located will fall within the limits of the board. The table is set up with the plotted

point a over A , and oriented by placing the alidade along the adopted direction ab and sighting on B .

The distance AB is obtained by stadia or measured directly with a tape and is laid off to scale from a , thus locating the point b on the map which corresponds to B on the ground. While the table is at a , it is good practice to locate also point e . The table is next moved and set up at B . It is oriented by placing the side of the alidade base along the line ba and turning the table about

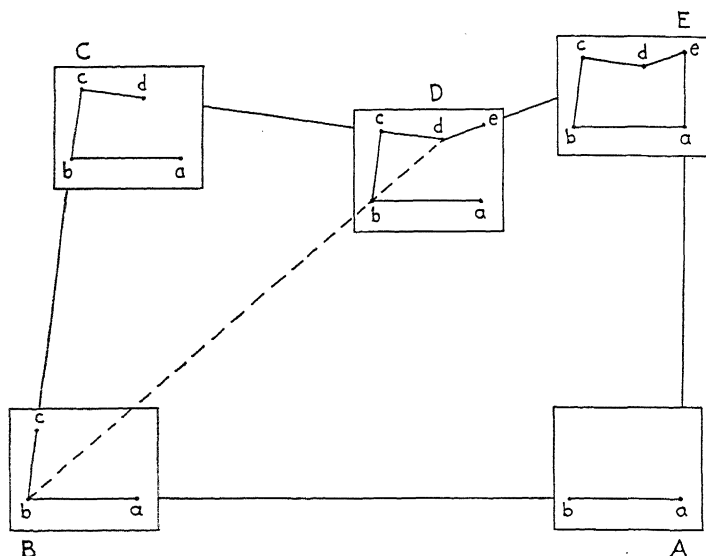


FIG. 133. TRAVERSING.

its vertical axis until A is bisected with the vertical hair. The table is then clamped in this position. The alidade, with its edge through b , is moved until C is bisected. A line is drawn along the alidade edge, and the stadia distance BC is laid off to scale. Thus c is located. By repeating the process at C , D and E , the traverse is completed. At Sta. A a check on the precision with which the orientation has been carried around the traverse is obtained as follows: After the table is set up at E and oriented by the line ed , the edge of the alidade base is placed through e and a , and it is then determined how closely the line

of sight falls on A . The discrepancy, if any, indicates the error in orientation or in measurements of distances.

Other checks may be obtained when stations are intervisible. For example, if B and D are intervisible, after the table is oriented at D , the alidade base may be placed along db . In this position the line of sight should bisect B .

226. Locating Points by Intersection. Fig. 134 illustrates the method of locating points by intersection without actually

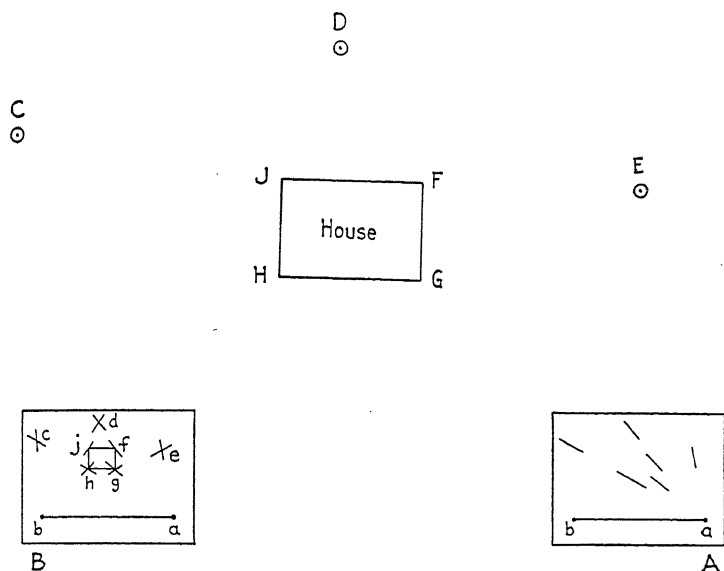


FIG. 134. INTERSECTION.

occupying them. For this purpose the distance between the two ground points, such as A and B , must be known. This line ab is plotted to scale on the plane-table sheet in such a position that all points which it is necessary to locate will plot within the map limits. The table is set up at A , and oriented by placing the alidade along ab and sighting B . With the alidade pivoted at a , the points C, H, G, F, D and E are sighted in turn and rays are drawn along the edge of the alidade. Somewhere along these rays will be the points c, h, g, f, d and e . The table is then

moved to B , set up, and oriented by sighting A . With the alidade pivoted at b , the points C, D, J, H, G and E are sighted and rays are drawn. The points at which these rays intersect the corresponding rays drawn from a , as at c, d, h, g and e , mark the map location of the points sighted in the field. It will be noticed that the back corners J and F of the house, which is assumed to be rectangular, cannot be located by intersection from A and B . To locate F , a perpendicular is drawn to hg at g . This perpendicular intersects at f the ray drawn from a in the direction toward F . Point h is located in a similar manner.

The intersection method is used for locating plane-table stations graphically, and for locating points that are inaccessible or to which it is difficult to obtain a linear measurement.

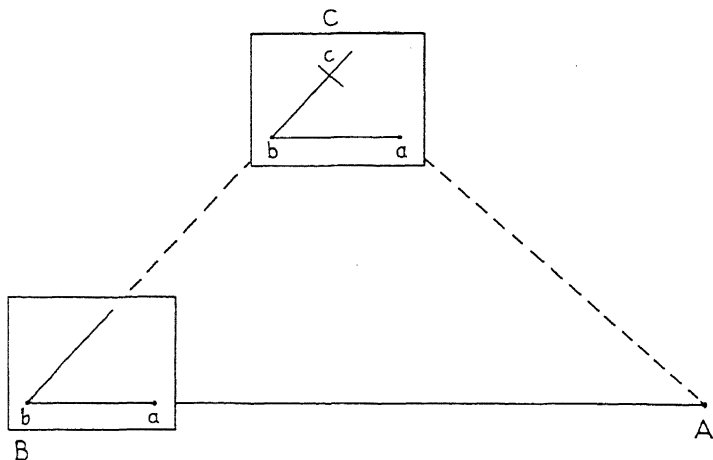


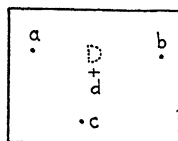
FIG. 135. RESECTION.

227. Locating Points by Resection. The method of resection is another way of locating points. For this method it is necessary to have two points that have been established and plotted on the map. To illustrate, let the two known points in Fig. 135 be represented by A and B , and the corresponding points on the map by a and b . Assume that the table is set up at B and oriented by sighting on A . Point C is to be located by resection (not by intersection or by measuring any distances). Place one

edge of the alidade through b , and sight C . Draw a line of indefinite length along the edge of the alidade through b . The map point c is somewhere in this line. Move the table to C , set up and orient by placing the edge of the alidade along the line of indefinite length from b , and sight B . Then, with the alidade pivoted about a , sight A and draw a line along the edge of the alidade through a that will intersect the line of indefinite length from b . The intersection of these lines is at c , the location of C on the map.

A
△

B
△



C
△

FIG. 136. THREE-POINT PROBLEM.

228. The Three-Point Problem.* If the plane table is set up over any point from which three established points may be seen, the position of the point over which the table is set may be determined from the plotted positions of the three established points. Thus in Fig. 136, let A , B and C be any three field stations whose plotted positions on the plane-table sheet are at a , b and c . Let D be any unknown point in the field from which A , B and C are visible. Set the plane table over the unknown point D . The problem is to find the location of d on the sheet, corresponding to D on the ground.

The solution of the three-point problem is a graphical one. In *Lehmann's Method*, the most commonly used in practice, the

* See Higher Surveying by Breed and Hosmer, Vol. II, Chap. V, for more complete treatment.

position of d is found by trial. In this method the plane table is set up at the unknown point D , leveled, oriented by estimation, and clamped. If this first estimated orientation were correct, resection lines from A , B and C would intersect at point d . As the table, however, was oriented only approximately, resection lines from A , B and C probably will not intersect at a point, but will form a triangle of error $a'c'$, $a'b'$, $b'c'$, Fig. 137, the size of which will depend on the error in orientation. From the triangle of error the true position of the point d may be estimated very closely and indicated by a dot marked on the sheet. The edge of

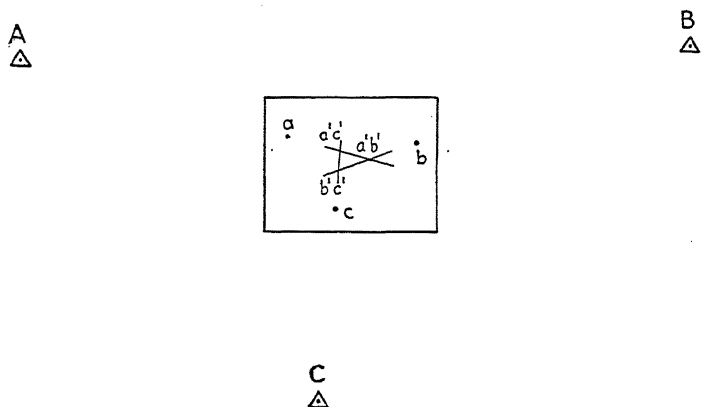


FIG. 137. TRIANGLE OF ERROR.

the alidade is now placed through this newly adopted point d and the most distant plotted point a , and the table oriented by sighting A . Resection lines are again drawn from B and C . If the latest estimated position of d was in error, a smaller triangle of error than the first will be obtained. By successive trials the triangle may be made so small that it is practically a point.

In estimating the correct position of the point on the map from the triangle of error, the following geometric relations will be found useful.

(1) If the plane table is inside the triangle ABC , the point d is inside the triangle of error.

(2) If a circle is passed through any two plotted points as a , b , and the intersection of the two resection lines from a and b , as $a'b'$ in Fig. 137, it will also pass through the true position of d , for the angle made by any two resection lines from a and b is the angle ADB . Similarly, a circle through a , c , and the intersection $a'c'$ of the resection lines from a and c will pass through d . Hence the intersection of these two position circles will give the

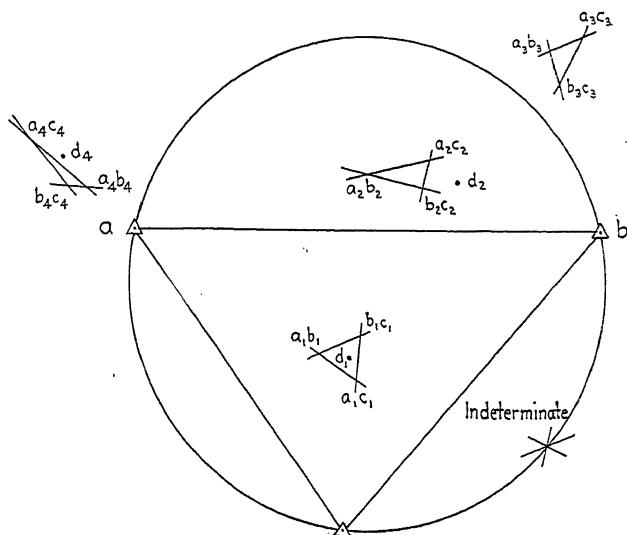


FIG. 138. DIFFERENT POSITIONS OF THE TRIANGLE OF ERROR.

position of the point sought. A similar circle through b , c and $b'c'$ will pass through the intersection of the first two circles. In practice these circles are not actually drawn; their positions are estimated by eye in judging the correct position of d .

(3) The perpendicular distance of d from any resection line is proportional to the distance of the table from the station from which that line is drawn.

Fig. 138 shows where the plotted position d of point D may be expected to lie with respect to the triangle of error, when D lies

inside the triangle abc , when it lies within the circle passing through abc , and when it lies outside the circle.

There is one case, however, where the three-point problem will not apply, i.e., if D is on a circle passing through A , B and C . In this case an indefinite number of locations for d would be found.

229. CONTROL OF PLANE-TABLE SURVEYS — HORIZONTAL CONTROL. In making plane-table surveys the plane-table stations may be located graphically on the map by the traversing method, by intersection, and by resection, as previously explained. However, the location of stations by purely graphical methods is to be avoided over extended areas where the scale of the map is comparatively large. The accumulation of errors made by using graphical methods may cause large errors in the location of the plane-table stations, since each successive station is dependent on the location of the previous station. For this reason plane-table surveys are usually controlled by transit and tape traverses, by stadia traverses, or by triangulation. When such control is provided, it is carefully plotted on the plane-table sheet before the plane-table work is started in the field. When transit and tape traverses are used, the table is set over one station and oriented by sighting another station, the alidade base lying along the line on the map connecting the plotted positions of these two points. Stations of minor importance may be located graphically, using the table as previously described, and should be checked whenever possible. For instance, a graphical plane-table traverse may start from one station on a transit and tape traverse, and a check may be obtained by running to another station on the same traverse; or the traverse may be run between two triangulation stations. If the control is triangulation, these stations may be occupied and the table oriented by sighting another station; or the table may be located and oriented by use of the three-point problem.

230. Vertical Control. In plane-table surveys where elevations are desired, it is necessary to determine the elevation of the station over which the instrument is set or the elevation of the instrument itself. The elevations of the stations may be found by:

- (1) direct leveling from bench marks or from station points of known elevation,
- (2) stadia distance and vertical angle to a point of known elevation or to a B.M. near the station,
- (3) vertical angle to a triangulation point of known position and elevation.

The first method is particularly applicable when the plane table is used in connection with a transit and tape traverse for which the elevations of the stations have been determined by B.M. leveling; or where B.Ms. have been established near the traverse control stations. The striding level is used to make the line of sight truly horizontal when the rod-readings are taken.

The second method is used for determining the elevations of new stations where no control has previously been established, such as would be the case if the control traverse were developed by use of the plane table as the survey progressed. An original elevation is obtained from a point of known elevation or from a B.M. and it is carried forward by the method of stadia leveling. (See Art. 217.)

The third method is used in determining the elevation of a plane-table station whose position has been determined by the three-point problem. The elevations of the three stations must be known. Vertical angles are observed to definite points on the signals which are erected at these stations. The vertical distance of such points above the station mark must be known, so that the elevation of the points sighted may be computed. The index correction must be carefully observed or eliminated. The horizontal distance from the table to the signals may be scaled from the map with sufficient accuracy for computing the elevation. By multiplying each scaled distance by the natural tangent of the vertical angle observed to that stations, the difference in elevation between the point sighted on the signal and the alidade is obtained.

If, in the example on the next page, the H.I. of the alidade is 4.4 ft., the elevation of the station occupied by the plane table is $646.1 - 4.4 = 641.7$ ft. Note particularly that if the horizontal distances from the plane table to the stations are greater than

about one-half mile, a correction for curvature and refraction should be applied in computing the difference of elevation. (See Art. 100.)

EXAMPLE

STA.	HOR. DIST.	VERT. ANGLE	INDEX CORR.	NAT. TAN	DIFF. ELEV.		HT. PT. SIGHTED	ELEV. ALIDADE
					PT. SIGHTED AND ALIDADE	ELEV.		
A	2065	+1° 16'	+03'	.02298	-47.5	680.7	13.4	646.6
B	1550	-0° 55'	00'	.01600	+24.8	609.2	11.6	645.6
C	1060	+2° 26'	-02'	.04191	-44.4	676.5	14.0	646.1
Mean								646.1

231. Locating Detail with the Plane Table. The methods used in making plane-table surveys and in locating detail should be modified according to the purpose and scale of the map. Large-scale maps (10 to 50 ft. to the in.) such as those, for example, which might be needed by a landscape architect, should be controlled by a transit and tape traverse. A sufficient number of B.Ms. should be established, so that the elevations of control points may be readily determined by direct leveling. The plotted control points on the plane table should be set vertically and with care over the corresponding station points on the ground, as described in Art. 223. Distances to definite points should be determined by taping rather than by stadia, since errors of one or two feet in stadia distances will be appreciable on the scales here considered; or such points may be located by intersection. Indefinite points, such as outlines of shrubbery, etc., may be located by stadia. Contours should be accurately drawn, preferably by locating a series of points on the contour, as explained in Chap. IX.

For maps of intermediate scales (100 to 500 ft. to the in.), the control may be either a triangulation system, a taped or stadia traverse, or a combination of the two. For obtaining detail, additional plane-table stations may be established by plane-table traversing, intersection, resection, or even by using the three-point problem. It is not necessary to center the plotted point over the stations as carefully as in the case of the larger scale maps. Detail may be located by stadia measurements or by intersection. Topographic control points may be determined

by stadia, and contours sketched from these by interpolation, as explained in Chap. IX.

On very small-scale maps the control is obtained principally from triangulation points, sometimes supplemented by taped traverses. Plane-table stations are established chiefly by the three-point problem. Much detail is located by intersection, and elevations are found by vertical angles and by distances scaled from the map.

PROBLEMS

1. On a comparatively level piece of ground three stakes *A*, *B* and *C* are set in line. $AB = 100$ ft. and $AC = 1000$ ft., both distances being measured with a steel tape. A transit is set at *A* and a stadia interval of 0.98 ft. observed on a stadia rod held on *B*. The rod is then held on *C* and the observed interval is 9.96 ft. If the $(f + c)$ of the instrument is 1.0 ft., compute $\frac{f}{i}$ in each case.

2. With a transit and steel tape stakes are set every 100 ft. along a fairly level line for a distance of 1000 ft. With the instrument set at a distance equal to $(f + c)$ in the rear of the line stadia intervals are observed on a rod held on stations as follows:

STANCE	STADIA INTERVAL
100	1.01
200	2.02
300	3.04
400	4.05
500	5.05
600	6.06
700	7.08
800	8.09
900	9.10
1000	10.10

Compute the average stadia interval factor $\frac{f}{i}$.

3. The following readings were observed on a stadia rod. $(f + c) = 1.0$ ft. not included in Dist.

Pt.	DIST.	Az.	VERT.
	$\overline{\wedge}$ at $\square A$		ANGLE
1	365	$10^{\circ} 15'$	$-1^{\circ} 00'$
2	589	$25^{\circ} 40'$	$+3^{\circ} 18'$
3	943	$38^{\circ} 35'$	$+5^{\circ} 41'$
4	780	$39^{\circ} 45'$	$+8^{\circ} 18'$

(a) If $\frac{f}{i} = 99.0$ compute the horizontal and vertical distances.

(b) If $\frac{f}{i} = 100.0$

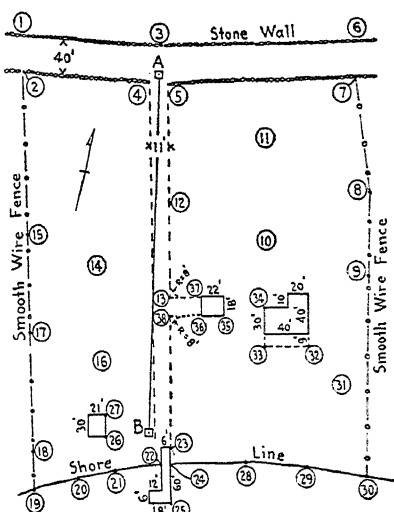
4. The following are the notes of a stadia traverse. Compute the horizontal distances and elevations. The elevation of $\square A$ is 169.7 ft. The stadia interval $\frac{f}{i} = 100$ and $(f + c) = 1.0$ ft.; latter is not included in the recorded distance.

PT.	DIST.	AZIMUTH	VERT. ANGLE	DIFF. ELEV.	HOR. DIST.	ELEVATION
		$\overline{\wedge}$	at $\square A$	H.I. = 4.95		169.7
$\square B$	262		$89^\circ 16'$	$+3^\circ 12'$		
		$\overline{\wedge}$	at $\square B$	H.I. = 5.36		
$\square A$	262			$-3^\circ 12'$		
$\square C$	438		$135^\circ 52'$	$+4^\circ 18'$		
		$\overline{\wedge}$	at $\square C$	H.I. = 5.08		
$\square B$	439			$-4^\circ 17'$		
$\square D$	668		$140^\circ 01'$	$+7^\circ 54'$		
		$\overline{\wedge}$	at $\square D$	H.I. = 5.50		
$\square C$	666			$-7^\circ 55'$		
$\square E$	534		$148^\circ 59'$	$-3^\circ 33'$		

5. In the following stadia notes

- Compute the horizontal distances
- Compute the elevations
- Plot on a scale of 1 in. = 40 ft.; contour interval 5 ft.

PT.	DIST.	AZIMUTH	VERT. ANGLE	DIFF.		REMARKS
				ELEV.	ELEV.	
B.M.	$\overline{\Lambda}$	at $\square A$	H.I. = 5.07	86.21	(f + c) = 1.0 not incl. in	
				85.33	Dist. $\overline{\Lambda}$ at $\square A$.	
					Oriented, Mag. S.	
					B.S. on B.M. = 5.95.	
1	141	95° 35'	+1° 07'			
2	124	78° 45'	+1° 20'			
3	36	164° 30'	-0° 30'			
4	6	49° 05'	+0° 03'			
5	5	292° 00'				
5	211	243° 15'	-0° 00'			
7	196	254° 35'	-0° 03'			
8	226	275° 35'	-0° 41'			
9	263	292° 05'	-1° 23'			
10	149	305° 00'	-2° 32'			
11	106	276° 25'	-0° 55'			
12	85	342° 45'	-2° 25'			
13	175	344° 00'	-2° 43'			
14	163	9° 40'	-1° 53'			
15	156	38° 10'	-0° 45'			
$\square B$	279	347° 22'	-3° 23'			
$\square A$	$\overline{\Lambda}$	at $\square B$	H.I. = 5.23			
16	68	120° 25'	+3° 23'			
17	134	105° 30'	+4° 26'			
18	120	62° 10'	+3° 21'			
19	139	43° 00'	+0° 08'			
20	90	44° 45'	-0° 29'			
21	48	35° 35'	-0° 45'			
22	25	336° 30'	-1° 24'			
23	10	265° 00'	-2° 39'			
24	31	327° 15'	-0° 03'			
25	61	336° 10'	-2° 08'			
26	45	336° 10'	-0° 02'			
27	45	65° 00'	-0° 07'			
27	49	101° 30'	+3° 13'			
28	72	280° 15'	-0° 56'			
29	147	270° 00'	-0° 28'			
30	229	271° 10'	-0° 18'			
31	213	247° 05'	+1° 43'			
32	138	234° 20'	+2° 25'			
33	102	226° 15'	+3° 07'			
34	126	210° 45'	+3° 23'			
35	102	199° 00'	+4° 14'			
36	91	187° 30'	+4° 45'			
37	108	183° 55'	+4° 22'			
38	85	174° 25'	+4° 57'			



CHAPTER XI

AERIAL SURVEYING*

232. AERIAL PHOTOGRAPHIC SURVEYING is employed in producing maps for highway, railway, waterway, pipe-line and transmission line location; for irrigation, water supply, flood control, river and harbor improvements; for city zoning and planning, traffic studies, and assessors plans; for soil conservation, geological research, and timber estimates; for topographic, military, and exploratory surveys.

Aerial photographs are classed as vertical, oblique or composite. In taking vertical photographs, the optical axis of the camera is held as nearly vertical as possible at the instant of exposure; in oblique photographs the camera axis is purposely inclined to the vertical and may or may not include the horizon; in composite photographs, taken with a multi-lens camera, one photograph is usually vertical and the others are oblique. Some multi-lens cameras have no vertical chamber. The oblique views of composite photographs are rectified (transformed into vertical prints) before being used for plotting. Then all prints will appear as verticals, although the oblique prints will no longer be rectangular in shape.

233. Cameras. Aerial cameras are precision instruments, and are either of single- or multi-lens type. The latter consists of two or more cameras mounted *en bloc*. All exposures are made simultaneously. For taking vertical and composite photographs, the cameras are mounted in gimbals attached to a frame rigidly fastened to the airplane, and have a level for leveling the focal plane at the instant of exposure. Exposures are made through a hole in the floor of the plane.

* See Higher Surveying by Breed and Hosmer, Vol. II, 1938 Ed., Chaps. VIII and IX, for complete text on this subject.

In taking single oblique photographs the camera is sometimes held in a standard attached to the edge of the cockpit and may be moved in any direction.

Cameras used for aerial surveying are of the fixed focus type. At the instant of exposure the film is held flat in the focal plane of the camera by means of a vacuum back or a glass pressure plate. The *principal point* (where the optical axis intersects the film) and coordinate axes on the photograph are located by notches or collimating marks in the focal plane which are photographed on the negative at the instant of exposure.

234. View finders are commonly used with the cameras. For taking obliques the view finder is equipped with a sighting arm, which may be set at any desired angle, and the camera is held at the required depression angle by directing the sight at the horizon. A vertical view finder is used in taking vertical photographs; it is mounted over a hole in the floor of the plane, between the camera and photographer. This instrument is essentially a camera with a ground glass in its focal plane through which the terrain is viewed; it is leveled by means of an attached circular bubble, and it may be rotated about its optical axis. This vertical view finder has several important functions. It serves to determine the instant when a point is vertically under the camera, the angle of crab (the angle between the longitudinal axis of the plane and the direction of flight) so that the taking camera may be adjusted along the true line of flight, and to the interval of time between exposures. The ground glass is ruled with lines $\frac{1}{2}$ inch apart in the direction of flight, and normal to these lines two other lines are ruled. To determine the "angle of crab," the view finder is turned about its optical axis until objects on the ground appear to move parallel to the lines ruled on the ground glass along the direction of flight. In order that successive pictures shall be directly one in front of the other, the camera is turned about its optical axis an amount equal to the "angle of crab."

The interval between exposures is determined by the time it apparently takes a ground point to travel between the transverse lines ruled on the ground glass plate.

235. Taking the Pictures. When taking the photographs of the area to be mapped, the airplane is flown back and forth over parallel straight lines, called "flight lines," and usually at a predetermined altitude. The proper distance between the flight lines is computed and the lines are plotted on a map before the flight, as an aid to the pilot in keeping the plane on its course. The altitude is given by an "altimeter" installed in the plane. If vertical photographs are to be used for compiling maps, it is best to show the principal point (approximate middle of photograph) of the preceding print in the following print. Exposures are therefore so timed that the overlap between successive pictures in the direction of flight will be 60 per cent; this overlap is sometimes called *forward lap* or *end lap*. To avoid an hiatus between adjacent strips of photographs, the flight lines are so spaced that the photographs will also overlap; this is called *side lap* and it is usually from 30 to 50 per cent.

236. Relation between the Scale of Print and Altitude of Camera. Strictly speaking, there is no such thing as the scale of a photograph where the ground has varying elevations (hills and valleys); it is only when the ground is level and when a truly vertical photograph has been taken, that the print has a definite, uniform scale. Any elevation on the earth may be selected as a datum, and, if the elevation of the camera is known at the instant of exposure, the scale of the print may be computed for that chosen datum.

There are two ways of expressing the scale of an aerial photograph: (1) by the *representative fraction* (*R.F.*); (2) by the *scale factor*. The *R.F.* is the usual scale; it is expressed by a fraction in which the numerator is unity. It is the ratio of a distance on the map to the corresponding distance on the ground, and is expressed in this text as s . The scale factor, expressed as S , denotes the number of feet on the ground corresponding to one inch on the map.

In Fig. 140, if a distance D on the ground (datum) is represented by d on the print, the *R.F.* scale $s = \frac{d}{D}$. It will also be

seen that $\frac{d}{D} = \frac{f}{H}$, where f is the focal length of the lens, H is the

altitude of the plane above the datum, and f and H are in the same units. If the scale factor (feet per inch) is desired then $S = \frac{H}{f}$, where H is in feet and f in inches. Both formulas show the relation between the scale of the print and the altitude of the plane above the datum.

EXAMPLE: Compute s and S when $f = 10$ in. and $H = 10000$ ft., i.e., 1 in. = 1000 ft.

$$s = \frac{f}{H} = \frac{10}{10000 \times 12} = \frac{1}{12000},$$

$$S = \frac{H}{f} = \frac{10000 \text{ ft.}}{10 \text{ in.}} = \frac{1000}{1}.$$

From these formulas the altitude may be computed at which the plane must fly above a definite datum in order to give the print a desired scale.

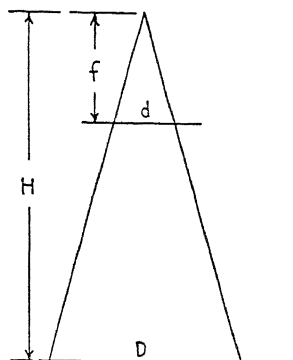


FIG. 140. SCALE FACTOR.

237. Time Interval between Exposures. As already stated, the time interval between exposures may be determined by observing the time required for a point in the field of view of the view finder to move from one of the transverse lines on the ground glass plate to the other. If, however, the speed of the plane, the focal length of the lens, the altitude of the exposure station, the size of the print, and the end lap are known, the interval between exposures can be computed.

EXAMPLE. Speed of plane = 120 miles per hour; $f = 10$ inches; altitude of exposure station 10,000 ft.; size of print 7 × 9 inches (the 7 inch dimension of print being in the direction of flight); end lap 60%.

$$10000 \times 12 = 12000$$

Total distance covered by the 7 inches on print : $7 \times 12000 = 7000$ ft. Net distance covered by one print = $7000 \times 40\% = 2800$ ft. Speed of plane in feet per second = $\frac{120 \times 5280}{60 \times 60} = 176$ ft. Interval between exposures = $\frac{2800}{176} = 15.9$ seconds.

238. Number of Negatives Required to Cover a Given Area. When the dimensions of the area to be photographed, the scale of the print, the size of print, and the end and side overlaps are known, the number of prints required to cover a given area may be computed.

EXAMPLE: Size of area 15.5 miles \times 20 miles; prints 7 \times 9 inches; scale 1:2000; end lap 60%; side lap 30%.

Total area covered by one print = $\frac{7 \times 12000}{12 \times 5280} \times \frac{9 \times 12000}{12 \times 5280} = 2.26$ sq. miles.

Net area covered by one print = $2.26 (1.00 - .60) (1.00 - .30) = 0.633$ sq. miles.

Required number of prints to cover area : $\frac{15.5 \times 20}{0.633} = 490$ prints.

To this value, add a sufficient number of prints to cover the margins.

239. ERRORS AND DISTORTIONS IN AERIAL PHOTOGRAPHS. These errors may be divided into two groups. The first group includes those errors caused by the camera or by the production of the prints. In this group may be considered camera construction and adjustment, lens distortion, film shrinkage, and print paper shrinkage.

The second group includes errors in scale caused by the inability to maintain a constant altitude, change of scale caused by relief, displacement of objects not in the datum, and displacement or distortion caused by tilt.

240. Errors and Distortions in Cameras, Film, and Printing Paper. The cameras used in aerial surveying are ruggedly built to withstand rough handling and temperature changes. They are factory adjusted. Lenses as now made are free from any distortion that would cause appreciable error in the negatives.

The amount of changes and distortions in film and printing paper should be a minimum, and the handling in developing, washing, drying, and printing should be carefully done. Film known as supersensitive panchromatic topographic base is now obtainable. Changes in this film are usually within the limits allowable in plotting. Printing paper will shrink, but if the shrinkage is uniform, the effect of the change can be corrected; in double weight printing paper the shrinkage is practically uniform. Changes in film and printing paper may be determined by comparing the distances between the reference marks with the corresponding marks on the camera.

241. Scale Errors. When taking aerial photographs for mapping, the plane is frequently flown at a precomputed altitude. Because the altimeter, which is used for indicating the altitude, is not a precise instrument, and because meteorological conditions vary, the altitude will vary, thus causing a change in the scale of the photographs during flight. However, under good flying conditions the altitude should not vary more than 100 ft.

242. Definitions. In making scale corrections use is made of the following terms: The *Nadir* or *Ground Plumb Point* is the point on the ground vertically beneath the lens at the instant of exposure. Its image on the print is called the *plate nadir* or *plate plumb point*. *Principal Point* is the point at which the optical axis of the camera intersects the film. If this point is not shown on the print, it is taken at the intersection of the lines between the reference marks (which are at the corners or along the sides of the print) on the margin of the print. When the optical axis is truly vertical, the nadir and principal point coincide. When the plate has some tilt, which is usually the case, these two points do not coincide. A point approximately midway between the nadir and principal point in a tilted photograph is called the *isocenter*, or *center of distortion*. The positions of the plumb point and isocenter can be located on the print provided the amount and direction of tilt are known. These may be determined if the position and elevation of three or more ground control points appearing in the print are known.

243. Displacement Caused by Relief. Because all photographs are perspectives, all objects in the photograph except the plumb point and those in the datum plane are displaced.

In Fig. 141 the top of the chimney *A* will be shown on the datum at *A'*. The displacement on the datum is *A''A'*, and on the print,

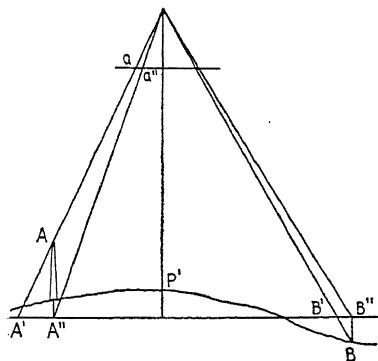


FIG. 141. DISPLACEMENT DUE TO RELIEF.

$a''a$. The displacement of B is $B''B'$. The plumb point P' has no displacement. It will be observed that the displacement of points below the datum is inward. Since the displacement is in a vertical plane, the **error on the print is radial from the plate plumb point**, which in a truly vertical photograph coincides with the principal point.

The displacement on the print is calculated as follows: The hill A (Fig. 142), the height of which above the datum is h , appears on the print at a ; the ground displacement is A_1A_0 , and the displacement on the print is aa_0 . By measuring the distance

Pa on the print and dividing it by f , the angle $\alpha = \text{angle } A_0AA_1$ may be found.

$$a_0a : A_0A_1 = f : H$$

$$a_0a = \frac{f}{H} A_0A_1 = s \cdot h \tan \alpha.$$

Another formula for computing the displacement is derived as follows: In the last equation

$s = \frac{f}{H}$ and $\tan \alpha = \frac{Pa}{f}$. Substituting we obtain

$$a_0a = \frac{f}{H} \cdot h \cdot \frac{Pa}{f} = Pa \cdot \frac{h}{H}.$$

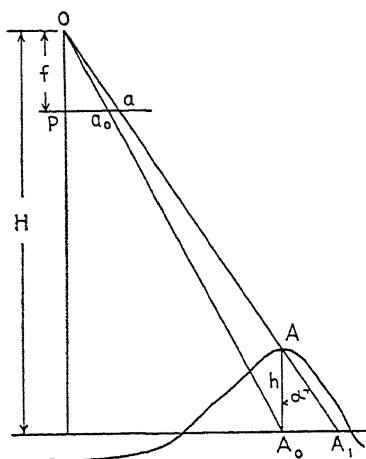


FIG. 142. DISPLACEMENT.

EXAMPLE: $H = 10000$ ft., $f = 10$ in., $h = 500$ ft., Pa measured on the print = $3''.075$.

Then by the first formula the displacement is

$$a_0a : 10000 \times 12 \cdot 500 \cdot 12 \cdot \frac{3.075}{10} = 0.154 \text{ in.};$$

and by the second formula

$$a_0a = 3.075 \times \frac{500}{10000} = 0.154 \text{ in.}$$

Thus by computing the displacement of a point on a print, we can, if desired, plot the correct position of the point on the print.

244. Tilt. In taking vertical photographs the ideal position for the optical axis, at the instant of exposure, is truly vertical; i.e., the principal point and plumb point coincide. This condition, however, seldom exists. The angle of inclination of the optical axis and the vertical is called the *tilt*. Experience shows that the tilt may be kept within 1 degree, and seldom exceeds 3 degrees. When tilt exists, the film, which is at right angles to the optical axis, is not truly horizontal. This causes the image of the object on the film to be displaced. If the angle of tilt is known, the amount of displacement may be computed; but there is no simple way of determining the angle of tilt. To determine the tilt of a print, the image points of at least three ground control points, the positions of which have been determined by ground surveying methods, must appear on the photograph. Then the tilt may be determined analytically, graphically, or by rectifying instruments. Displacement caused by tilt is radial about the isocenter.

245. Effect of Displacement on Plotting Caused by Relief and Tilt. Since the displacement caused by relief is radial from the plumb point, and that caused by tilt is radial from the isocenter, the positions of these points are indeterminate unless the tilt is known. In plotting, however, relief and tilt are both assumed to be **radial from the principal point**. With this assumption, the resulting error in plotting is negligible when both relief and tilt are small.

246. GROUND CONTROL. Ground control is the term applied to those points identifiable in the photographs the horizontal positions and elevations of which, referred to the selected datum, are known. Their locations and elevations are determined by ordinary ground methods of surveying. The amount of ground control may consist of many or of few points depending on the accuracy required and the mapping method employed.

In deciding how the control points should be located, the position of three points at least must be known in order to fix independently the position of a vertical photograph from which a planimetric map is to be compiled. When stereoscopic processes are to be used, at least two known points in position and elevation and a third point known in elevation must be available in the area of common overlap of prints. In both of these instances the con-

trol should be so positioned that the strongest possible geometrical figure can be drawn with either the single photograph or the overlap. Thus in deciding how control should be located, consideration should be given to the proposed use of the photographs. Notice that triangulation stations are usually elevated points that are subject to large relief displacements in the pictures. Traverses, on the other hand, may be run along highways, in valleys, or near to the level of the selected datum, and are therefore nearly free from this objection.

In taking levels, points should be selected which will be easily identified in the pictures and they should be so placed that they will give the best determination of tilt.

Sometimes the photographs are taken before the ground control has been established. When this is the case, points which have little or no displacement and can be identified on the photographs, such as intersections of roads with other roads, railways, and streams, should be chosen for control and then located in the field by ordinary survey methods.

In districts where there are no roads or other physical objects which can be easily identified on the photographs, it will be helpful to lay on the ground white cheesecloth, about a yard wide, in the form of crosses or circles; they should be located in open spaces where they will show up well on the photographs. These cheesecloth stations can be joined by measurements to the ground control either before or after the photographs are taken.

247. PHOTOGRAPHIC MAPS. The term photographic map is a misnomer, because, in general, a photograph cannot be both a photograph and a true map, or projection. The photograph is a perspective, and all points in it that are above or below the datum are therefore displaced on the picture (Fig. 141). When pictures are pieced together to form a continuous representation of the ground, the result is a *Mosaic*.

In spite of the geometric defects of mosaics, they are extremely useful for certain studies, and are more readily understood by the non-technical man than are engineers' plans. For preliminary engineering investigations, engineers often like to have, in addition to the mosaic, a complete set of photographs for stereoscopic inspection.

The following terms are used to define the different types of photographic maps:

Uncontrolled Mosaic, a representation of the ground made by matching aerial photographs without reference to ground control points;

Controlled Mosaic, a representation of the ground made by bringing aerial photographs to a uniform scale and fitting them to ground control stations;

Planimetric Map, a map showing the natural or cultural features (or both) in plan only, often called "line map";

Topographic Map, a map made from data derived from aerial photographs which show contour lines derived by ground surveying;

Stereometric Map, a relief map made by the application of the stereoscopic principle to aerial or terrestrial photographs; also called stereotopographic map.

248. Orienting Prints and Compiling Maps by Graphical Methods. All photographs are **true perspectives**, whereas maps are **orthographic projections**. The perspective, or photograph, is a view in which the rays of light from the object points converge to a point, the lens of the camera, and diverge from the lens to the negative. With the exception of the ray of light that lies in the optical axis of the lens, the rays are not perpendicular to the negative. An orthographic projection is a map on which all objectives are represented as they would appear if the eye were on a line everywhere perpendicular to the plane of the map. The map compiler makes the orthographic projection map from the perspectives (photographs).

249. Radial Control. The *Radial Triangulation Method* is commonly used to develop an orthographic map from several aerial photographs of the same terrain; it is sometimes called the *Radial Intersection Method*, or *Radial Intersections* and *Radial Control*. This method is based on the principle that the photograph gives the true horizontal angles regardless of change of scale or displacements of high points, because **such errors are radial from the plumb point, or nadir**. In plotting, the assump-

tion is made that relief displacements are radial from the principal point.

Suppose that on three consecutive overlapping pictures the three principal points are P_1 , P_2 , P_3 (Fig. 143), and that from each principal point a radial is drawn on the corresponding

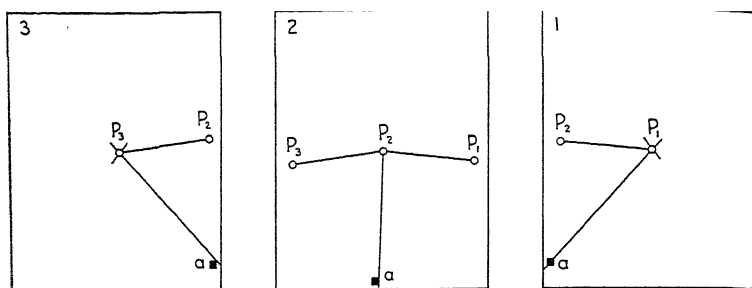


FIG. 143. RADIAL LINES.

print to the corner of a building a . By superimposing the pictures (Fig. 144) it is seen that all three lines intersect at a common point, but that this point does not coincide exactly with any of the positions of the building on the pictures as we would expect, since the angles P_2P_1a , P_1P_2a , etc., are true angles, and their sides are radial from the plumb point.

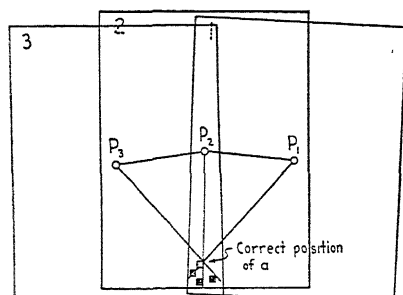


FIG. 144. INTERSECTION OF RADIALS.

to the camera being at different heights or to the elevation above the datum of the ground on which the building stands. If the three radial lines so drawn have no common or approximate intersection, there is probably an error in the plot-

No appreciable error was caused in plotting by using the principal point for the plumb point. The true position of the corner of the house is at the intersection of the three radial lines. The different positions of the building on the different prints may be due

ting; or one of the pictures may be distorted by excessive tilt and should be rejected when identified. If the point in question appears on four or five prints, it is easier to identify the print that is in error.

In applying the principle of radial intersection to a strip of aerial photographs, the first step is to determine the approximate scale of the prints. This is done by comparing known distances on the ground with the corresponding distances on the prints. When the approximate scale has been determined, a map projection of the area is made to this scale, and all ground control points are located on the projection. The photographs are next examined, and the ground control points are identified and marked on all the prints on which they appear. In Fig. 145 the ground control points are shown on photographs I and II at T_1 , T_2 and T_3 . The principal point of each photograph is next identified and marked on all preceding and succeeding photographs in which it appears. In Fig. 145 these points are shown at P_1 , P_2 and P_3 .

Although the principal point is easily located on each individual print, it frequently happens that it cannot be accurately identified with respect to the terrain on the adjacent prints, because it may not be a readily identifiable point. If this is the

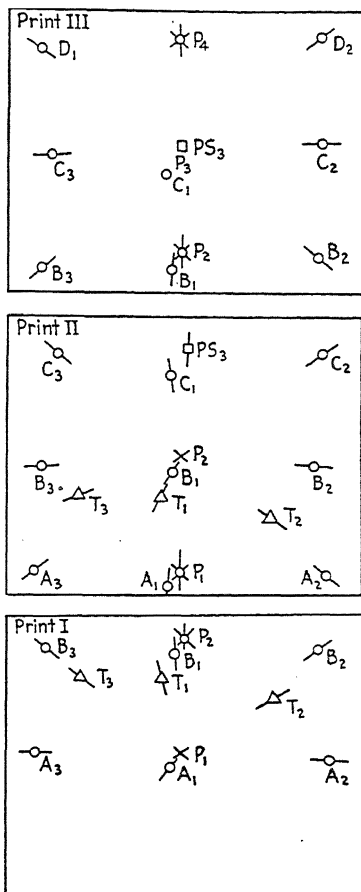


FIG. 145. CONTROL POINTS.

case, an identifiable point near the principal point is chosen on one print and the same point identified on the adjacent prints, and this is used as if it were the principal point. When such a point is substituted for the principal point, the error in plotting caused by the substitution is negligible. In Fig. 145, print III, PS_3 is chosen in place of P_3 ; it is also shown on print II.

The picture control points are next identified and marked on all prints in which they appear; they should be well defined, and at least 9 should be chosen on each print. However, the number of picture control points will depend on whether the terrain is flat, medium, or heavily accidented; the greater the accidentation is, the greater will be the number of picture control points required for correct plotting. Three should be chosen across and near the bottom of the print, three across and near the center, and three across and near the top. For example, in Fig. 145, print II, the picture control points are A_1, A_2, A_3 ; B_1, B_2, B_3 ; and C_1, C_2, C_3 . A stereoscope greatly aids in properly selecting the points.

After the ground control points, the principal points or their substitutes, and the picture control points have been located, radial lines are drawn on each print from the principal point or its substitute to all control points and to the photograph centers shown on the print, as in Fig. 145.

The orientation of the successive pictures is based on the principle of resection. Intersection of the radials indicates their orthographic projection. The tracing cloth solution of the three-point problem is used for this purpose. (See Art. 333.)

The first photograph is oriented under the tracing cloth in such a way that the radial lines from the center to the ground control points will fall through the plotted positions of these points on the tracing. The center of the print is then correctly oriented and is so marked on the tracing. Radial lines are drawn on the tracing to the selected picture control points and to the photograph centers shown on the print. The first print is now removed, and the second is placed under the tracing and oriented in the same manner. Radials are drawn from the center to the picture control points and to the centers, as in the first print. Some of the radials on the tracing from the first

print will be intersected by some of the radials drawn on the tracing from the second print. These intersections may now be used for orienting the third print. Fig. 146 is the plot of radial control of the three prints shown in Fig. 145, which is used for orienting the prints for preparation of the finished map. The foregoing process is continued until a new set of ground control points has been reached, when a check is obtained. If the points on the tracing do not check with the ground control points, they are adjusted to correct the errors in their locations.

In locating each successive print, the radial line to its center and the radials through the previously established ground control or picture control points are used. If all of these do not fit, and no gross error is evident, each radial is shifted slightly. Because of errors resulting from the assumption that displacements caused by relief and small angles of tilt are radial from the principal point, perfect checks cannot be expected, but the tracing may be so adjusted that these errors can be equally divided and a compromise position adopted.

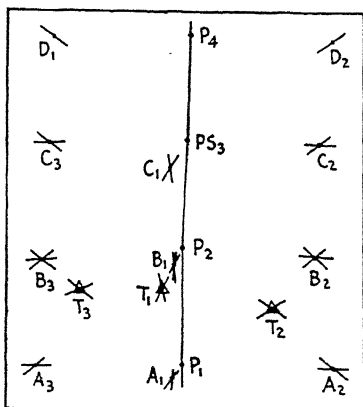


FIG. 146. RADIAL INTERSECTIONS.

250. Controlled Mosaics. From control established either by ground survey methods or from a radial plot, it is possible to rectify the individual photographs and bring them to a common scale between control points. With these scale-rectified prints a mosaic may now be laid which will contain very small displacement errors.

251. Transferring Details to the Tracing. After the skeleton of the map has been prepared, the detail is filled in from the original photographs. In each case the photograph is placed under the tracing cloth and oriented with respect to those points the positions of which are now fixed. Detail is taken off along the line

joining any two of these points. The print is so oriented that one point on the photograph lies under its map position and the second point lies on the line joining the two points on the map. Differences in scale between print and plot are then corrected by estimation during the compilation process, due regard being taken for any known relief differences along this line.

In the case of controlled mosaics, there will be no apparent difference in scale between the positions of the control points on the print and on the plot. Detail can be taken from the mosaic or from rectified prints, and a planimetric map drawn. The operator must use his best judgment, based on knowledge of the terrain, in selecting the positions of those details where mismatches occur as a result of relief.

A projection camera is now in use, which is like an enlarging camera; into this a photograph may be inserted and its image cast on a drawing board. The drawing board may be tilted by three leveling screws. The map projection is placed on the board, and small sections of the photograph are made to fit the control points by tilting the board and varying the enlargement of the photograph; in this way detail may be transferred to the map.

252. Contours. There are two methods of obtaining the contours on a map made from aerial photographs. The first is by means of the ordinary ground methods (plane-table or stadia), after the horizontal locations have been established with the aid of aerial photographs. The second is by the use of the **stereoscopic principle** applied to overlapping photographs.

If the first method is used, there must be enough lines of direct levels to furnish all necessary bench marks, as well as enough points located horizontally for the plane-table work. If the second method is used, there must be sufficient ground control to enable the operator to so orient his photographs in the stereoscope that correct location of points on the contours can be obtained.

253. STEREOSCOPES. A stereoscope is an instrument, composed of lenses, prisms or mirrors, or a combination of them, for observing two photographs of the same object taken from different viewpoints. As the photographs are viewed in a stereoscope, the right-hand view with the right eye and the left-hand view with the left eye, the two views merge and the observer

perceives a model of the object in three dimensions. By viewing overlapping photographs through a stereoscope, a clear-cut relief picture can be obtained which gives a clearer vision of the shape of the terrain than can be obtained by examining each picture separately.

254. Stereoscopic Instruments. Several kinds of stereoscopic instruments have been constructed to aid in the compilation of maps from these overlapping photographs, or *stereo-pairs*. In the stereocomparator type, the measurements are made on the image points on the photographs for the purpose of determining the coordinates of each of those points; they are then plotted independently. Universal instruments such as the *stereoplanigraph*, the *autograph* and the *aerocartograph* combine in one operation stereoscopic observing, measuring, and map plotting. These last instruments consist essentially of a measuring system, an optical system, and a drawing system; the photographs used in them are positives or negatives made on glass plates or films. Some of these instruments are adapted either for vertical or oblique photographs.

These instruments are so complicated that their description and manipulation cannot be given in a text of this brevity. That the reader may get some idea of the principles involved, a few of the principles underlying their manipulation will be mentioned. For a detailed explanation of these instruments and their manipulation, the reader is referred to the bibliography at the end of this chapter.

The stereo-pair photographs are observed through lenses having the same characteristics as those in the taking camera. The holders are adjusted until the photographs have the same positions one to the other and to the ground that they had at the instant of exposure in the taking camera. By means of two telescopes, similar to binoculars, consisting of a complex train of lenses and prisms, the observer may view the two photographs, which present a stereoscopic model of the ground. Two marks, one in the optical system of each telescope, appear to the observer as a single mark when looking through both telescopes. This mark, commonly known as the *floating mark*, appears to be in space, but by manipulation of the instrument this floating mark

moves and it may be made to contact with any point on the model. By mechanical system the horizontal projection of the image point is plotted on the map. Since ground control points must appear in at least one of the stereo-pair, it should be apparent that if the instrument is set for a definite elevation and the floating mark is moved and kept in constant contact with the surface of the model, the pencil will trace a contour on the map.

An *anaglyph* is a kind of picture produced when the overlapping portions of two photographs are printed one over the other in complementary colors, usually in red and blue green. When viewed through spectacles (filters) of like complementary colors, a model in three dimensions, similar to that seen in the stereoscope, is observed. The *Aero Multiplex Projector* is an instrument for compiling maps which utilizes the anaglyphic principle.

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CHAPTER XII

LINES AND GRADES

255. LINES AND GRADES is a term commonly used to include surveying for the purpose of defining engineering works in plan and in elevation; it involves setting stakes or other reference marks as guides for construction. Marking these lines and elevations is called "*giving the lines and grades.*"

For example, the construction of a concrete building requires that lines be established to fix the locations of the centers of the columns and the outside lines of the footings and walls; the elevation (often called "the grade") of each floor and of the basement and the roof are also required. In building a sewer the center line and the elevation of its flow line (the inside of the bottom of the sewer) must be marked.

256. Lines. The lines required to define any work will depend on the kind of project. In those projects where the stakes will become obliterated as the work proceeds, as, for example, stakes set at the corners of a building, it is common practice to set them on an offset line where they will be preserved during the construction period.

In laying out a dwelling the actual building lines are defined by strings stretched between nails driven in batter-boards (Art. 263). Another method frequently used is to stake out offset lines parallel to, and one or two feet away from (outside), the faces of the structure. Such lines are preserved during the construction. For projects such as highways, sewers, pipe lines, and drains, the center lines are staked out. Of course, these center stakes will be dislodged during construction. They are readily replaced, however, by making use of additional stakes located on extensions of the various lines, i.e., beyond or outside of the actual zone of construction.

257. Grades. The word "grade" in the expression "lines and grades" usually, although not always, means an elevation. The grade of a bridge seat means its elevation above some datum plane. The grade of a sewer means the elevation of its flow line at a given station; but the word "grade" when referring to a sewer, a conduit, a road, or any other linear structure that stretches out across the country may also mean the longitudinal slope (referred to the horizontal). If, for example, the slope rises 1 ft. in a horizontal distance of 100 ft., it is said to have a $+1\%$ grade. If the slope falls 1.32 ft. in 100 ft., it is recorded as a -1.32% grade, or *gradient*. The latter is a more definite word for rate of grade, but it is seldom used.

As it is used by engineers, "grade" therefore means the **elevation** when referring to a particular point or level plane; it also means the **rate of slope** with the horizontal when referring to the profile of a linear structure like a highway or sewer.

When wooden stakes are used for grade marks they may be driven so that their tops will be exactly at grade, or at a full number of feet above or below; or they may be driven to a firm bearing and a crowfoot \wedge marked on the face of the stake to indicate the grade elevation; or the crowfoot may be a whole number of feet above or below the grade, in which case it is marked thus $\overline{\wedge}$ (grade is 1 ft. below), or $\underline{\wedge}$ (grade is 2 ft. above).

In hard pavements such as those in city streets and sidewalks where it is difficult to drive stakes, it is customary to use ship spikes about $\frac{1}{2}$ " square. These are driven flush with the surface and the elevations of their heads are determined. All grades are referred to the elevations of the tops of these spikes. If these spikes are placed at an exact offset distance from the line of the structure, they can also serve for lines as well as for grades.

258. Setting Grade Stakes. The detail of setting grade points at desired elevations is illustrated in Fig. 147. Point *A* is alongside of a foundation where a grade stake must be set to define the top of the base plate of a column which must be exactly at the desired grade elevation, 157.21 ft. The surveyor's level is set up and a backsight of 6.42 ft. is taken on a B.M., the elevation of which is 155.18. The resulting H.I. is $155.18 + 6.42 = 161.60$ ft. The grade elevation 157.21 subtracted from the

H.I. 161.60 is 4.39, which is the reading a rod would have if we imagine the base plate has been set accurately in place and the rod held on top of it. This is called the *grade rod*, or the *rod-reading for grade*. If the rod is held against the face of stake *A* and moved vertically until the observed reading is 4.39 ft., the bottom of the rod will be at the grade elevation desired, i.e., 157.21 ft.

If it is desired to set the top of the stake exactly at grade, the stake is carefully driven until the rod, when held on top of it, reads the grade rod-reading. The top of the stake is then at grade. When the grade line is below, or is so far above, the surface of the ground that it cannot be marked directly on the

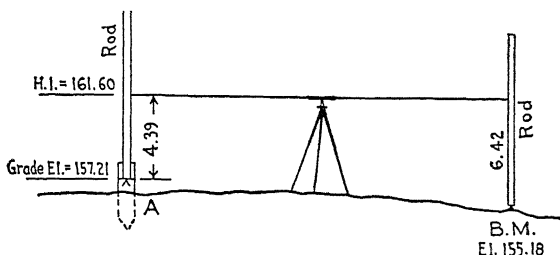


FIG. 147. SETTING GRADE STAKE.

stake, the top of the stake is set a whole number of feet above or below the grade line and marked accordingly, as explained in Art. 257.

259. Staking Out a Column Footing. The lines and grades are first used to define the excavation, and then to set the forms for the concrete footing. In Fig. 148, the column center stake *E* is accurately set by transit and tape, from dimensions given on the plan. Stakes *A*, *B*, *C* and *D* are then set with the transit so that the lines connecting them will define a rectangle 2 ft. outside, and parallel to, the sides of the lowest course of the foundation. Each of these stakes *A*, *B*, *C* and *D* is driven to exactly the grade of the top of the foundation, after which the exact points *A*, *B*, *C* and *D* are marked on the stakes by small nails or tacks. The positions of the tacks in the stakes are checked by measuring diagonals *AD* and *BC*, which should be equal. Lines are stretched between *A*, *B*, *D* and *C*.

The excavation is started 4'-6'' square by marking lines on the ground 2 ft. inside from lines AB , BD , DC and CA . This assumes that the earth will be sufficiently cohesive for the sides of the excavation to act as forms for the lower layer of the foundation. Similarly, the forms for the two layers of the foundation above the base course are placed by measuring in 3' and 3'-6'' respectively from the strings.

The grade of the top of the foundation is fixed by bringing the top of the foundation form up to the bottom of a straightedge that rests along the diagonals AD or BC .

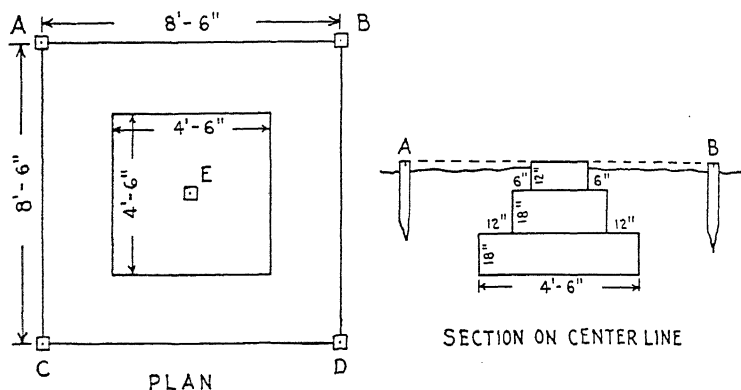


FIG. 148. LINES AND GRADES FOR COLUMN FOOTINGS.

Another way to stake out a column footing is by the use of batter-boards, as explained in Art. 263. If bolts are to be used to anchor the foot of the column to the concrete base, they are located and placed as described in Art. 262.

260. Staking Out Curb Lines and Grades. In giving lines and grades for curbstones or forms for concrete curbs, stakes (or heavy spikes) are set that can be used for both line and grade. If the stakes were set along the face line of the curb they would be disturbed when the trench was excavated. For this reason they are usually set in the sidewalk along an offset line, at some definite interval of from 1 to 3 ft. from the outside face of the curb. The intervals between the stakes are usually from

25 to 50 ft. or closer, and stakes are set to mark the beginning and the end of a curve.

In setting the stakes along the offset line, they are driven only enough to be firmly set in the ground. The line is not marked on them at this time. After they have been set on the offset line, their tops are driven to grade by the level party, using the method explained in Arts. 257 and 258; or they are marked or sawed off at the proper grade. The line is then carefully marked on the tops of the stakes by tacks which are lined in with the transit.

When the offset line has been staked out, the face of the curb (or the form) is located by measuring the offset distances from the tacks in the tops of the stakes, and the grades are transferred from the tops of the stakes by means of a carpenter's level.

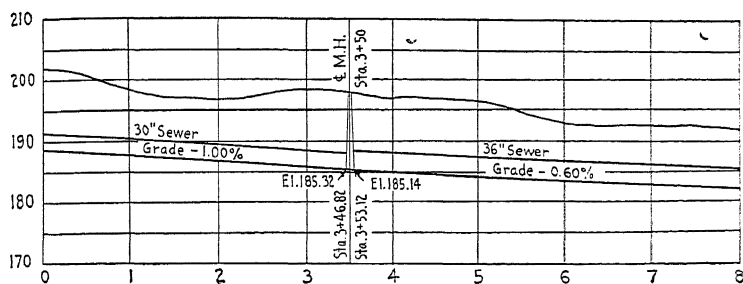


FIG. 149. SEWER PROFILE.

261. Staking Out Sewers. In staking out a sewer, its center line is defined on the surface of the ground and stakes are set every 25 to 50 ft. along the line. In hard surfaces, spikes instead of stakes are used, and in surfaces where spikes cannot be driven, chisel cuts are made in the pavement.

Profile levels of the surface are run along the center line and are plotted on profile paper (Art. 106). The proposed grade line of the sewer is then plotted on the same profile, as in Fig. 149. The grade line of a sewer is the flow line, i.e., the lowest line on the inside of the sewer. From the profile the approximate cut at each station is readily obtained by computation (subtracting the grade elevation from the ground elevation) or by

scaling from the profile; these values should be given to the foreman to guide him in excavating the trench.

As the excavation of the trench progresses, a number of stakes will be displaced. From stakes previously set outside the excavated area, the center line is rerun over the trench as follows: Planks are placed on edge across the trench at each station, the ends of the planks resting with their edges on the surface of the ground (Fig. 150). To hold them in position, they are nailed

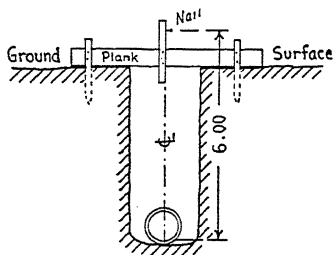


FIG. 150. BATTER-BOARDS FOR SEWER.

to heavy stakes which are driven in the ground. Sometimes the planks are supported by piling earth around their ends. A strip of wood about $1'' \times 2'' \times 30''$ is firmly nailed to each plank; the strip is in a vertical position with one edge exactly on the center line of the sewer. A small nail is partially driven into the edge of the vertical strip, at a full number of feet above

the invert (flow line). The distances from the invert to the nail should be a full number of feet, such as 5.00 or 6.00 ft.

Instead of using a vertical strip nailed to the plank, some engineers prefer to set the top edge of the plank at a full number of feet above the grade of the sewer, and then drive a nail in the top of the plank to mark the center line of the sewer. With this exception, the same procedure is followed as that already explained.

If a stretching line joins the center-line nails, it will be parallel to the grade line of the sewer. In order to set the sewer pipe at the proper distance below the string, a "grade pole" is used. This is a straight wooden pole with saw cuts or notches made in it, every foot throughout its length. Rigidly attached to the bottom of the pole is a right angle metal bracket, which may be extended into and rest on the invert of the pipe. Care should be taken to hold the pole vertically when the pipe is being tested for grade; and to keep the string from sagging between the nails. If the string is left out overnight it is likely to absorb moisture;

although it may be fastened taut in the morning, it will begin to sag as soon as it starts to dry out.

262. Staking Out an Engine Bed. All stationary engines require firm base supports and precise alignment of the driving element. Small units usually have a metal base provided with bolts for attaching them firmly to the floor of a building or to a concrete foundation. Larger units may have two or more separate foundation supports. The foundations must be so constructed that the engine bed will be exactly level and the anchor bolts will fit exactly the holes in the metal base of the unit. It is the duty of the surveyor to set the lines and grades for the concrete foundation and to define the locations for all anchor bolts. The control lines of the foundation should be marked by permanent points, far enough away from the engine to be free from any possibility of disturbance. All other lines and grades should be located sufficiently near to the actual work for the convenience of the mason and yet not so near that they will be obliterated during the process of building the foundation. The first requisite is an accurate interpretation of the dimensions on the foundation plans; these dimensions are usually expressed in feet and inches. (See Art. 10.)

Lines for the inside of the forms, used for the concrete foundation, should be marked by crosses cut in the concrete floor or by nails on batter-boards which are firmly attached to posts in such a way that they will be convenient for use by the carpenter who constructs the forms and by the mason who pours the concrete foundation. (See Art. 259.)

Sometimes the foundations do not have vertical sides but are battered (the base of the foundation is wider than the top, and the sides slope). In this case batter-boards may be marked, both for the bottom and the top dimensions of the foundation. It is of the utmost importance that the carpenter understands the surveyor's marks, which can be made plain by markings such as "neat line of base at floor level," or "1 ft. outside top of foundation."

It is customary to set each anchor bolt in a short vertical steel pipe of greater diameter than the bolt; the pipe is then embedded in the concrete, so that after the concrete has set there is still an

opportunity to bend the bolts slightly if necessary to make them fit the holes in the base of the unit. It is also customary at first to pour the concrete to a grade a little below the exact elevation of the top of the foundation and then to set the unit and true up its base by driving metal wedges between the top of the new concrete base and the base of the unit, thereby bringing the base precisely to the desired grade and making it precisely level. To establish this elevation the batter-boards are set exactly at the desired elevation. This should be tested by levels. Cement grout (Portland cement, water and sand) is then poured into the forms; it fills the space between the pipes and the anchor bolts and also the narrow space between the temporary top of the foundation and the base of the unit, thereby fixing firmly in place the anchor bolts and giving the base a uniform continuous support on the top of the foundation.

Lines and grades for the foundation may be marked on nearby parts of the structure in which the unit is to be housed; these are sufficient for setting concrete forms, but when the final alignment of the unit is made and its base is leveled, the surveyor's transit (and level) should be employed.

263. SETTING BATTER-BOARDS FOR A DWELLING. Since the first step in the construction of a building is the cellar excavation, it is evident that if the exact corners of the building were marked, these marks would soon become displaced. It is therefore customary to define the lines and grades of the building by marking them on batter-boards, located 3 to 4 ft. outside the limits of the foundation walls. These batter-boards are composed of ordinary boards nailed horizontally to posts (usually 2" \times 4" joist), driven firmly into the ground. The top of the batter-board is level with the top of the masonry of the cellar wall, and the lines defined by nails in the top of the batter-boards are the outside face of the wall which supports the sill of the building.

Fig. 151 illustrates how the mason plumbs down from strings stretched from one batter-board to the opposite one along the line of the building.

The architect usually determines the height of the cellar wall above the ground (top of cellar wall or elevation of first floor). It may be marked, or it may be stated as so many

feet above the level of the sidewalk; or the actual elevation may be stated.

The location of the building on the lot is given either by plan or by orders from the architect or the owner. Not infrequently the front line is to be on line with the other buildings on the street and the side line is to be a certain number of feet from one of the side lines of the lot.

The process of setting the batter-boards is as follows: Referring to Fig. 152, the surveyor first accurately sets temporary stakes in the ground (with nail at exact corner) at points *A, B, C, D, E, F, G* and *H*. Then he measures the diagonals of all rectangles

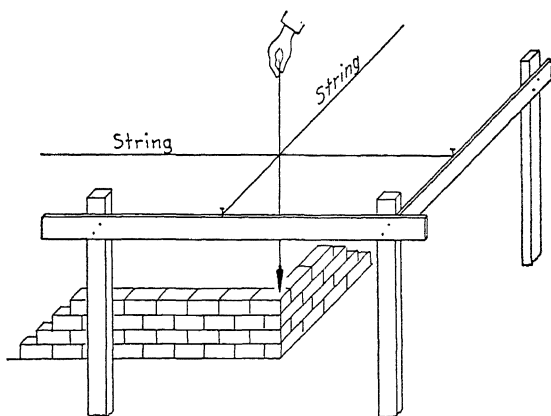


FIG. 151. USE OF LINES AND GRADES FROM BATTER-BOARDS IN CONSTRUCTING CORNER OF CELLAR WALL.

for a check; this should **always** be done. Next, he sets up the six batter-boards indicated, drives the stakes vertically, and fastens the boards exactly horizontal, at the grade determined by means of the engineer's level. For example, if the elevation of the top of the cellar masonry is to be 1 ft. above the sidewalk elevation at the property line, the level is set up and a rod-reading is taken on the sidewalk at the property line, say, 4.32. The reading of the rod when held on the batter-board should therefore be 3.32. The bottom of the rod should be marked on each batter-board stake when the level reading is 3.32. The batter-

board should be nailed at one end to the stake in such a position that the top of the board is at the line marked. The top of the other end of the board is held at the line on the other stake and the nail is driven in just far enough to keep the board from falling off. Then the rod should be held on top of the batter-board at about the place where the line of the building will come; if it reads 3.32, the nails are driven home. If not, the board is moved up or down slightly until the rod reads 3.32 and then the board is securely nailed to the stake. After all the batter-boards are in place they should be checked roughly by sighting across them; they should all appear to be at the same level. Because of the

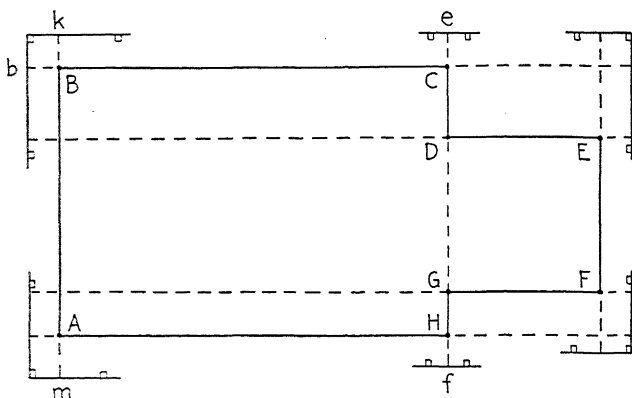


FIG. 152. SETTING BATTER-BOARDS FOR A DWELLING.

slope of the ground some of them may have to be set a definite number of feet above or below grade.

Set the transit at *C*, sight on *B* and set nails *b* and *c*. With transit still at *C*, sight *H* and set nails *f* and *e*; these nails are driven in the top of the batter-board, until their heads are out about $\frac{1}{2}$ ". Similarly, set up the transit at *D*, *G* and *H*, and finally at *A* and *F*, setting in the batter-boards all the remaining nails which are necessary to define all the lines of the building. When the cellar wall construction begins, strings are stretched taut between these nails, as from *b* to *c*, *e* to *f*, etc., and thus this network of strings hanging overhead gives the lines and grades

for the mason's guidance in constructing the cellar walls (Fig. 151).

After setting up the transit at *A* and sighting on *B*, it is a simple process to set point *k*; but the transit at *A* cannot be focused on the batter-board at *m* because it is only 3 or 4 ft. away. One way to locate *m* is to set with the transit a point 10 ft. or so from *A* on line *BA* produced, and then stretch a string from *A* to that point and plumb up from it to locate *m*. A quicker and more practical way is to set *m* on line with *A* and *B* by eye. The observer should stand 4 or 5 ft. outside the batter-board and sight past the plumb-line hanging from the transit at *A* toward the point *B*. He will then find that he can define *m* within 0.01 ft. readily, because he is now far enough from *m*, *A* and *B* to focus his eye on all three and to see them all distinctly.

It is good practice to set reference points on the main lines of the building, entirely outside of the area disturbed by the construction.

264. GIVING LINES AND GRADES FOR A LARGE CITY BUILDING. On valuable city property, buildings frequently occupy nearly all of the land and abut directly on the street lines. The frame and floors of such buildings are usually of steel or of reinforced concrete. The established lines and grades for column centers, wall faces, and floor levels are laid out by a survey party which often is required to mark and check lines and grades throughout much of the construction period.

At the start, levels should be carefully taken on, and offsets from accurately defined lines at well-defined points should be measured to, all existing construction abutting on or near the property. These same levels and lines should be rerun while the work is in progress to discover any settlement or movement of adjacent buildings while the settlement is still small; the necessity for preventive measures will then be discovered before serious damage results. If settlement or movement of any kind does occur, a continuous record of such changes should be kept.

During the progress of the work the surveyor will set grades and lines for all excavation work and for masonry foundations of basement walls and column footings, mark all column centers

on the completed footings, set lines and grades for forms for concrete walls and piers, check their positions before the concrete is poured, mark grades for each floor level at all columns, check

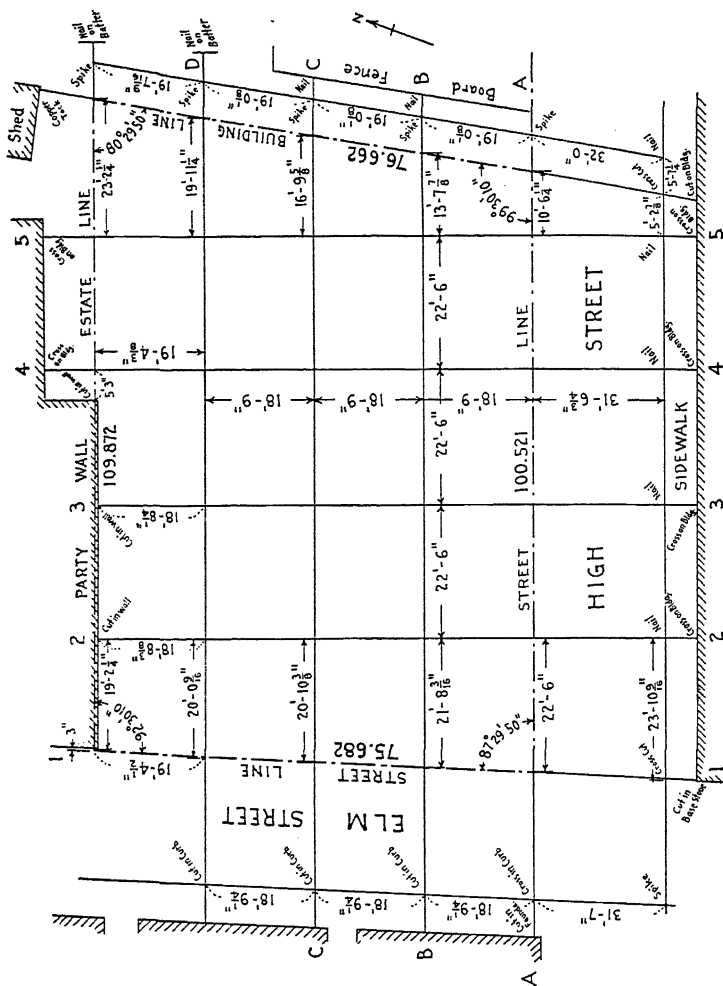


FIG. 153. COLUMN CENTERS FOR OFFICE BUILDING.

the grade of forms for floors before they are poured, and check the elevation at many points throughout each floor, before and

after the forms supporting the floors have been struck, and for a month or more thereafter, in order to determine whether the green floor slab has sagged.

The surveyor's plan of the building site is represented in Fig. 153; the distances between column centers and between reference lines are shown in feet, inches and fractions; the controlling dimensions of the property are shown in feet and decimals. This is the reference plan which has coordinated the architect's plan with the property lines which have previously been surveyed. The surveyor actually runs the column lines on the ground and marks reference points on each of the column lines on the sidewalk curb on the opposite side of the street (Elm St.), or on the face of the building on the opposite side of the street (High St.), on the property wall on the north side of the property, and on the fence and batter-boards which have been set up on the east side. Thus, each column line is referenced at both ends to points which are well outside of the construction area. Reference marks for levels are also made at the ends of each column line, preferably at some full number of feet above the City Datum. All these reference marks are cut into the walls or curbs, or are marked by stout spikes in such a way that they will be permanent, at least through the construction period. There should be a sufficient number of marks to make them easily visible from all parts of the building site.

A sketch of the reference-point locations should be given to the superintendent of construction; this sketch should be so arranged as to admit of only one, and that the correct, interpretation. The superintendent usually prefers the dimensions in feet, inches and fractions (usually to $\frac{1}{8}$ '').

In laying out large plants involving many buildings, connecting pipe lines, and tracks, the entire layout may be referred to a rectangular coordinate system. If this is done, the property lines, buildings, and all points needed for purposes of construction are tied together, and points of known coordinates may be set by measuring from any convenient station in the system. (See Arts. 145, 146 and 128.)

265. Staking Out a Culvert. A culvert is usually a straight waterway of limited length which passes under another structure,

such as a highway. The plan of the highway specifies the station on the highway center line at which the center line of the culvert crosses and also the angle between center lines of culvert and highway.

Referring to Fig. 154, which represents a box culvert, the transit is set up at *A*, the angle $72^{\circ} 30'$ turned off, and four reference stakes, or other fairly permanent marks, are accurately set on the center line of the culvert and on both sides of the proposed highway, sufficiently far away to lie well outside the limits of the culvert construction. Points *B* and *C* may be set, say,

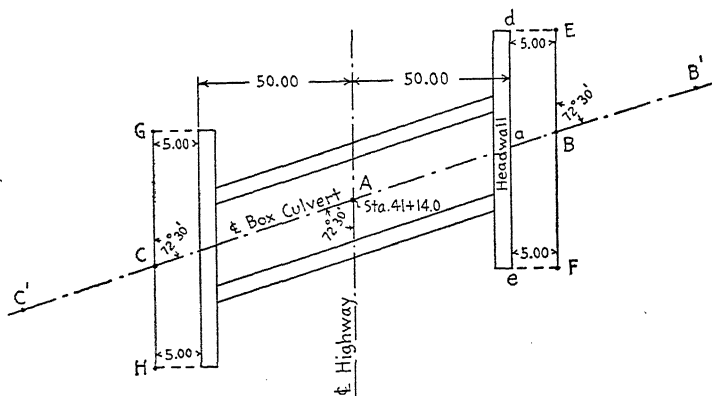


FIG. 154. LINES FOR A CULVERT.

5 ft. (measured at right angles) outside the face of the headwalls of the culvert; this is $55.00 \div \sin 72^{\circ} 30' = 58.72$ ft. from the center line of the highway along the center line of the culvert. Points *B'* and *C'* may be 100 ft. or more from *B* and *C* respectively. Should points *B* or *C* be obliterated during construction, a center line can be run between *B'* and *C'* without the necessity of setting up on *A*, which may be occupied by the formwork of the culvert. The distances between *B* and *B'*, and *C* and *C'*, should be measured accurately and recorded in the field-notes.

At *B*, lay off line *EF* parallel to the center line of the highway (if the headwalls are so designed) and set stakes *E* and *F* at points which will be directly opposite, and at right angles to, the ends of the headwalls. These offsets will be exactly 5.00 ft.,

provided that no mistake has been made in computing AB and AC . The distance $BE = ad - 5.00 \cot 72^\circ 30'$; $BF = ae + 5.00 \cot 72^\circ 30'$. Similarly, CH and CG are computed.

The stakes thus set are sufficient to locate in plan the forms for the headwalls and for the barrel of the culvert.

Grade stakes for the bottom of the flow line of the culvert should be set near B and C , and for the bottoms and tops of the headwalls they should be set near E , F , G and H , where they can be readily transferred to the headwall forms by the use of a carpenter's level and a straightedge.

266. Staking Out Bridge Abutments. The first step in this process is to define accurately the center line at the proposed bridge site either by finding the original survey points on each side or, if they have become lost, by reproducing these points from ties. With the instrument sighting along the center line, reference points, such as A , B , F and S in Fig. 155, should be set on both sides of the bridge site. These points should be substantial hubs, firmly driven into the ground in locations where they are least likely to be disturbed, yet are conveniently near the work.

The plans will furnish the dimensions of the abutment and its station on the center line. The station may be given at D (Fig. 155) where the center lines intersect, or at the face of the abutments at C and E . If the latter are not given, they may be computed from Sta. D . The lines that are to be staked out are the faces (or neat lines) of the abutments and of the wings. If the front faces of the abutments and the wings are battered (sloped backward from the vertical), as is the case in Fig. 155, the face line directly under the bridge seat or coping is laid out. The contractor must be informed of this fact, so that he may lay out the proper offsets to the line of the base of the face of the wall, and to the face of the footings (foundation). The lower abutment in Fig. 155 should be staked out as follows:

Locate point C by taping from the nearest established station on the center line. Check the measurement by repetition or by independent measurement from another station point. Find the precise stations of the reference points A and B and record them in the notes. Set up over C , take a backsight on center

line, lay off the angle of skew $68^{\circ} 35'$ by method of Art. 60, and set reference points L and K . Point K should be just outside the construction area; point L should be near the right-of-way line, and if possible a still more distant point should be set for

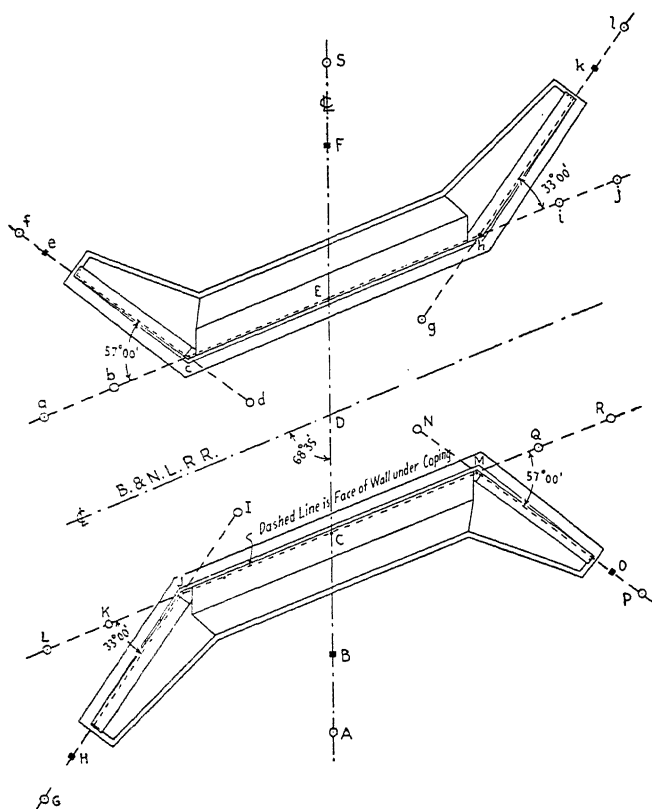


FIG. 155. LINES FOR A BRIDGE ABUTMENT.

a foresight in case both L and K become lost. Determine the dimension CJ from the plans, set point J on line CKL by tape measurement, sighting from C . Now move the instrument to J , sight on L , or on a more distant point if available, lay off angle of the wing ($33^{\circ} 00'$), and set points H and G on line beyond the end of the wing. Reverse the telescope and set point I in

prolongation of the line GHJ . Measure the distances JK , KL , JH , GH and JI and record them in the note-book.

By a similar process the right half of the abutment may be staked out, setting points M , Q , R , O , P and N . Great care must be taken in the fieldwork, since errors in the layout will lead to costly mistakes in construction. Checks on the positions of the points set should be computed and applied. For example, the coordinates of points G , H , L , K , O , P , Q and R could be computed with reference to the center line as an axis and with C as an origin. The computed dimensions could then be checked by direct measurements in the field. The relative positions of the two abutments may be checked by measuring the perpendicular distance between faces of the abutments, i.e., between lines LR and aj . The two lines should be parallel and the distance between them should be equal to the abutment spacing.

A substantial bench mark should be established at the site, and grades should be given on stakes near the proposed work. In Fig. 155, grade and line are given on the stakes shown by a black square and line only, on the stakes shown by a circle. As the form work progresses elevation should be given for the bridge seat.

When the construction work starts, the points J and M will be lost. It may prove desirable to lay out an offset line parallel to line $LKQR$ and, say, 2 ft. away from it, i.e., outside of the excavation area.

267. STREET LINES AND GRADES. Lines and grades of city streets are usually defined by municipal ordinance or by usage. The side lines of a street are called "street lines," and they define the boundary between public and private ownership. They are often marked by stone or concrete monuments placed at the street line corners, at the beginning and end of curves, and at angles in the street line.

In many cities it is customary to permanently mark the street lines by offset lines. These offset lines are frequently, but not always, parallel to the street lines. They are usually located along the sidewalk at 2 to 4 ft. from the street line; far enough from the actual property lines so that they will not be disturbed when buildings are constructed that run up to the

street line. Another method of locating street lines where the buildings are not exactly on the line is by swing offsets (Art. 325). In this method several offsets are taken from the street line to permanent parts of the buildings located along the street, defining the street line beyond question.

It is the duty of the city engineer to establish center lines for sewers, paving, and underground conduits; and to make all surveys pertaining to public improvements along a street, including running the street lines. Such survey data are kept in the form of field-notes in a carefully preserved note-book or on plans which show the surveys made for various purposes.

When making a survey of private property, the private surveyor will run the front line (the street line) where the city engineer has established it. He will obtain the data for this purpose from the office of the city or town engineer. In some cities these street lines will be rerun by the city engineer on application. Since this often causes delay, the private surveyor usually reruns the lines himself.

For private streets which have not yet been accepted by the city, the defined street lines can usually be found on record in the city engineer's office or in the office of the surveyor who laid out the street.

268. Street Grades are usually defined by city ordinance. As a rule they relate to the center line of the roadway pavement and are accompanied by a cross-section of the roadway and sidewalks, so that the established grade along the inside line of the sidewalk may be readily computed. For example, a practical cross-section grade for a 60-ft. street is one in which the sidewalks are each 8 ft. wide and the roadway is 44 ft., thus providing two lanes for parking and two lanes for traffic. The grade of the center of the street may be, say, 5'' above the gutter, the face of curb 6'', and the crosswise slope of sidewalk $\frac{1}{4}$ '' per ft. In this case the grade at the property line will be $-5'' + 6'' + 2'' = 3'' = 0.25$ ft. higher than the established grade at the center of the street.

Street grades are frequently defined by starting at a "grade station," say, at the center of two intersecting streets and running to another grade station at the intersection of some other street.

Between these two grade stations there may be many changes of grade and vertical curves, all of which will be defined as illustrated by the following grade ordinance:

"Beginning at grade station 170 (Pleasant St.) at the junction of the center lines of Pleasant St. and Clark Road, at grade 62.00, the grade line falls 0.50 per 100 for 726 ft. to grade 58.37 — thence rises 0.82 per 100 for 322 ft. to grade 61.01 — thence falls 0.50 per 100 for 142 ft. to grade 60.30 — thence falls by a vertical parabolic curve for 100 ft., as follows:

STA.	ELEV.
181 + 90.....	60.30
182 + 15.....	60.08
182 + 40.....	59.66
182 + 65.....	59.05
182 + 90.....	58.25

thence falls 3.60 per 100 for 239 ft. to station 185 + 29, at the junction of the center lines of Pleasant St. and Clark Rd., grade 49.65."

269. CURVES. The horizontal alignment of highways, railroads, canals, many city streets, and property lines consists of straight lines connected by curves. The straight lines between the curves are called "*tangents*." These curves are commonly arcs of a circle. Occasionally they are a combination of circular arcs of different radii, and sometimes they are spirals.

The vertical alignment of highways and railways consists of straight grade lines and parabolic curves tangent to them. The straight grade lines are where the rate of grade is uniform. Where the rate of grade changes, a vertical curve is introduced in order to ease the transition from one grade to the following.

Both horizontal and vertical alignments are usually determined in the office, after a study of plans of the site. The surveyor reproduces on the ground the alignment adopted in the office.

270. ELEMENTS OF A CIRCULAR CURVE. Fig. 156 shows a circular curve connecting the tangents *A* and *B*.

Radius of the curve is designated by *R*.

Vertex, *V*, or point of intersection, *P.I.*, is the point where the two tangents to the curve intersect.

Intersection angle, *I*, is the deflection angle between the tangents, and is equal to the angle at *O* between the radii.

Point of Curvature, *P.C.*, is the point where tangent *A* ends and the curve begins.

Point of Tangency, P.T., is the point where the curve ends and the tangent *B* begins.

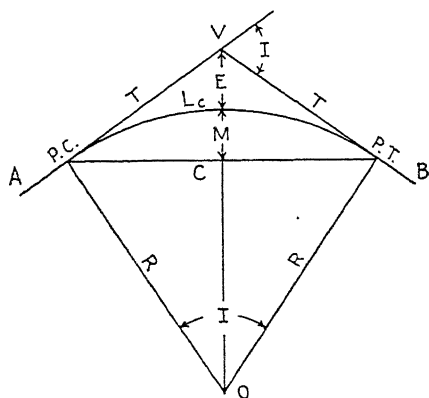
Length of Arc, L_c , is the length of the arc from the *P.C.* to the *P.T.*

Tangent Distance, T , is the distance along the tangent from the vertex, *V*, to the *P.C.* or the *P.T.*

Long Chord, C , is the straight line distance from the *P.C.* to the *P.T.*

External Distance, E , is the distance from the vertex to the mid-point of the curve.

Middle Ordinate, M , is the distance from the mid-point of the curve to the mid-point of the long chord.



The relations between these various elements are

$$T = R \tan \frac{I}{2}$$

$$E = R \operatorname{exsec} \frac{I}{2}$$

$$M = R \operatorname{vers} \frac{I}{2}$$

$$C = 2R \sin \frac{I}{2}$$

$$L_c = R \times \text{circular measure of } I.$$

FIG. 156. ELEMENTS OF CIRCULAR CURVE.

271. Methods for Designating Curvature. In city streets and property surveys and in some highway surveys a circular curve is designated by its **radius**, and the length of the curve is the **actual arc length**. The curve may also be defined by the “**degree of curve**”; this has several meanings, two of which are commonly used: (1) when the length of curve is the actual arc, the degree of curve is the angle subtended by an **arc** of 100 ft.; (2) when the length of curve is measured by a series of 100-ft. and fractional chords, the degree of curve is defined as the angle subtended by a **chord** of 100 ft. The latter definition is commonly used in railroad practice and also in some highway surveys.

Arts. 272-6 deal with the **actual arc method** for defining a circular curve; Arts. 277-8 deal with the **chord-stationing method**.

272. Staking Out a Circular Curve by Arc Method. In laying out a curve by the arc method, stakes are set at the *P.C.*, *P.T.* and at all the full stations on the curve. If the curve is sharp, intermediate points are desirable at plus 50- or plus 25-ft. stations. A curve may also be laid out independent of any system of stationing by dividing it into a number of equal arcs; or, it may be laid out in arcs which are a full number of feet long with a sub-arc of shorter length at one end. Again, equally spaced points may be laid out on the arc by making all chords (except the last one) equal, and a full number of feet in length.

273. Deflection Angle Method. The most common method for locating points on the curve is that known as the "deflection angle" method. It is the only method that will be discussed in detail in this text. For other methods, see standard texts on curve layout.*

In speaking of curves, a **deflection angle** means the angle between a tangent and a chord. In Fig. 157 the angles VDa , VDb and VDE are deflection angles to points a , b and E respectively. In staking out the curve the deflection angles and the chord lengths Da , ab and bE must be known. The computations of the deflection angles and the chords are explained in Art. 274.

In staking out a curve by this method, the vertex V is chosen. or the tangents A and B , Fig. 157, are produced in the field and a stake is set at their point of intersection V , by the method explained in Art. 67. The angle I is then carefully measured by double reversal and checked (Art. 57). With the value of the radius R , already selected or shown on the plan, and the measured value of I , the tangent distance T is computed by the formula $T = R \tan \frac{I}{2}$. Stakes are then set at the *P.C.* and *P.T.* by measuring the distance T along the tangents from V .

* For further details on curve layout, see standard texts on this subject. Chord-stationing method is described in *Railroad Curves and Earthwork* by C. F. Allen, McGraw-Hill Book Co., New York. The arc-stationing method is covered in *Route Surveys* by Harry Rubey, 1938 Ed., Chap. XVIII, The Macmillan Co.

The deflection angles and corresponding chord lengths are then computed for the points to be located on the curve.

To lay out the curve, the transit is set at the *P.C.*, and sighted on *V*, the circle reading 0° . The first deflection angle VDa is set off on the circle of the transit; the line of sight will be along Dx . Point *a* on the curve is somewhere in the line Dx . It is located by setting a stake at the chord distance Da from *D*, along the line Dx . When point *a* has been established, the deflection angle VDb is set off on the circle of the instrument. Point *b* is located by finding the point of intersection of the line of sight from the transit and the chord length ab , measured from *a*. When all points have been set along the curve, a field check is obtained on the linear measurements by measuring the last chord distance bE , and comparing it with the computed length of that chord. A check is obtained on the angles by laying off the deflection angle to the *P.T.* and observing whether the line of sight passes through that point, which was previously set by laying off *T* from *V*.

274. Computing the Deflection Angles by Actual Arc Method. In computing the deflection angles use is made of the following theorems of plane geometry:

- (1) The angle at the center of a circle is measured by the arc it subtends.
- (2) The angle between a tangent and a chord the vertex of which lies in the circumference is measured by **half** the subtended arc.
- (3) The angle between two chords the vertex of which lies in the circumference (an inscribed angle) is measured by **half** the subtended arc.

In Fig. 157 the deflection angle VDa $DOa = \frac{\alpha_1}{2}$; VDb

$DOb = \frac{\alpha_2}{2}$ and $VDE = \frac{DOE}{2}$

To compute the angles DOa , aOb and bOE , the distances along the arc, Da , ab and bE , and the radius of the curve must be known.

There are several ways of computing these angles. If the

arc $Da = 50$ ft., and the radius $R = 500$ ft., then by proportion.

$$\frac{\alpha_1}{360^\circ} = \frac{50}{2\pi 500}$$

$$\frac{\alpha_1}{I} = \frac{50}{\text{length of arc } DE}$$

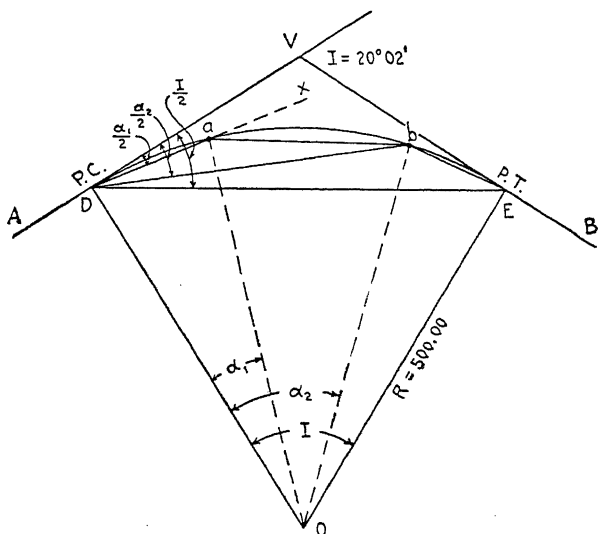


FIG. 157. DEFLECTION ANGLES FOR LAYING OUT A CURVE.

A more direct method for obtaining arcs from angles, or vice versa, is provided in Table VII which gives the length of circular arcs for radius unity. The arc is first divided by the radius, thus obtaining the circular measure of the angle. The angular value is then taken from the table, as follows:

$$\frac{50}{500} = 0.1000000$$

Table VII gives $5^\circ = .0872665$

Subtracting gives .0127335

43' .0125082

Subtracting gives $\underline{.0002253}$

$$46'' = .0002230$$

$$\alpha_1 = 5^{\circ} 43' 46'$$

$$\frac{\alpha_1}{2} = 2^\circ 51' 53''$$

A convenient expression to remember for the arc in terms of the radius and the central angle is the following:

$$\text{Arc} = .0174533 R \times \text{angle in degrees},$$

where .0174533 is the radian measure of one degree of angle.

275. Computing the Chord Lengths by Actual Arc Method. The chord length corresponding to the arcs may be computed by trigonometry through the central angles and the radius of the curve. In Fig. 158, which is a part of Fig. 157,

$$\text{Chord } Da = 2(500) \sin \frac{\alpha_1}{2} = 2(500) \sin 2^\circ 51' 53'' = 49.98 \text{ ft.}$$

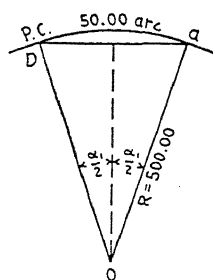


FIG. 158.

It will be observed that the deflection angle $\frac{1}{2}\alpha_1$ was computed to the nearest second. In practice these angles are seldom laid off closer than one-half minute. The reason for **computing** the angles so close is for exactness in computing the chord lengths. Where the chords are short compared with the radius, a small change in the angle at O, Fig. 158, may cause a comparatively large error in the computed chord length.

276. Another simple method of computing the chords is to compute the **difference** between the arc and chord and subtract this difference from the arc length. Except for the sharpest curves the following approximate formula will give the difference with sufficient precision by slide-rule computation.

$$l - c = \frac{c^3}{24 R^2} \quad \text{or} \quad \frac{c^3}{24 R^2} \quad \text{approx.}$$

where l = length of arc, c = length of chord, and R = radius.

In the above illustration,

$$\text{chord } Da = 50 - \frac{50^3}{24 \times 500^2} = 49.98 \text{ ft.}$$

EXAMPLE:

In a circular curve, $I = 48^\circ 24' 00''$, $R = 200$ ft.

Compute: (a) tangent distance, (b) length of arc $P.C.$ to $P.T.$, (c) deflection angles and chord lengths to set points 40 ft. apart along the arc, starting with the $P.C.$ (See Fig. 159.)

$$\text{Tangent Distance, } T = R \tan \frac{I}{2} = 200 \tan 24^\circ 12' = 89.884 \text{ ft.}$$

Length of Curve (Table VII)

$$\begin{array}{r} 48^\circ = .8377580 \\ 24' = .0069813 \\ \hline .8447393 \\ \hline 200 \\ L = 168.948 \text{ ft.} \end{array}$$

Central angle: for arc of 40 ft.

$$\begin{array}{r} 40^\circ = .2000000 \\ 200 \\ \hline .1919862 \quad 11^\circ \\ .0080138 \\ \hline .0078540 \quad 27' \\ .0001598 \quad 33'' \end{array}$$

$$d = 11^\circ 27' 33''$$

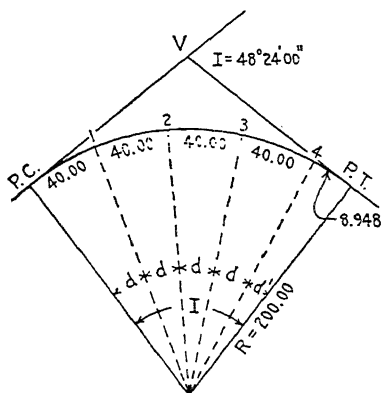


FIG. 159.

for arc of 8.948 ft.

$$\begin{array}{r} 8.948 \\ \hline .0447400 \\ .0349066 \quad 2^\circ \\ .0098334 \\ \hline .0095993 \quad 33' \\ .0002341 \quad 48'' \end{array}$$

$$d' = 2^\circ 33' 48''$$

Chord lengths corresponding to a 40-ft. arc.

$$\begin{aligned} C &= 2R \sin \frac{d}{2} \\ &= 2(200) \sin 5^\circ 43' 47'' \\ &= 39.936 \text{ ft.} \end{aligned}$$

Corresponding to an 8.948 ft. arc.

$$\begin{aligned} C' &= 2R \sin \frac{d'}{2} \\ &= 2(200) \sin 1^\circ 16' 54'' \\ &= 8.948 \text{ ft.} \end{aligned}$$

$$\text{Alternate: } l - C = \frac{40^3}{24 \times 200^2} = .067; \quad C = 40.00 - .067 = 39.933 \text{ ft.}$$

$$l' - C' = \frac{8.95^3}{24 \times 200^2} = .0007; \quad C' = 8.95 - .0007 = 8.95 \text{ ft.}$$

Deflection Angles*		Chord Lengths
o ' "		
To point 1 = $\frac{d}{2}$	= 5 43 47	39.94 ft.
2 = $\frac{d}{2} \times 2 = d$	= 11 27 33	39.94 ft.
3 = $\frac{d}{2} \times 3 = \frac{3d}{2}$	= 17 11 20	39.94 ft.
4 = $\frac{d}{2} \times 4 = 2d$	= 22 55 06	39.94 ft.
P.T. = $2d + \frac{d'}{2}$	= 24 12 00	8.95 ft.

Check the deflection angle to the P.T. by noting that it should equal $\frac{I}{2}$.

277. Laying Out a Curve by Chord-Stationing Method. In railroad practice and in some highway practice the curvature is designated by the *degree of curve*, meaning in this practice the angle at the center of the curve subtended by a **chord** of 100 ft. (Fig. 160.) Hence $\sin \frac{D}{2} = \frac{50}{R}$.

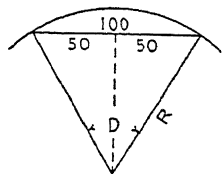


FIG. 160. DEGREE OF CURVE—RAILROAD PRACTICE.

For usual values of D , radii are nearly inversely proportional to the degrees of curve.

for a 1° curve, $R = 5729.65$ ft.

for a 2° curve, $R = 2864.93$ ft., $\frac{1}{2} R$ for a 1° curve approx.

for a 3° " , $R = 1910.08$ ft., $\frac{1}{3} R$ " " 1° " "

for a 4° " , $R = 1432.69$ ft., $\frac{1}{4} R$ " " 1° " "

for a 5 " , $R = 1146.28$ ft., $\frac{1}{5} R$ " " 1° " "

for a 10 " , $R = 573.69$ ft., $\frac{1}{10} R$ " " 1° " "

In railroad and highway surveys it is customary to use the continuous system of stationing, with stations 100 ft. apart.

* The deflection angle in seconds for 1 ft. of arc = $103, 132.4 \div R$ in ft. Deflection angle in radians = arc $\div 2R$. Making arc unity, and changing angle to seconds, gives deflection angle in seconds = $\frac{1}{2R} \times \frac{180}{\pi} \times 3600 = \frac{103, 132.4}{R}$.

Referring to Fig. 161, the *length of curve* is made up of

- | | |
|---|----------|
| (a) Sub-chord from <i>P.C.</i> to next full station | 60.2 ft. |
| (b) Four full chords of 100 ft. each | 400.0 |
| (c) One sub-chord from the last full station
on the curve to the <i>P.T.</i> | 24.8 |

on the curve to the <i>P.T.</i>	<u>24.8</u>
Total length of curve from <i>P.C.</i> to <i>P.T.</i>	485.0 ft.

The length of curve is found by the formula $L = 100 \frac{I}{D}$.

$$L^* = \frac{100 (24.25)}{5} = 485.0 \quad T = R_s \tan 12^\circ 07' 30'' = 246.3$$

$$\left. \begin{aligned} \text{Sta. } P.C. &= \text{Sta. } V - T \\ \text{Sta. } P.T. &= \text{Sta. } P.C. + L \end{aligned} \right\}$$

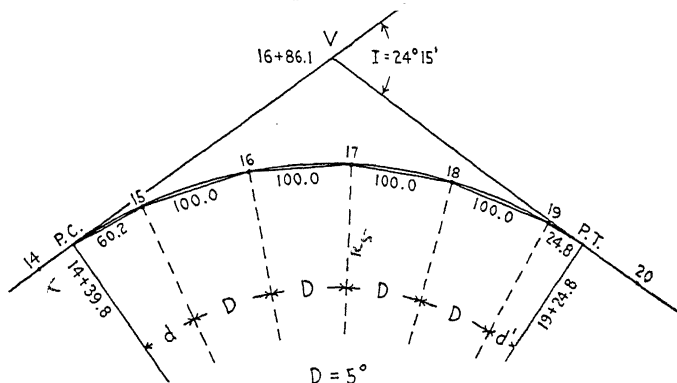


FIG. 161. CHORD-STATIONING METHOD.

The deflection angle corresponding to a chord length of 100 ft. is $\frac{D}{2}$, and for any sub-chord it may be found by proportion.

$$\frac{60.2}{100} \times 5^\circ = 3^\circ.01 = 3^\circ 00'.6, \text{ and } \frac{1}{2}d = 1^\circ 30'.3.$$

Similarly,

$$d' = \frac{24.8}{100} \times 5^\circ = 1^\circ.24 = 1^\circ 14'.4, \text{ and } \frac{1}{2}d = 0^\circ 37'.2.$$

* The difference in L computed as above and the actual arc length is 0.16 ft.

Fig. 162 shows the form of field-notes for this curve. Notice that the deflection angle to the $P.T.$ = $\frac{I}{2}$.

Station	Points	Description of Curve	Deflection Angles
21			
20			0
	⊙ + 24.8 $P.T.$		12 07.5
19		$R = 1146.3$	11 30.3
18		$L = 485.0$	9 00.3
17		$T = 246.3$	6 30.3
	⊙ + 86.1 $V.$	$I = 24^{\circ} 15'$	
16		$D = 5^{\circ}$	4 00.3
15			1 30.3
	⊙ + 39.8 $P.C.$		
14			
13			
12			

FIG. 162. FIELD-NOTES FOR CURVE LAID OUT BY RAILROAD PRACTICE.

278. When the Entire Curve Cannot be Run Out from the $P.C.$ It frequently happens that the entire curve cannot be laid out from the $P.C.$ because of some obstruction, as shown in Fig. 163. When this condition exists, set as many stations as possible from the $P.C.$ Then move the transit to the last station set and run the curve from that point out to the end.

In Fig. 163 Stas. 15, 16 and 17 have been set from the $P.C.$ In order to set Stas. 18 and 19 and check on the $P.T.$, the instrument is set over Sta. 17. With the circle reading 0° , invert the telescope and sight the $P.C.$ Plunge the telescope so that it is pointing along the line bd . Lay off the angle $dbf = \text{angle } aef$, which is the deflection angle that would have been used if it had been possible to sight Sta. 18 from the $P.C.$

The proof that $dbf = aef$ is as follows: line ac is drawn tangent to the curve at Sta. 17. Since ea and ab are tangent to the curve, angle $aeb = abe$. The angle cbf equals the angle bef , since both are measured by one-half the arc bf . Therefore $dbf = aef$. To set Sta. 19 and check on the $P.T.$, the deflection angles computed for those points from the $P.C.$ are laid off from the line bd .

gent and distances to these offsets measured along the tangent from the *P.C.* In Fig. 164, the offset $a = R \text{ vers } \alpha$ and the distance from the *P.C.*, $t = R \sin \alpha$. In terms of the radius and chord

$$a = \frac{C^2}{8R} \quad t = C - \frac{u^2}{2C}$$

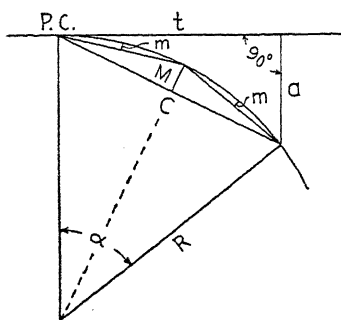


FIG. 164.

282. Setting Points on a Curve by Middle Ordinates. Another method of laying out a curve is by offsets measured at right angles to a chord at its center point, i.e., "*middle ordinates.*"

In Fig. 164, $M = R \text{ vers } \frac{\alpha}{2}$, $C = 2 R \sin \frac{\alpha}{2}$. In terms of the chord and the radius, $M = \frac{C^2}{8R}$ (approximately). A curve may be laid out by erecting ordinates to successively shorter chords,

as shown in Fig. 164, $m = \frac{\left(\frac{C}{2}\right)^2}{8R} = \frac{C^2}{32R} = \frac{M}{4}$ (approximately).

283. Compound and Reversed Curves. A compound curve is a series of simple circular curves running in the same general direction, one following the other and having a common tangent where they join. A reversed curve is formed by two circular curves having a common tangent where they join, but lying on opposite sides of this common tangent.

A compound curve is laid out like a simple circular curve from the *P.C.* to the point of compound curvature, *P.C.C.* At the *P.C.C.* the transit is set up, circle set at half the central angle of the first section of the compound curve and a sight taken on the *P.C.* If the circle is now turned to 0° , it should be sighting along the common tangent at *P.C.C.*, and the next section of simple curve (of different radius from the first section) may be laid off by deflection angles, just like any other simple curve.

In laying out a reversed curve the same general method is used, viz., the first section is laid out like any simple curve. At the point of reversal, *P.R.C.*, set up the transit, manipulate it so that it sights 0° along the common tangent, and then lay off the second simple curve on the opposite side of the common tangent.

284. Spirals. On railroads and high-speed highways, spirals (or transition curves) are usually introduced between the straight and curved parts of the alignment, thus providing safer and easier riding.

These transition curves are either true spirals or they are a series of compound curves. Where they depart from the tan-

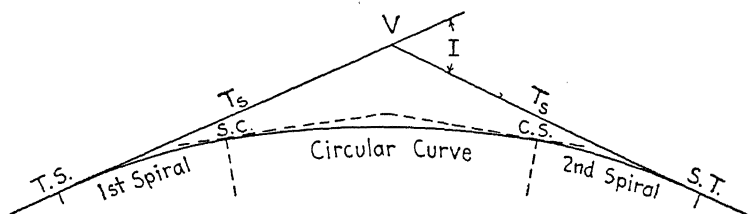


FIG. 165.

gent they are very flat and gradually grow sharper until they have the same radius as the circular curve to which they are to connect in a common tangent.

In laying out these spirals the tangent distance T_s from the vertex to the point where the spiral starts (the *T.S.*) is computed and laid off on each tangent measuring from the vertex (Fig. 165). The first spiral may then be laid out by chords and deflection angles from the main tangent. When the end of the spiral (the *S.C.*) is reached the instrument is moved to the *S.C.* and the circular curve is laid out to its end at the *C.S.* by chords and deflection angles from an auxiliary tangent at the *S.C.* The second spiral may be laid off from the *S.T.* exactly as the first spiral was laid off from the *T.S.* It should check on the *C.S.*

The process of computing T_s and the chords and deflection angles of the spiral is too lengthy to include in this text. They will be found in standard textbooks on curve layouts.

285. VERTICAL CURVES. Vertical parabolic curves are commonly used on railroads and highways at points where the rate or direction of grade changes, such as where a downgrade is followed by an upgrade. The parabola is useful for this purpose because the vertical offsets required between the straight grade lines and the curve can be easily computed, and knowing these vertical distances lends itself to the usual fieldwork of setting grades by means of the level and rod.

The equation of the parabola is

$$y^2 = 4px, \quad \text{or} \quad y^2 = \text{a constant times } x$$

From the above equation it is evident that the offsets x vary as the square of the distances y . Use is made of this simple relationship in computing vertical curve problems as follows:

$$\frac{x_1}{x_2} = \frac{y_1^2}{y_2^2}, \quad \frac{x_1}{x_3} = \frac{y_1^2}{y_3^2}, \quad \frac{x_1}{x_4} = \frac{y_1^2}{y_4^2}; \quad \text{etc.}$$

When the offsets are taken at equal intervals, i.e., $y_1 = y_2 = y_3 = y_4$, etc., then

$$x_1 = \frac{x_2}{4}; \quad x_1 = \frac{x_3}{9}; \quad x_1 = \frac{x_4}{16}; \quad \text{etc.}$$

In Fig. 166, AV and VB are two grade lines intersecting at V . A vertical parabolic curve CDE is introduced, extending equal horizontal distances on both sides of V , with points of tangency at C and E . The elevations of the stations along the curve are required. VF is a diameter of the parabola (a line to which the offsets are parallel). When the parabola extends the same horizontal distance on both sides of V , the diameter VF is vertical and therefore all the offsets taken parallel to it are also vertical. These offsets are all proportional to the square of their distances from C along CV . But since the horizontal projection of CV is divided into the same number of equal distances as is CV , these horizontal projections are proportional to the inclined distances along the tangent CV . Hence, the vertical offsets cc' , etc., are proportional to the squares of the horizontal distances to them, measured from C .

$$\text{Elev. } F = \text{Elev. } C + \text{Elev. } E - \frac{348.52 + 340.52}{2} = 344.52$$

$$\text{Elev. } D = \frac{\text{Elev. } F + \text{Elev. } V}{2} = \frac{344.52 + 336.52}{2} = 340.52$$

$$VD = \text{Elev. } D - \text{Elev. } V = 340.52 - 336.52 = 4.00$$

$$aa' = ff' = \frac{VD}{16} = \frac{4.00}{16} = 0.25$$

$$bb' = ee' = \frac{VD}{8} = \frac{4.00}{8} = 1.00$$

$$cc' = dd' = \frac{9VD}{16} = \frac{9(4.00)}{16} = 2.25$$

Adding the offsets to the elevations of a' , b' , c' , etc., the grade elevations of the required points along the curve are obtained. (Fig. 167.)

One of the properties of the parabola is that the second difference should be equal; a check is thus obtained on the calculations. In computing the first and second differences, the signs + or - must be taken into account. This check is illustrated in the last two columns of Fig. 167.

Sta.	Straight Grade Elev.	Offsets	Grade Elev. Curve	1st Diff.	2nd Diff.
23					
22 = E	340.52	0.00	340.52		
+50	339.52	0.25	339.77	-0.75	+0.50
21	338.52	1.00	339.52	-0.25	+0.50
+50	337.52	2.25	339.77	+0.25	+0.50
20 = V	336.52	4.00	340.52	+0.75	+0.50
+50	339.52	2.25	341.77	+1.25	+0.50
19	342.52	1.00	343.52	+1.75	+0.50
+50	345.52	0.25	345.77	+2.25	+0.50
18 = C	348.52	0.00	348.52	+2.75	

FIG. 167. COMPUTATION OF ELEVATIONS ON A VERTICAL PARABOLIC CURVE.

286. Highest and Lowest Points on a Vertical Curve. It is sometimes necessary to find the station of the highest point on a summit vertical curve, or of the lowest point on a sag (valley) curve. The distance from the beginning of the curve to the high (or low) point is $\frac{Lg_1}{g_1 - g_2}$, where L is the total length of the curve in stations; g_1 and g_2 are the rates of grade at the beginning and end of the curve respectively. Upgrades are assumed to be plus and downgrades, minus. Referring to the vertical curve in Fig. 166, the distance from Sta. 18 to the low point of the curve is $4 \times \frac{-6}{-6 - 2} = 4 \times \frac{3}{4} = 3$ stations; i.e., the low point occurs at Sta. 21 + 00. In many cases the low (or high) point does not fall at a full station, as it happened to do in this example.

287. Length of Vertical Curve for Required Sight Distance. In highway practice the length of the vertical curve over a summit is controlled by the minimum "sight distance" required for safe driving. It is intended to be the horizontal distance between the driver's eye in each of two cars approaching the summit from opposite directions. This is assumed to be 4 to 4.5 ft. above the pavement. The vertical curve may be either longer or shorter than the sight distance, depending upon the abruptness of the change of grade and length of sight distance required.

When the vertical curve is longer than the sight distance $L = \frac{S^2 (g_1 - g_2)}{8h}$, where L = length of vertical curve in stations, S = sight distance in stations, g_1 and g_2 = per cent of approach and following rates of grade (upgrades are plus and downgrades minus), and h = height of eye in feet above the pavement at each end of the sight distance.

When the vertical curve is shorter than the sight distance the two ends of the sight distance fall on the straight grades and the formula then becomes

$$L = \frac{2 (g_1 - g_2) S - 8h}{g_1 - g_2}$$

It is not always possible to tell in advance which of these two formulas applies. A trial solution with one formula, how-

ever, will show whether the sight distance is greater or less than the length of curve.

EXAMPLE: If a $+4\%$ grade is followed by a -2% grade, and $h = 4.5$ ft., and $S = 500$ ft., find L .

$$g_1 - g_2 = +4 - (-2) = 6$$

Assuming L longer than S ,
$$L = \frac{5 \times 5 \times 6}{8 \times 4.5} = 4.17 \text{ sta.}, \text{ or } 417 \text{ ft.}$$

Since this trial value for L is shorter than S , the wrong formula was used. Using the formula for L shorter than S ,

$$L = \frac{2 \times 6 \times 5 - 8 \times 4.5}{4.00 \text{ sta.}, \text{ or } 400 \text{ ft.}}$$

288. STAKING OUT PROPERTY LINES OF A CITY LOT. A survey is required to set stakes, to be later replaced by monuments, at the corners of the lot $ABCD$, Fig. 168. Assume that the transit may be set at the corners and that measurements may be made directly along the property lines.

From the city engineer's office the following data were obtained: width of Broad St.; description of locations of monuments 74 and 75 on Broad St. and 50 and 51 on North St., defining the offset lines; ties to the monuments for checking their positions, that is, for determining whether they have moved; offsets from offset lines to the street lines; angle between the offset lines.

The above data, together with the length of the sides and angles of lot, are shown on the sketch, Fig. 168.

There are several ways in which the corners A , B , C and D can be located; a simple method when all lines are unobstructed is as follows:

Set transit at Mon. 51 and sight Mon. 50; set two points m and n on line, approximately opposite A and D ; set transit at Mon. 75 and sight Mon. 74; set two points o and p on line, approximately opposite A and B ; with transit at p , sight Mon. 74, lay off a right angle, and set p' , 57 ft. from p . In a similar manner, set o' from o . Repeat the above process along North St. by setting the transit at m to locate m' , and at n to locate n' the distance $mm' = nn' = 37$ ft.

Point A is next located by finding the intersection of the lines $n'm'$ and $p'o'$ (see Art. 67). With the transit at A , sight p' and measure $AB = 130.00$ ft. and set B . Then sight n' , measure $AD = 135.00$ ft. and set D . It is advisable to measure

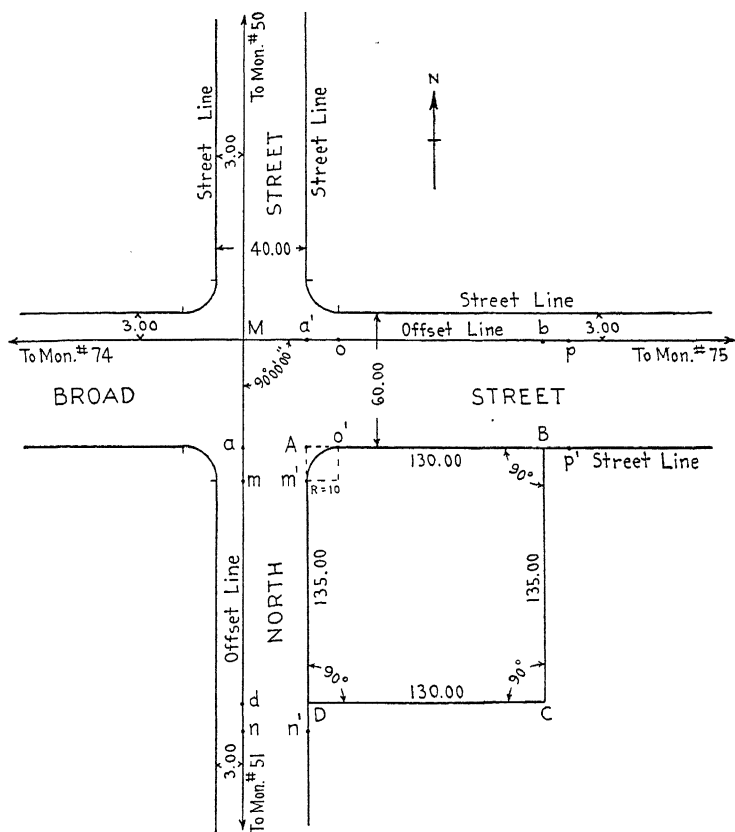


FIG. 168. CORNER CITY LOT WITH NO OBSTRUCTIONS.

also the angle BAD for a check. To locate C , set transit at B , sight A , lay off 90° (see Art. 60) and set C on line 135.00 ft. from B . For a check, C may be located also from D , and the diagonals AC and BD should be equal.

A second method is to locate the intersection M (see Art. 67) of the offset lines. With the instrument at M , sight Mon. 51 and lay off the distances $Ma = 57$ ft. and $ad = 135$ ft. To locate A , set the transit at a , sight Mon. 51, lay off a right angle and measure 37 ft. Without disturbing the transit, locate B in the line of sight by laying off the distance $AB = 130$ ft. By a similar procedure, locate D and C by setting the instrument at, and measuring from, point d . AD and BC should be measured for checks.

It will be observed that this problem is essentially one of moving the offset lines parallel to themselves from their original locations to the positions AB and AD .

In the first method six set-ups of the transit and four accurate tape measurements are required to establish the positions of the lines AB and AD . To locate points A , B and D , nine set-ups are required, and six lines have to be accurately measured. However, no angles have to be laid off with precision to locate A , B and D . The right angles at m , n , o and p need only be measured to the nearest minute, since the length of measurements (37 and 57 ft.) are small.

In the second method, in order to locate A , B and D , five set-ups of the transit are required, two angles have to be laid off accurately, and six precise tape measurements have to be made.

By studying the problem, other ways of locating A , B and D will present themselves.

289. In the above survey it was assumed that there were no obstructions to interfere with making both angular and linear measurements directly on the property lines. If a building abuts on the property lines, the corner of the building may be located from auxiliary traverse lines established for this purpose. Sometimes it will be found that the walls of the building do not coincide with the property lines. The actual lines of the building may be slightly over the street line or they may be set back from the street (property) line. In Fig. 169, the solid lines are the property lines; the walls of the building coincide with these lines except where indicated by the dashed lines. Referring to "Detail at Corner B " in Fig. 169 the corner B' of the building encroaches on the street area by $B't$, and on the property to the

east by Bs . The corner C' of the building is set back from the property corner C by the distance CC' . The setbacks and encroachments are assumed to be small, as is the usual case. In making the survey it is desirable to locate the actual corners of the property where it is possible and to determine the amount of encroachments and setbacks, if any.

The procedure in making this survey is as follows:

Point M , the intersection of the offset lines, is first located. (See Art. 67.)

With the transit at M , sight Mon. 51. Locate points a , d and m on line by measuring $Ma = 57$ ft.; $ad = 135$ ft.; and $dm = 8$ ft. The distance dm need be only of sufficient length to insure a sharp, distinct image when the telescope is focused at that distance.

With the transit still at M , sight Mon. 75 and set point p on line, at a distance of $37 + 130 + 8 = 175$ ft. from M . Likewise set point q on line, $50 \pm$ ft. beyond p .

The instrument is then set up at a . Point A is located by laying off a right angle (using method of repetition) with the offset line M -Mon. 51 and measuring $aA = 37$ ft. Similarly, D is located by setting the transit at d , turning a right angle with the offset line and measuring $dD = 37$ ft. The distance AD is measured along the face of the building for a check.

The transit is then set up at m , a right angle is carefully laid off with the offset line, and points e and f are set on line at distances from m of 167 ft. and 175 ft. respectively.

Before locating C , point f should be checked. This is done by setting the transit at p , laying off a right angle with the offset line, and measuring the distance pf . The angle laid off should pass through f , and the distance pf should be 200 ft. With the instrument still at p , locate point r on the property line, the distance $pr = 57$ ft. The instrument is next set up at q , a right angle laid off with the offset line, and point v located on the property line, the distance $qv = 57$ ft.

To locate C , the transit is set at e , a right angle laid off with em , and the distance $eC = 8$ ft. measured. The distance DC' is measured along the wall and found to be 129.91 ft., which indicates that C' is set back from the property corner point C by 0.09 ft.

$B't$, which may be taken as the encroachment of B' on the public property. $B't$ could be determined directly by a swing offset.

The distance $C'B'$ is measured and found to be 135.10 ft.

In the foregoing discussions all distances are measured or laid off to the nearest 0.01 ft. To attain this precision, the tape should be standardized and used under the conditions of standardization. In laying off the right angles different precisions may be used. In laying off the right angle at q to locate v , an angle to the nearest minute is amply sufficient. To be consistent with the linear measurement, the right angle at p to locate f should be determined to at least the nearest 5 seconds. To locate A and D from a and d , the right angles should be laid off to at least the nearest 25 seconds.

290. STAKING OUT THE LOT SUBDIVISION OF A BLOCK. In Fig. 170 A, B, C, D and E are intersections of the center lines of streets bounding the group of eight lots as shown. AE and CD are 40-ft. streets, and ABC and DE are 60-ft. streets. The dimensions of all the lots have been computed in the office; it is now desired to stake out the lot corners in the field. Points A, B, C, D and E were referenced to permanent objects by the method of Art. 119. If any of the original points are still in place, their locations should be checked from the ties to see that they have not moved. If any points have been lost, they should be replaced by means of the ties.

To stake out the lot frontages on street ABC , proceed as follows: set the instrument over B and sight on A . Measure 77.56 ft. from B towards A and set a point accurately on line at a , the P.C. of the center-line curve. From A measure distance $Ab = 17.07$ (see Fig. 171), and set b on line AB opposite d . Check the distance $ba = 203.80 - 77.56 - 17.07 = 109.17$ ft. With instrument still at B , sight on C and set points m and n in the same way in which a and b were set. Move the instrument to m , sight on C , lay off 90° , and measure 30.00 ft. to set l . Now move the transit to a , sight on A , lay off 90° , and set point c 30.00 ft. from a . Set up on c and sight on a swing offset of 30.00 ft. from A or b . Measure 26.75 ft. from c and set i ; measure $26.75 + 64.34 = 91.09$ ft. from c and set g (P.C. of rounding curve); then measure $91.09 + 18.08 = 109.17$ ft. from c and set

point d . Compute the chords and deflection angles for setting points j , k and l on the curve by the method of Arts. 274-6; with the instrument still at c , sighting on d , invert telescope and set points j and k , and check on location of l , previously established. Set up at l and place o and p in the same way in which g and d were set.

To set the back corners of lots 1 to 4 inclusive, set up at C , sight on D and set points q and s . Cn and Cq are computed in the

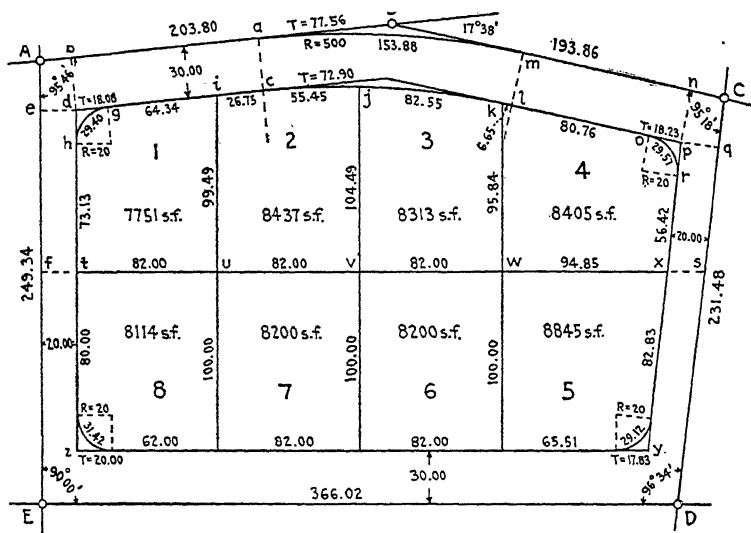


FIG. 170. BLOCK SUBDIVISION.

same manner as that illustrated in Fig. 171; s is on the extension of the back line of the lots. Check the position of p by the dimensions $np = 30.00$ and $pq = 20.00$. Next set up the transit at A , sight on E and set points e and f ($Ae = 28.13$ in Fig. 171, $ef = 73.13 + 18.08 = 91.21$). Move the instrument to f , sight on s and lay out distances $ft = 20.00$, $tu = 82.00$, $uv = 82.00$, $vw = 82.00$, and $wx = 94.85$; xs should equal 20.13 . If there is any discrepancy in the total measurement of the line fs , ft should be made exactly 20.00 ft. and xs exactly 20.13 ft. The discrepancy should not be more than few hundredths and

should be distributed equally among the several lot dimensions. Discrepancies arise due to the difference in tape-lengths between the time of the original survey and the time at which the lots are being staked out. Likewise there is always some error of closure in a traverse such as $ABCDE$ which must be distributed. It is common practice to distribute this among the lots rather than to alter the street widths. Measure $th = 73.13$ and check $hd = 18.08$. Measure the depths of all lots such as iu , iv and kw and see that they check dimensions shown on plan.

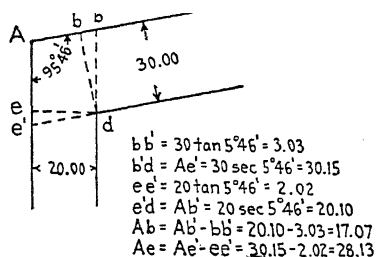


FIG. 171.

The frontages of lots 5 to 8 inclusive may be located by first setting points z and y in a manner similar to that described above. Ordinarily only the ends of the rounding curve need be set, such as at g and h . If more points are required on these curves, they may be determined by locating the center and swinging an arc of 20 ft., or by stretching the chord hg and locating points on the curve by the method of middle ordinates or by offsets from the tangent. (Arts. 281-2).

For permanent reference, monuments should be set at d , c , l , p , x , y , z and t .

291. LINES AND GRADES IN TUNNELS.* In defining the lines and grades for a tunnel, great precaution must be exercised in transferring lines and grades into the tunnel and in extending them through the tunnel as the construction progresses, because all tunnels (except those that are open cut and cover) are driven from at least two places usually far apart, and sometimes also from very short lines established at the bottom of intermediate shafts.

Surveying for tunnel work falls into two parts: (1) transferring the lines and grades from the surface to the tunnel, (2) carrying them through the tunnel as construction proceeds.

* Mine Surveying is treated in Elementary Surveying by Breed and Hosmer, Vol. I, Chap. XII.

The process herein described relates to a typical project where a deep tunnel is to be driven a mile or two underground in both directions from a central shaft and from the ends also.

The center line of the tunnel is carefully run, and checked, over the hill from portal to portal, and marked exactly at the proposed shaft. The lines and grades are extended into the tunnel at each portal, as in ordinary surveying, all precautions being taken to manipulate the transit and level so as to eliminate the effect of every error in instrumental adjustment.

292. Transferring Lines and Grades from Surface to the Tunnel. When the tunnel must be driven either in one or in both directions from a vertical shaft, the direction line on the surface must be transferred to the tunnel; this is usually done by the use of long plumb-lines hung in the vertical shaft which is sunk directly over the center of the tunnel.

293. Plumbing Down the Shaft. In this process the plumb-bob must be heavy enough to straighten out all bends in the wire to which it is attached; piano wire is most suitable for such plumb-lines. In a deep shaft it is advisable to have the wire wound on a reel having a base plate that is firmly fastened to a stout plank which spans the top of the shaft. From this reel the wire runs over a pulley the axis of which is controlled by an adjustment, so that the plumb-bob wire may be set **exactly on line**.

When the shaft excavation approaches the bottom of the tunnel, two plumb-lines *A* and *B* are hung on the center line of the tunnel, and as close to the sides of the shaft as is possible. A hole is drilled in the wall of the shaft behind each of the wires at a point a few feet above the roof of the tunnel, and a wooden plug is driven into each hole; the plug should be almost long enough to touch the wire. The line *AB* defined by the wires is transferred by eye to the plugs behind the two wires and is marked exactly by spads driven into the plugs. (Fig. 172.) This gives a line *AB* that is sufficiently accurate for the start of the tunnel. Later, a more precise determination of the line *AB* will be made, as will be explained.

The elevation of each plug is determined by leveling, using in place of a rod a steel tape to which is attached a 20-lb. weight.

The upper end of the tape is fastened with C-clamps or with some other suitable device to an upright plank at the top of the shaft, and the elevation of a foot-mark near the top of the tape is determined by direct leveling from a B.M. The tape should be calibrated, so that its length when hanging in this position and supporting a 20-lb. weight will be known.

The level is set up in the tunnel, and a reading taken on the suspended tape. This tape measurement, together with the reading taken on the upper part of the tape with a level set up on the surface, gives the data from which the H.I. of the level in the tunnel is computed. The rod is then held upside down on the spads in the two wooden plugs *A* and *B* that have been placed in the lower part of the shaft; from these the elevations of points *A* and *B* are accurately determined, thus giving two B.Ms. that can be used in the tunnel. Leveling

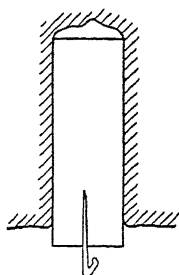


FIG. 172.
SPAD USED IN
TUNNELING.

from these two B.Ms., other B.Ms. are set within the tunnel on steel rods securely fastened in drill-holes within the tunnel; these B.Ms. are used to extend levels into the tunnel as the construction work progresses, by the usual method of leveling.

Starting from the shaft the driving of the first 75 to 100 ft. of tunnel is controlled by extending by eye the line defined by the two plumb-lines hung from the spads in the shaft. After about 100 ft. of the tunnel has been driven, plugs are established on the center line in the roof of the tunnel, 50 ft. on both sides of the shaft. This is done by again hanging two plumb-lines from the surface; but this time the lines are kept a little farther from the sides of the shaft and are checked to see that no drops of water are striking the wires at any point along their length. This time the plumb-bobs are immersed in oil to damp the swing.*

* The oil must be of the correct specific gravity and viscosity to check the motion of the bob, but should not be viscous enough to hold it in a position off the true vertical, which is possible if the proper oil is not used. A mixture of castor oil and light machine oil is satisfactory. The hole in the cover of the oil bucket should be just large enough to prevent the swing of the wire from touching the edge and small enough to prevent drops of water in the shaft from splashing in the oil and deflecting the bob.

The transit is then set up near the forward end of the tunnel, not necessarily at the actual end of tunnel construction, at a point away from the nearer plumb-line by an amount equal to about three times the distance spacing AB of the plumb-bobs. The transit should be equipped with a slow-motion *lateral head adjuster*, by which the movement normal to the center line of the tunnel can be controlled. The transit is then "jiggled in" on line (Art. 66), the final adjustments being made with the lateral head adjuster. This process may take an hour or two in a deep shaft, because the plumb-lines will still continue to swing in spite of the effect of the damping buckets. The mean position of the plumb-line wires must be established by taking many observations on the plumb-lines when they are farthest one way and the farthest the other way on their swings. Only one wire may be sighted at a time. A box with a light in it is usually placed behind the plumb-wire to illuminate the wire. When the transit is on line, a spad is driven into a wooden plug in the roof of the opposite heading (the extension of the tunnel) exactly on line; the process is then repeated by moving the transit to the opposite heading and placing the plug in the roof of this other heading. If only a single heading is being driven from the shaft, two plugs are located in this heading from a single set-up; these points are used to project the center line forward as the tunnel construction progresses.

Using the definite marks in the two roof plugs thus located as a base-line, the alignment of about a thousand feet of tunnel in both directions may be controlled. When 1000 ft. of tunnel in both directions has been completed, the final center line is transferred from the plumb-lines A and B to the tunnel.

294. Special plugs are now put in the tunnel roof on the center line of headings at about 50, 350 and 650 ft. from the shaft. To these plugs are fastened brass scales consisting of a horizontal bar about 8 in. long, graduated to any convenient scale, usually tenths and hundredths of feet. On this bar is a rider with a vernier reading to 0.002 ft. and with a slot under the zero of the rider from which a plumb-line may be hung. (Fig. 173.)

The long plumb-lines from the surface are again hung in the shaft and checked for water-drops on the wires and also for air

currents. This survey work in the tunnel will take a full day; no one should pass up or down the shaft during that time; the work and all air blowers must be shut 'down. A survey party must remain on the surface to keep the two wires at the surface always exactly on line.

The transit in the tunnel is set up in one heading as before, and, after "jiggling in" on line, the short plumb-lines hanging from the three brass scales in the opposite heading are set on line and the scale readings recorded. An independent setting of the transit is then made in line with the plumb-lines, and

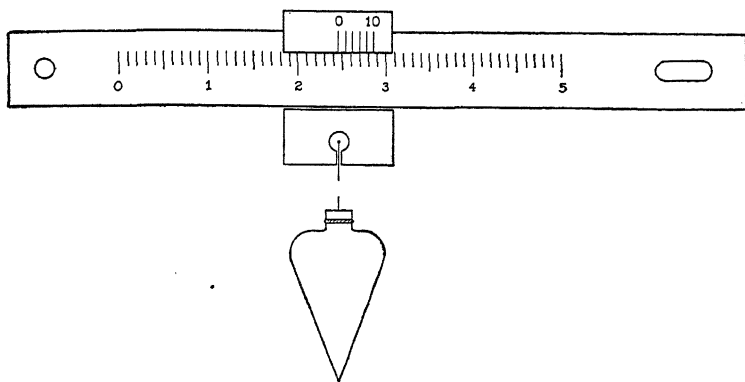


FIG. 173. BRASS SCALE.

readings are again taken and recorded at each scale; this process is repeated until a sufficient number of sets of readings (from 25 to 30) have been made for one heading. This will usually be enough to establish accurately the line in each heading. It may require five or six days to take these readings; it may take from five to ten weeks to finish the work, depending upon the frequency (Sundays) with which the construction work is shut down for a day.

The three brass scales *C*, *D* and *E* are drawn on cross-section paper, as in Fig. 174. They are plotted at distances apart on the paper corresponding to their distances apart in the tunnel. The zero-points of scales *C* and *E* are plotted on the **same vertical line** of the cross-section paper. The readings are plotted

on scales *C* and *E* (only 4 of the 25 or 30 are shown in Fig. 174 for illustration). Straight lines are drawn between the corresponding readings 1, 2, 3 and 4 on scales *C* and *E*. The corresponding readings taken on scale *D* are plotted on the edge of a separate piece of paper, which we will call scale *D*; this scale

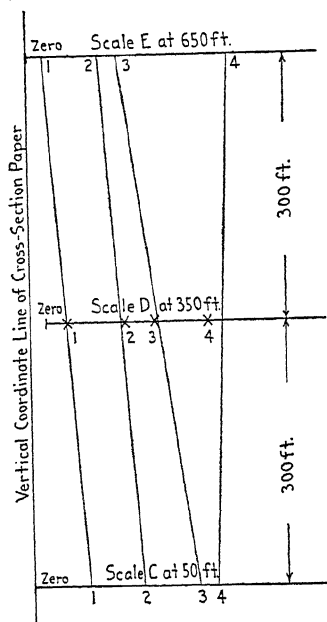


FIG. 174.

is then laid on the horizontal line where scale *D* is plotted on the sheet and is moved laterally until the corresponding plotted points fit the lines 1-1, 2-2, etc. It will be observed that the plotted points on *D* (at the crosses) all fall on, or close to, the proper lines, except the 4th readings; these readings will be discarded because there is an apparent error among them; they should lie on, or nearly on, the line 4-4.

The purpose of this plot is to **eliminate those observations in which there is obviously an error.** This leaves a set of readings on each of the three scales which appears to be consistent. Using this set of readings only, the mean of the readings of *C*, *D* and *E* are computed and the riders on *C*, *D* and *E* are **locked** at their

respective mean readings by means of set screw in back of rider, thus defining the **accepted** center line of the tunnel.

The line thus defined may be used to transfer the center line into the opposite heading by setting the transit under the plumb-bob at the first scale in the known heading, backsighting on the plumb-line at the third scale, and double inverting the transit (Art. 64). A few repetitions should be enough to locate the line in the opposite heading. The first three scales in each heading then become the permanent base-line for both headings. Three scales are used so that a check may be made when-

ever the line is to be carried forward for the purpose of checking against the possibility that one of the scales may have been knocked out of position by the workmen.

The line should be established as soon as possible, because the temporary bucket used for transporting men and materials up and down the shaft will probably soon be replaced by permanent skips. After the skips are in place it is usually more difficult to obtain a good determination of the line. It is advisable, however, for the engineer to see that no brace or structure of any sort is allowed to interfere with the space required for hanging the plumb-lines in the shaft.

Each heading now has its own base-line and, should structures be required at the foot of the shaft obstructing the view from one heading to the other, the line in each heading may be run independently.

295. The final grade elevation is determined by locating two permanent B.Ms. near the shaft, as explained above. These are pieces of steel (usually broken pieces of drill steel) grouted into bedrock, if possible, at an angle of about 45° in the side of the tunnel, above water level. The invert (bottom) of tunnel is usually wet during construction and may have a foot or more of water in it; this requires placing all survey marks above the water line. Because of the construction activity, the bench marks must be on the side of the tunnel, and the center-line marks must be in the roof. In all level work two B.Ms. are used for a check, thus eliminating error due to the possibility of one being disturbed during construction activities. The elevations of these B.Ms., located along the tunnel as it progresses, are obtained by direct leveling.

296. Extending Lines and Grades Within the Tunnel. Since the visibility near the heading is usually poor, the center line must be so marked that the nearest station shall be not over 100 ft. from the heading, where the tunnel driving is going on. If the tunnel is timbered, the spad is driven into the timber, but if the tunnel is in bedrock, a hole must be drilled in the roof and a wooden plug driven to which the spad is nailed, as in Fig. 172. The drill-holes are placed at a distance slightly under 100 ft. apart, so that the distance between plugs may be measured in a single tape-length.

The **approximate** line is determined by setting up at the last plug, backsighting on next to the last plug, and inverting the telescope. To set up at a new plug ahead a plumb-line is hung from the spad in the last plug and the instrument is set under the plumb-bob; when the telescope is in a horizontal position the small hole in the top of the telescope tube, marking a point directly over the vertical axis, should be under the point of the plumb-bob. The hole for the new plug is then drilled vertically into the roof by means of a stopper (a machine for drilling holes overhead), and the plug is driven in solidly. The line is then marked on the plug by double inverting from backsight on the next plug back, and a spad is driven into the plug exactly on line. A plumb-line is next hung from the new spad, and the distance between plugs is taped. Using the line defined by the brass scales, the new plug line should be checked and corrected, if necessary, in order to place the plugs **exactly** on the line **defined by the locked scales**. A new scale is installed usually at every third plug. The scale line is always behind the plug line, since it is not safe to install a scale within 300 ft. of the heading; if heavy blasting is necessary in the heading, scales even farther back may be damaged. When a new scale is installed, it is located by putting its center on the line determined by extending the scale line already defined. The transit is then set up at the last scale on which a known reading has been set, and a backsight is taken on the most distant locked scale that is visible. Readings are taken on the scale ahead, usually by three double invertings. The mean of these readings is then taken as the best value for the line, and the rider on the new scale is **locked** at that position. It will be noticed that projecting the center line forward is usually done by backsighting and inverting the telescope. This is necessary on account of the poor visibility; the transit has to be set at the last definitely located point, otherwise the point ahead which is to be placed cannot be seen.

297. Grades. Grades are carried on the **lowest point on the plug** (not on the spad, which may be knocked out or pulled out and have to be replaced). This is done by setting the level between the plugs, inverting the level rod, and holding it on the last plug for which the elevation is known. The elevation of

the new plug is then determined, and usually the level readings are checked on the second plug back to be sure that the plugs have not been loosened by the construction activity. Since visibility may not be good for over 10 ft., turning points must not be set too far apart; they will usually be chosen on the air or water pipes along the side of the tunnel.

A B.M. is located every 500 ft. Levels are run from the last B.M. the elevation of which is known; they should be checked by running the levels back to a different B.M. in order to be sure that the B.Ms. have not been disturbed. This constant checking, by using at least two different points in all the surveying operations, is necessary, because with all the heavy equipment, timbering, trimming, and other activities, the survey points are frequently disturbed, and, if disturbed only a slight amount, the movement might not be noticed and the position of the B.M. or plug might erroneously be assumed to be correct. As soon as a new B.M. has been established, the elevations of the plugs ahead of it are checked.

There is no check on the distances other than careful taping. The only errors that taping can introduce in a straight tunnel are errors in the grade of the tunnel; this will only be affected by errors of a hundred feet or so in the distance, since the rate of grade is low, and slight errors in the stationing can be corrected when the tunnel is "holed through."

When working in a heading that is being drilled, it is impossible to hear any speech; flashlight signals must be agreed on before the surveying work is begun. The plumb-line can be seen if the flashlight is held behind it, pointed at a slight angle away from the direct line to the transit. When pointed directly at the transit, the glare obscures the plumb-bob. In reading the level rod the axis of the flashlight case should be parallel to, and close to, the rod with the lens of the flashlight held close to the line of sight.

298. Bends in Tunnels. If there is an angle or bend in the tunnel, additional precautions have to be taken at such points. The alignment is first determined by setting off the angle on the transit at the bend and starting off on the new line. After a sufficient amount of tunnel has been driven, the alignment is

checked by measuring the angle to the last scale by repetition, and then getting the exact scale reading by correcting the setting by the angular error and distance from angle point (vertex) to the scale. In order that the line shall be correct, the vertex point must be exact in stationing. This is accomplished by precise taping.

If the tunnel is on a curve such that the vertex falls outside the tunnel, two or more deflections must be measured between the straight tangents adjoining the curve. The problem is similar to that encountered when there is an obstacle on a curve and an intermediate set-up is required. (See Art. 278.)

PROBLEMS

1. Data for a circular curve are as follows:

$$I = 63^\circ 17' \quad R = 800 \text{ ft.} \quad V = 27 + 36.23 \quad \textit{Arc Stationing}$$

- (a) Find station of *P.C.* and *P.T.* to $\frac{1}{10}$ ft.
- (b) Find deflection angles (to $\frac{1}{2}$ minute) for setting all full stations on the curve with the transit at the *P.C.*
- (c) Find chord lengths (to 0.01 ft.) for sub-chords at each end of curve and chord length for a full arc station (100-ft. arc).

2. (a) A 5-degree curve is to connect two tangents which intersect at an angle of $51^\circ 23'$. If the station of the *P.C.* = $12 + 42.71$, find the station of the *P.T.* and the deflection angles (to $\frac{1}{2}$ minute) required to set all the stations on the curve by the chord-station method.

(b) An obstacle prevents running the curve beyond Sta. 17 with the transit at the *P.C.* With the transit at Sta. 17, what angle should be set on the plate when backsighting on the *P.C.* and what are the deflection angles required to set the remaining stations on the curve and the *P.T.*?

3. A circular curve has an angle $I = 41^\circ 34'$, radius 700 ft. and *P.C.* = $21 + 50.00$ *Arc Stationing*.

(a) Find right-angle offset from the *P.C.*-*V* tangent to Sta. $23 + 00$ on the curve.

(b) Find also the distance from the vertex to the point on the tangent from which the offset is to be laid off. Distances to $\frac{1}{10}$ ft.

4. Referring to Fig. 170

(a) Compute lengths nC and Cq to .01 ft.

(b) Compute the middle ordinate to set the middle point on the rounding curve *or*.

(c) Assuming in lot 5 that the frontage 65.51 and the depth 100.00 are correct, check the remaining dimensions of lot 5 and its area.

5. STA.	GRADE ELEV.	The theoretical grade line of a highway is straight from Stas. 62 to 72; and from Stas. 72 to 87. A vertical (parabolic) curve is to connect these grade lines. Vertical curve is 800 ft. long (extending 4 stations each side of vertex).
62	131.78	
72	124.58	
87	144.83	

Find (a) Elevations to 0.01 ft. at each full station on vertical curve.

(b) Elevation at Sta. 74 + 30 on the vertical curve to 0.01 ft.

6. STA. GRADE ELEV. (a) A vertical (parabolic) curve 300 ft. long
 8 + 00 90.35 connects the straight grade lines between
 13 + 50 121.15 the stations indicated, extending 150 ft. each
 17 + 50 98.75 side of vertex. Find elevations on vertical
 curve to 0.01 ft. at each 25-ft. point from Stas. 12 to 15 incl. Check results by
 principle of "second differences."

(b) Find "clear sight" distance over the summit of the vertical curve in part (a) to nearest foot. Assume "height of eye" to be 5 ft.

(c) What length of vertical curve (to nearest foot) would be needed to connect the grade lines of part (a) in order that "clear sight" distance would be 500 ft.?

CHAPTER XIII

CROSS-SECTIONS — VOLUMES

299. CROSS-SECTIONS DEFINED. The term "cross-section" has several different but related meanings in surveying work. In one sense the term refers to short profiles of the ground taken at right angles to the center line of a linear structure, such as a railway, highway, dam, or canal. In another sense the term refers to the figure (in a vertical plane) at right angles to the center line which is formed by the intersection of the outlines of the proposed structure with the ground line (as in Figs. 176 and 181). In some types of surveys (such as those made for borrow-pits, or for the location of contours), a rectangular grid is laid out on the ground, and elevations are taken at the grid corners. This likewise is called cross-sectioning. (See Art. 303.)

300. Ground Cross-Sections for a Highway, Dam, or Canal. The shape of the ground on the site of a linear structure, such as a highway, dam, or canal, is usually determined by running short profiles, called cross-sections, at **right angles to the center line** of the project. Cross-sections are usually taken at every 100-ft. station, and at intermediate points (pluses) **wherever the ground changes abruptly**. For highways, they are taken also at the starting points and at the ends of horizontal and vertical curves and at 25- or 50-ft. intervals on these curves; the cross-sections are extended to the right-of-way lines on both sides, and even farther if there is likelihood of the embankment or excavation extending beyond these limits.

The process of cross-sectioning is similar to that of profile leveling. The levelman obtains the height of instrument (H.I.) by taking a backsight on a bench mark (B.M.) of known elevation. Rod-readings (foresights) are then taken at a sufficient number of points on the cross-sections to determine the shape of the ground. The points at which rod-readings are taken are

located by taping the distance from the center-line station to the place where the rod is held. The rod is read to the nearest 0.1 ft. when held on the ground and to nearest 0.01 ft. on T.P.s. and B.M.s. Distances are usually measured to 0.1 ft. and with a metallic tape. To obtain checks on the level work a rod-reading should be taken on every available B.M. along the line, and the levels must always be extended to, and checked on, a B.M. after the last cross-section has been taken.

CROSS-SECTION NOTES County Road, Ferndale, Mass.					Winslow Rankin Ludlam	
Proj. 5-52					Right of Way 80 ft. Oct. 30, 1940	
Sta.	B. S.	H. I.	F. S.	Elev.	Left	Right
B.M.3	4.21	76.70	—	72.49	4.0 1.7 (75.0)	25 4.3 (72.4)
10+00					10 2.4 (74.3)	3.8 4.6 (72.9)
11+00					18 5.2 (72.1)	30 5.2 (71.5)
					4.0 8.7 (68.0)	
+43				Brook Valley	4.0 3.3 (73.4)	25 6.8 (69.9)
					12 4.2 (72.5)	6 5.3 (71.4)
					15 6.5 (70.2)	28 8.2 (68.5)
					4.0 10.4 (66.3)	

FIG. 175. CROSS-SECTION NOTES FOR A HIGHWAY.

A typical form of cross-section notes for a highway is shown in Fig. 175. Readings on B.M.s. and T.P.s. and the computed values of H.I.s. and elevations are recorded in the usual form for level notes and are entered on the left-hand page of the note-book. On the right-hand page all cross-section readings are recorded opposite the stations where they were taken. A heavy line is drawn down the middle of the right-hand page to represent the center line of the highway. Rod-readings taken to the left of the center line are recorded on the left of this line, and those taken to the

right of the center line are recorded on the right side of this line. In the field-notes the distance from the center line to the point where the rod-reading was taken is recorded above the rod-reading, and the computed ground elevation ($H.I. - \text{rod-reading}$) is recorded in brackets below the rod-reading. These elevations may be computed in the office after the survey has been made, but it is better practice to compute them as the work progresses, since an unreasonable or inconsistent elevation may reveal a mistake in a rod-reading which can be readily corrected while the survey party is still in the field.

Cross-section notes are sometimes so recorded that the stations read up the page, since this is the way they appear to the note-keeper as he stands facing in the direction of increasing stations; i.e., in the direction in which the survey is proceeding. There is less chance of recording readings on the wrong side of the center line, when using this method. When level notes are carried along with the cross-sections, as in Fig. 175, the level notes conventionally read down the page; therefore, the cross-section notes are also arranged with the stations reading down the page. Which ever system is used, as the observer faces in the direction of increasing stations, **left and right in the note-book must correspond with left and right in the field.**

301. Earthwork Cross-Sections. The simplest earthwork cross-section is one bounded by a level line at the grade of the road (called the "base"), two sloping lines representing the side slopes of the cut (or fill), and the ground surface. (See Fig. 176.) Railroad and dam cross-sections are usually of this simple type; highway cross-sections do not, as a rule, have level bases but are shaped to conform to the bottom of the pavement, ditches, and shoulders, as shown in Fig. 181. For preliminary earthwork calculations, the highway cross-section may be conveniently considered to have a level base placed at such a distance below the grade elevation of the pavement that the quantity of earthwork computed to the level line will be equivalent to that indicated by the irregular shape.

Simplified earthwork cross-sections are classed as level, 3-level, 5-level, and irregular sections, depending upon the number and location of the points determined on the ground, as shown in Fig.

176. At the place where the base passes from cut to fill (or vice versa), additional sections are required, as shown in Fig. 176. These are triangular in shape.

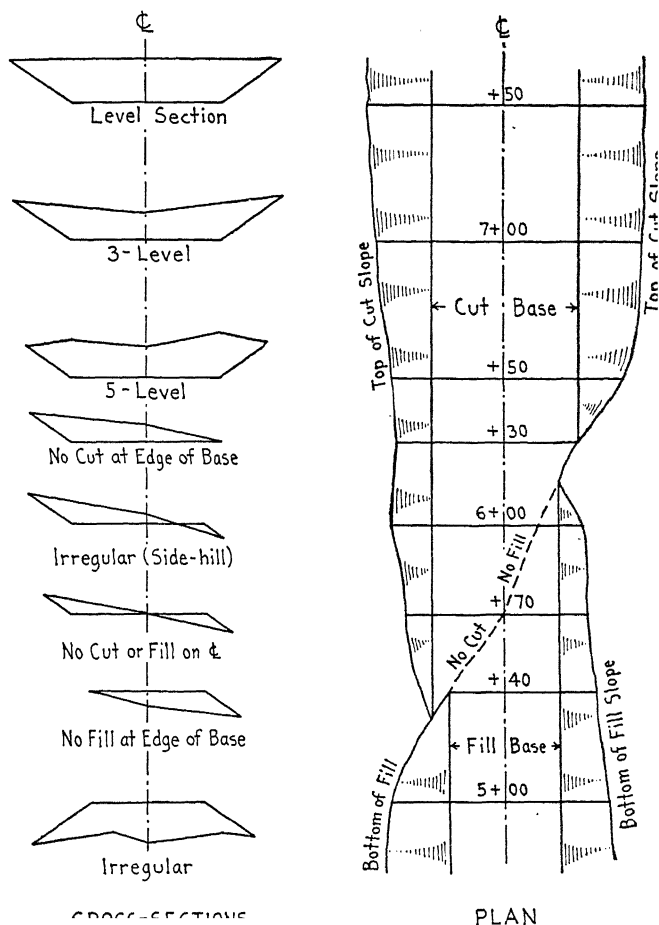


FIG. 176. PASSING FROM FILL TO CUT.

The grade elevation of the subgrade, or base of the section, is determined from the center-line profile of the structure. The difference between the elevation of the surface of the ground and

the base elevation at any station gives the cut or fill at that point. Positive differences indicate cut and negative differences indicate fill; commonly cuts are designated as plus and fills as minus.

The side slopes of the section are usually expressed as a ratio, such as $1\frac{1}{2}$ to 1; this means $1\frac{1}{2}$ horizontal to 1 vertical. The $1\frac{1}{2}$ to 1 slope is commonly used in railroad construction; the tendency in highway practice is toward flatter slopes, such as 2 to 1. Dams have still flatter slopes, 3 to 1 or even 6 to 1.

302. Setting Slope Stakes. It is customary to set stakes on the right and on the left of the center line at points where the finished side slopes will intersect the surface of the ground; these stakes are called "slope stakes." Their positions are usually found by trial in the field. For this purpose it is convenient to compute the rod-reading which would be obtained if the rod could be held on the finished grade. The height of the instrument (H.I.) at the site of the work is determined by leveling from a B.M.; the difference between the H.I. elevation and the grade elevation at any station on the center line is the "*rod-reading for grade*," or the "*grade rod*" as it is sometimes called. The rod-reading for grade may be either positive or negative depending upon whether the proposed grade is below or above the H.I. elevation, as illustrated in Fig. 177(a) and 177(b) respectively. When the H.I. is above the finished grade, as in Fig. 177(a), the cut or fill at any point is the difference between the rod-reading for grade and the surface rod-reading. It is the sum of the readings when the H.I. is below the finished grade, as in Fig. 177(b). Notice that the rod-reading for grade is computed to hundredths of a foot but used to the nearest tenth of a foot when combined with surface readings which are read only to tenths.

In setting slope stakes, such as *B* in Fig. 177(a), it is required to find the measured distance *AB* that equals the computed distance *AB*. To set the slope stake at *B* in Fig. 177(a), the surveyor keeps in mind the width of base (40 ft.), side slope ($1\frac{1}{2}$ to 1), and the center cut (7.5). At *B* there is actually no cut but it is customary to allude to the cut at *B* as being the vertical distance from the base to *B*. Thus if *B* were level with *A* the distance from the center stake to *B* would be $20 + 1\frac{1}{2} \times 7.5 =$

31.2. This computation is done mentally. The tape is stretched from the center stake, at right angles to the center line. At 31.2 ft. on the tape it is observed that the ground appears to be, say, 2 ft. lower than at *A*, which means that the cut at *B* is about 2 ft. less than at *A*, and the distance out to *B* is therefore $1\frac{1}{2} \times 2 = 3$ ft. less than 31.2, or 28.2. The rod is therefore held at the 28.2-ft. point and a surface rod-reading is taken. Suppose the reading is 3.6; the cut at that point then equals $+9.8 - 3.6 = 6.2$. The distance from the center line to the point where the

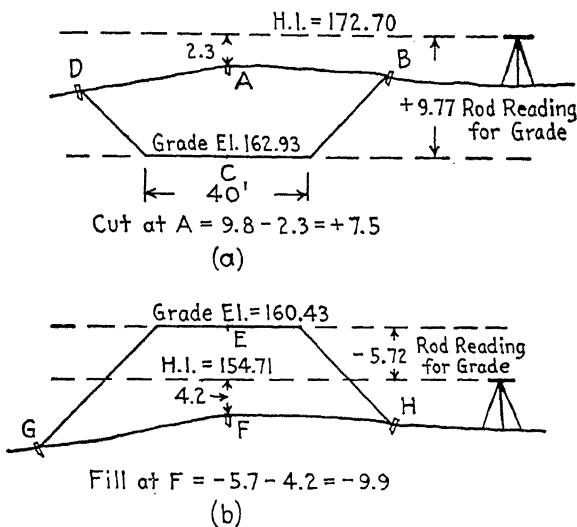


FIG. 177. DETERMINING CUT OR FILL AT CENTER STAKE.

side slope should intersect the ground is $20 + 1.5 \times 6.2 = 29.3$. This is 1.1 ft. farther from the center line than the point first selected. If the ground is level between the 28.2- and 29.3-ft. marks on the tape and the rod is moved to 29.3, the surface rod-reading will still be 3.6, in which case the stake *B* would be driven at 29.3 ft. But if it appears that the ground falls off, say, 0.2 ft., from 28.2 to 29.3, the cut will be 0.2 ft. less and therefore 0.3 ft. less distance out to *B*. Therefore the rod should be held for second trial at $29.3 - 0.3 = 29.0$. After two or three trials

the correct position of the stake can be determined; but with experience only one or two trials will be necessary. The stake *B* is then driven into the ground and the cut is marked on its face and station number on its back.

The position of the stake is recorded in the notes as a fraction, with the cut (or fill) in the denominator and the distance from

Cross-Sections of Millers Road Westbrook County, Conn.			
July 23, 1940			
Sta.	Surface Elev.	Grade Elev.	Cross-Sections
			Bases: Cut 40', Fill 36', Slopes $1\frac{1}{2}$ to 1.
+50	73.1	67.50	$\frac{28.4}{+5.6}$ +5.6 $\frac{28.4}{+5.6}$
7	70.5	67.00	$\frac{27.5}{+5.0}$ +3.5 $\frac{28.7}{+5.8}$
+50	70.1	66.50	$\frac{25.7}{+3.8}$ $\frac{20.0}{+4.8}$ +3.6 $\frac{20.0}{+5.8}$ $\frac{26.6}{+4.4}$
+30	68.5	66.30	$\frac{25.1}{+3.4}$ +2.2 $\frac{20.0}{0}$
6	67.3	66.00	$\frac{25.7}{+3.8}$ +1.3 $\frac{9.5}{0}$ $\frac{20.7}{-1.8}$
+70	65.7	65.70	$\frac{23.9}{+2.6}$ 0 $\frac{21.6}{-2.4}$
+40	63.5	65.40	$\frac{18.0}{0}$ -1.9 $\frac{22.8}{-3.2}$
5	59.8	65.00	$\frac{25.5}{-5.0}$ $\frac{10.0}{-3.7}$ -5.2 $\frac{23.0}{-3.9}$ $\frac{24.9}{-4.6}$

FIG. 178. CROSS-SECTION FIELD-NOTES.

the center line in the numerator. This is shown in Fig. 178, which is a complete set of notes for the cross-sections shown in Fig. 176. The fractions at the extreme left and right of the cross-section column show the slope stake locations. The figures in the center are cut or fill on the center line. Intermediate recordings on the cross-sections have been made at critical points on the ground, as at Stas. 5 + 00, 6 + 00 and 6 + 50 in the notes. The intermediate cross-sections at Stas. 5 + 40, 5 + 70 and

6 + 30 were taken where the fill side line, the center line, and the cut side line respectively pass through points of no cut or fill. These sections are utilized in earthwork calculations and will be described later. The positions of these sections must be found by a method of trial in the field. Obviously when there is no cut or fill at a point, the surface rod-reading and the rod-reading for grade must be the same. For example, to determine the station of no cut or fill on the center line, the plus station is first estimated by eye and the grade elevation at that plus station is computed. The difference between the H.I. elevation and the grade elevation at the plus station is the rod-reading for grade. The rod is then held on the plus station and a reading taken. If the reading is the same as the rod-reading for grade, the correct station was assumed; if not, another trial must be made using the result of the first guess as a guide.

In Figs. 176 and 178, Sta. 5 is an irregular section and Sta. 6 is a special type of irregular section which commonly occurs when part of the road is in cut and part in fill. The position of the point of no cut or fill must be determined on this cross-section so that cut and fill earthwork quantities may be computed separately, as described later in Art. 308. Sta. 6 + 50 is a 5-level section, Sta. 7 is a 3-level section, and Sta. 7 + 50 is a level section.

303. Cross-Sections for Borrow-Pit. The shape of the original ground may be obtained by dividing the area into squares and taking elevations at the corners of each square and at intermediate points where it is necessary to represent the true shape of the surface. These surface elevations are measured to tenths of a foot. The squares may be of any desired dimensions, usually 10 ft. to 100 ft. on a side, the size chosen depending on the ground slopes. The grid lines should be spaced close enough so that the general slope from one corner to the next will be approximately uniform. The grid is laid out with the transit and tape, and a temporary stake is driven at each grid corner. The corners of the grid are designated by rectangular coordinates, one set of parallel lines being marked by letters and the other by numbers, as shown in Fig. 179. The point x would be designated (C, 4); and if the squares are 30 ft. on one side the point

y would be $(D + 20, 3 + 10)$, and point z would be $(F + 10, 3)$. The notes are kept as in profile leveling except for the designations in the "Sta." column.

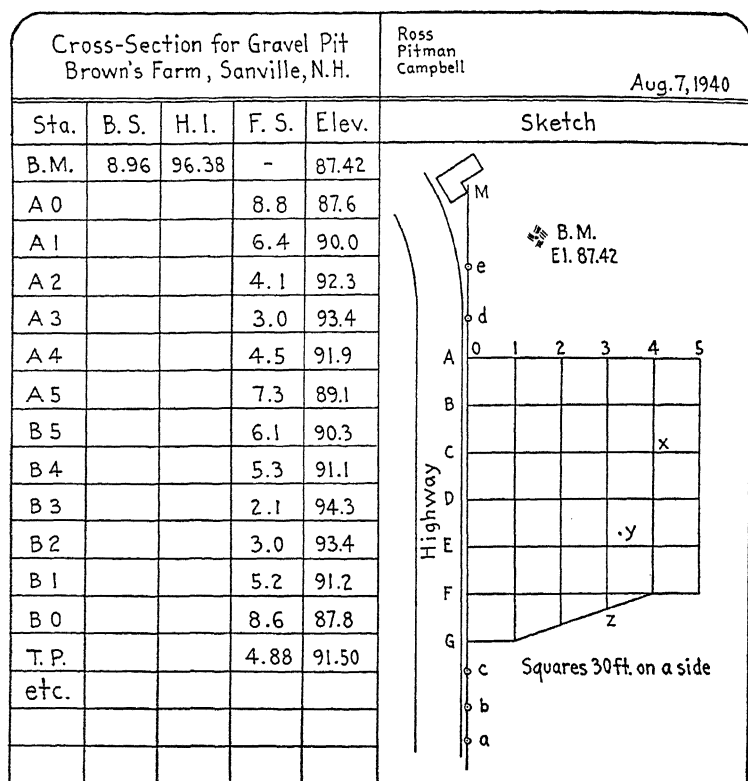


FIG. 179. FIELD-NOTES FOR BORROW-PIT.

After the excavation has been completed, the same grid is staked out once more, and the corner elevations are again determined. The difference in the elevations at a corner before and after excavation is the cut. Knowing the amount of cut at each corner and the size of the squares, the volume of excavation can be computed, as shown in Art. 309.

In laying out the grid at least one of its lines should be so staked out that it can easily be rerun. Two or three hubs should be set at each end of the line, well outside the area to be excavated, and in positions where there is no liability of their being disturbed; otherwise the ends of the line should be thoroughly tied in. (Art. 119.) In Fig. 179 the reference line is ae ; additional points c , b and d have been placed on this line. Furthermore, the direction of the line has been chosen so that it falls through the corner of the house at M , which serves as a distant foresight.

A suitable B.M. should be established at a convenient location where it will not be disturbed. If a B.M. of known elevation is not available nearby, an arbitrary elevation, such as 100.00, may be assigned to this B.M. and the fact recorded clearly in the notes. The B.M. should be described definitely in the notes, so that another surveyor can readily find it.

VOLUMES

304. VOLUMES. Determining the amount (volume) of earthwork and masonry required for construction projects is an important branch of practical surveying. In projects involving the movement of earthwork such as pipe lines, sewers, cellars, borrow-pits, grading, highways, and railroads, a knowledge of the volume is one of the items necessary in estimating cost. The tonnage in a pile of coal, the yardage in sand and gravel piles, the capacity of reservoirs, and the volume of masonry structures are all problems for the surveyor.

Masonry volumes may be computed by solid geometry from dimensions on the plans, but it is common practice to check the pay quantities by recomputing the volume from dimensions of the actual structure taken in the field. These solids are usually prisms, pyramids, cones, or portions of these solid figures.

Earthwork volumes are computed from the cross-sections taken in the field, as described in the previous articles. These cross-sections define parallel areas at definite stations and at definite distances apart along the center line of the project. The solid between these approximates a prismoid and is considered as such in computing earthwork volumes.

Two formulas are used for computing the volume of a prismoid:

- (1) End-Area Formula
- (2) Prismoidal Formula.

305. END-AREA FORMULA. In the end-area formula the volume is assumed to be equal to the average of the areas of the two ends multiplied by the distance between them, i.e.,

$$V = \frac{A_1 + A_2}{2} \times l$$

in which A_1 and A_2 are the areas of the two end bases and l is the distance between them. Although this formula, because of its simplicity, is almost universally used in computing earthwork volumes, it is not exact and in the majority of cases it gives a volume somewhat larger than the exact value.

306. PRISMOIDAL FORMULA. A prismoid is a solid bounded by planes, the end faces being parallel and having the same number of sides. The correct volume of a prismoid is given by the prismoidal formula, i.e.,

$$V = \frac{l}{6} (A_1 + 4A_m + A_2)$$

in which l is the distance between the two bases A_1 and A_2 ; and A_m is the area of the section midway between them. Notice that A_m is not defined as the mean of A_1 and A_2 . To find the area of the middle section, the dimensions of the middle section are first found by averaging the dimensions of the end sections; from these values the middle area is computed.

The average end-area method is almost universally used for computing earthwork quantities. Although approximate, its precision is usually consistent with the field methods used in determining the cross-sections upon which the calculations are based. The unit cost of ordinary excavation is relatively low and refinements in earthwork calculations are not justified in most cases. In computing masonry quantities, however, more precision is required, since a cubic yard of concrete costs many times as much as a cubic yard of earth excavation. The prismoidal formula is very useful in determining accurately the volumes in certain parts of masonry structures.

The greatest difference between the results of the two formulas will be found for tapering solids of greatly different end areas, as illustrated in the following example.

$$\text{In Fig. 180, } A_1 = \left(\frac{2.0 + 2.5}{2} \right)^2 = 4.5 \text{ s.f.}$$

$$A_2 = \left(\frac{2.0 + 5.0}{2} \right)^2 = 42.0 \text{ s.f.}$$

The approximate volume by average end-area method = $\left(\frac{4.5 + 42.0}{2} \right) 20 = 465 \text{ c.f.}$

The area of the mid-section = $\left(\frac{3.75 + 2.0}{2} \right)^2 = 20.1 \text{ s.f.}$, and the exact volume by prismoidal formula

$$\frac{20}{6} (4.5 + 4 \times 20.1 + 42.0) = 423$$

c.f. In this example the average end-area formula gives a result that is about 10 per cent too large. This example illustrates why the end-area method is not used in practice in computing the quantities of masonry.

307. EARTHWORK COMPUTATIONS. A common method for computing the quantity of earthwork is to plot the cross-sections to scale and obtain the areas from this plot with a planimeter (Art. 312) or by graphical methods.

The cross-sections are plotted to the same horizontal and vertical scale; 1 in. = 5 ft. and 1 in. = 10 ft. are commonly used. Special cross-section paper is available; this is ruled with heavy lines into one-inch squares and with lighter lines into one-tenth-inch squares. A heavy vertical ruling is selected to represent the center line for a series of cross-sections; it should be one that will admit of plotting the cross-sections so that they will not run off the edge of the paper. The elevations are then plotted at their respective distances to the left and right of this line. A heavy

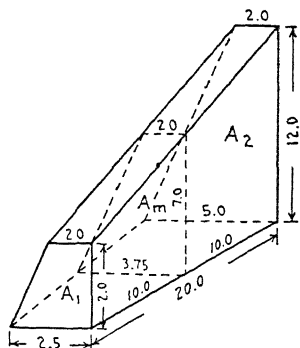


FIG. 180.

horizontal ruling near the bottom of the sheet is arbitrarily marked with an elevation (such as 70.00) and this line is used for plotting the elevations of first cross-section. The next cross-section is plotted above the first and its elevations are referred to another convenient horizontal line (as 70.00). Each cross-

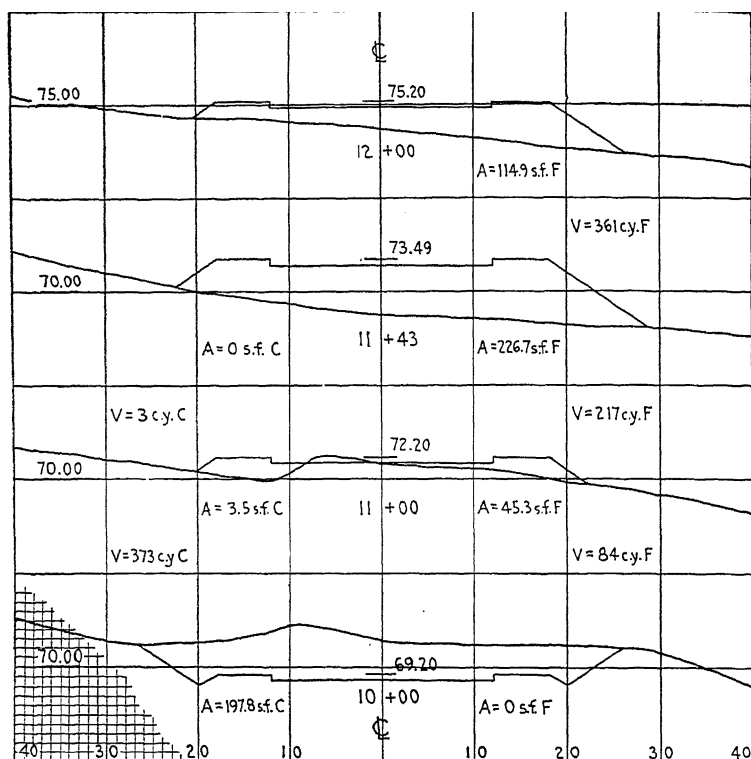


FIG. 181. CROSS-SECTIONS FOR HIGHWAY.

section has a separately designated datum line. (See Fig. 181, which is a plot of a portion of the cross-section notes shown in Fig. 175.) The plotted points are connected by a heavy free-hand line which is drawn to represent the shape of the ground.

For estimating earthwork quantities, the final cross-section is superimposed on the original ground line, as shown by light lines

in Fig. 181. The reference elevation for plotting the final cross-section is the elevation of the finished grade at the center line, shown by the short horizontal dash line, such as 73.49 for Sta. 11 + 43. The area between the original and final cross-sections is determined either with the planimeter, or by dividing the area into triangles and trapezoids and computing the area in parts, using dimensions scaled from the plot. Cut and fill areas are computed and recorded separately, as at Sta. 11 + 00 in Fig. 181 the cut area = 3.5 s.f. and the fill area = 45.3 s.f.

The volume of earthwork between adjacent stations is computed by the average end-area method and the cut and fill are recorded separately between sections, as in Fig. 181: between Stas. 11 + 00 and 11 + 43, the cut volume = 3 c.y. and the fill volume = 217 c.y.

308. Computing Earthwork from Cross-Section Notes. Earthwork quantities may be computed directly from the cross-section notes when they are kept in the form illustrated in Fig. 178. The areas of the sections are found mathematically, and the volumes by average end-area method. Referring to Fig. 178, the areas at Sta. 6 + 50 and Sta. 7 + 00 would be computed by dividing the figures into pairs of triangles, as shown in Fig. 182.

Area at Sta. 7 + 00 equals

$$\frac{3.5}{2} (27.5 + 28.7) = 98.3$$

$$\frac{20}{2} (5.0 + 5.8) = 108.0$$

$$206.3 \text{ s.f.}$$

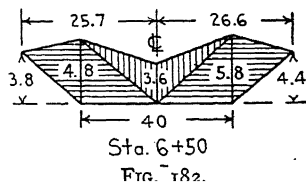
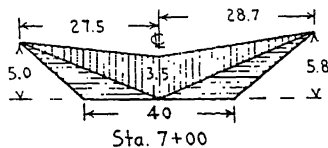
Area at Sta. 6 + 50 equals

$$\frac{3.6}{2} (40) = 72.0$$

$$\frac{4.8}{2} (25.7) = 61.7$$

$$\frac{5.8}{2} (26.6) = 77.1$$

$$210.8 \text{ s.f.}$$



Volume between $6 + 50$ and $7 + 00$ equals

$$V = \frac{50}{27} \left(\frac{206.3 + 210.8}{2} \right) = 386.2 \text{ c.y.}$$

The areas of other types of sections may be obtained by a similar process. Cut areas and volumes are kept separately from fill areas and volumes. In passing from cut to fill (or vice versa), as in the notes shown in Fig. 178, the zero area of fill at Sta. $6 + 30$ and the zero area of cut at Sta. $5 + 40$ must be included in the calculations of volumes. For example, the area of cut at Sta. $5 + 70$ is $\frac{2.6}{2} (20) = 26 \text{ s.f.}$, and at Sta. $5 + 40$ the cut is 0; therefore the volume by average end-area method between Stas. $5 + 40$ and $5 + 70$ is $\frac{30}{2 \times 27} (0 + 26) = 14.4 \text{ c.y.}$

309. Computing Borrow-Pit Excavation. Borrow-pit cross-sectioning and the method of determining the amount of cut at each corner are explained in Art. 303.

The volume of the excavated material removed from the borrow-pit is computed by assuming that the solid within each of the grid squares, rectangles, or triangles is a truncated prism, the volume of which is equal to the area of a right, or horizontal, section multiplied by the mean of the cuts (heights) at the corners. Expressed as a formula, the volume of a square or rectangular truncated prism is:

$$V = A \left(\frac{h_1 + h_2 + h_3 + h_4}{4} \right)$$

where A is the area of the right section and h_1 , h_2 , h_3 and h_4 are the cuts at the four corners of the prism; i.e., the cuts at the four corners of the cross-section grid squares (or rectangles).

Similarly, the volume of a triangular truncated prism is:

$$V = A \left(\frac{h_1 + h_2 + h_3}{3} \right)$$

When a number of prisms have the same right area, these may be combined and their volume computed in one operation by the

“assembly of prisms” method. If the volumes of a contiguous group of prisms of the same right area are computed separately, it will be seen that the right area and the divisor 4 are common to all the calculations. Likewise, some of the heights will be common to one or more of the calculations. Such computations may be combined and simplified by applying the “assembly of prisms” method, expressed as follows:

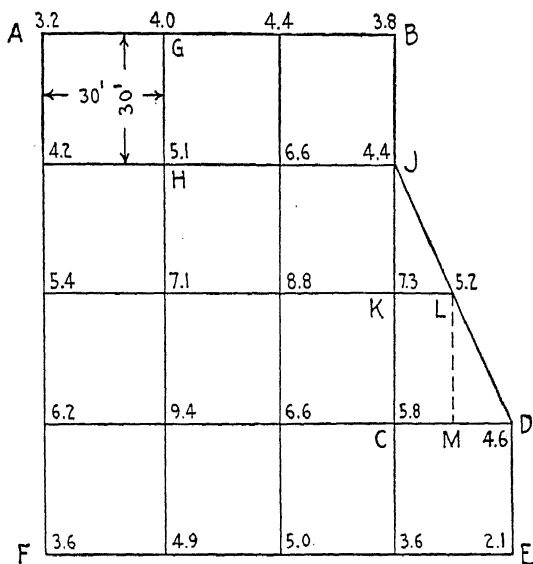


FIG. 183. BORROW-PIT.

$$V = A \left(\frac{\Sigma h_1 + 2 \Sigma h_2 + 3 \Sigma h_3 + 4 \Sigma h_4}{4} \right)$$

where A = area of one right section in the assembly
 Σh_1 = sum of all heights applying to one square
 Σh_2 = sum of all heights common to two squares
 Σh_3 = sum of all heights common to three squares
 Σh_4 = sum of all heights common to four squares.

In Fig. 183 the volume of the borrow-pit bounded by $ABCDEF$ may be computed by the "assembly of prisms" method as follows:

Σh_1	Σh_2	Σh_3	Σh_4
3.2	4.0	5.8	5.1
3.8	4.4		6.6
4.6	4.4		8.8
2.1	7.3		7.1
3.6	3.6		9.4
	5.0		6.6
	4.9		
	6.2		
	5.4		
	4.2		
17.3	49.4	5.8	43.6

$$V = \frac{900}{4} [17.3 + 2(49.4) + 3(5.8) + 4(43.6)] = 69,278 \text{ cu. ft.} = 2566 \text{ cu. yds.}$$

The volume bounded by JKL is a triangular truncated prism and may be computed as follows:

$$V = \frac{225}{3} (4.4 + 5.2 + 7.3) = 1268 \text{ cu. ft.} = 47 \text{ cu. yds.}$$

In computing the volume of the trapezoidal prism whose area is $KLDC$, it is customary to divide it into two triangular prisms by drawing diagonals or into a rectangular prism and triangular prism, depending on the slopes of the surface. The method to use should be indicated in the field-notes; it depends on the shape of the surface of the ground. If the latter method is indicated, the cut at $M = 5.2$ is found by interpolation.

The volume of $KLMC$ is

$$V = \frac{450}{4} (7.3 + 5.2 + 5.2 + 5.8) = 2644 \text{ cu. ft.} = 98 \text{ cu. yds.,}$$

and the volume of LDM is

$$V = \frac{225}{3} (5.2 + 4.6 + 5.2) = 1125 \text{ cu. ft.} = 42 \text{ cu. yds.}$$

Total volume of the borrow-pit

$$V = 2566 + 47 + 98 + 42 = 2753 \text{ cu. yds.}$$

310. Computing Earthwork from Contours. Estimates of the quantities of earth to be moved in a grading operation may be made from a topographic map of the area over which the grading is to be done. The shape of the proposed surface may be represented by dash-line contours to distinguish them from the full-line contours which represent the original ground. (Fig. 184.)

Where the two surfaces intersect are lines of "no cut and no fill," such as *CB* and *DE*. The portion to the left is cut, because the new contours are up-hill from the original contours. This cut is to be moved to the right to make the indicated fill (where the new contours are down-hill from those representing the present ground). It is assumed that the cut runs out to zero at *A* and *N*, and the fill at *J* and *U*.

Assume that the cut quantity within the area *ABCNDE* is to be computed. This may be done by computing the volume of earth between a plane at Elev. 10 and the original surface, and then computing the volume between the same plane Elev. 10 and the proposed surface; the difference between these two volumes is the amount of cut. By this method the areas *ABCNDE*, *FBCNDE*, *HPCNDQ* and *LCND* are found by planimeter (see Art. 312).

The End-Area Method is used to compute the four volumes, the height of each volume being 2 ft. This total volume (in cu. yds.) from Elev. 10 to the original ground would be expressed as follows:

$$\frac{2}{27} \left(\begin{aligned} &ABCNDE + FBCNDE \\ &+ FBCNDE + HPCNDQ \\ &+ HPCNDQ + LCND \\ &+ LCND + \text{zero} \end{aligned} \right)$$

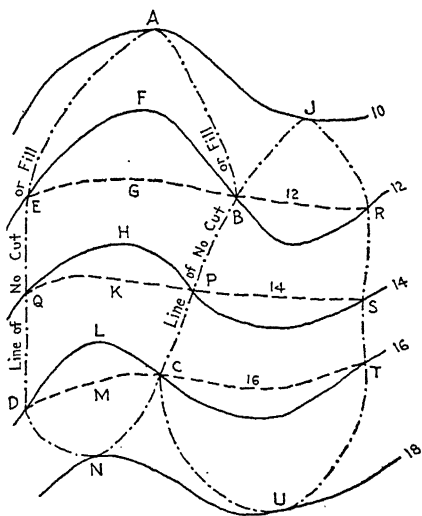


FIG. 184.

The volume lying between the plane at Elev. 10 and the proposed surface is found in the same manner, using the areas *ABCNDE*, *GBCNDE*, *KPCNDQ* and *MCND*.

Another practical method involves computing by one process the amount of cut between successive contours as follows: the

solid *CMDL-PKQH* has two parallel planes *CMDL* (at Elev. 16) and *PKQH* (at Elev. 14); the vertical distance between them is 2 ft. The volume of cut between them is, by end-area method, $\frac{CMDL + PKQH}{2} \times 2$. Similarly the volume between the next

pair of contours is $\frac{PKQH + BGEF}{2} \times 2$. The remaining volumes are pyramids; pyramid *BGEF - A* = $BGEF \times \frac{2}{3}$; and pyramid *CMDL - N* = $CMDL \times \frac{2}{3}$.

311. Computing Earthwork Required for a Tennis Court. Fig. 185 shows in plan a standard tennis court *ABCD*, 36' \times 78'. It is to be constructed on the sloping ground represented by the 1-ft. contours, by digging out the upper half and filling in the lower half of the playing space so that the subgrade will be level and at Elev. 26. On this subgrade the specially prepared soil, or pavement, of the court is to be laid so that its surface will have a slight slope for drainage. It is economical practice to make the subgrade planes parallel to the proposed finished grades of the court, but to simplify this illustration the subgrade is here made level.

To provide ample playing space the ground is to be made level for 10 ft. outside the side lines and 21 ft. behind the back lines of the double court (standard distance for professional matches). The rectangle *EFGH* is therefore to be constructed at Elev. 26, and the slopes of the cuts and fills are to be $1\frac{1}{2}$ to 1 (or 1 vertical on $1\frac{1}{2}$ horizontal). The problem is to find the amount of cut and fill involved. The cut must always be computed separately from the fill. The 26-ft. contour *JK* is a line of no cut or fill separating the cut and fill portions of the project. The simplest method of computing the earthwork in this project is to compute all cuts down to Elev. 26; then, to compute all fills up to Elev. 26.

Let us first consider the cut portion *JEFK*. This is to be excavated to a level plane at Elev. 26. The ground surface at *E* is 30.3; at *Q* it is 30.5; at *F* it is 30.1; and at *R*, *S* and *V* it is 30.0; the corresponding cuts at these points are 4.3, 4.5, 4.1 and 4.0. The volume of *EQSR* = $\frac{4.3 + 4.5 + 4.0 + 4.0}{4} \times \text{area}$

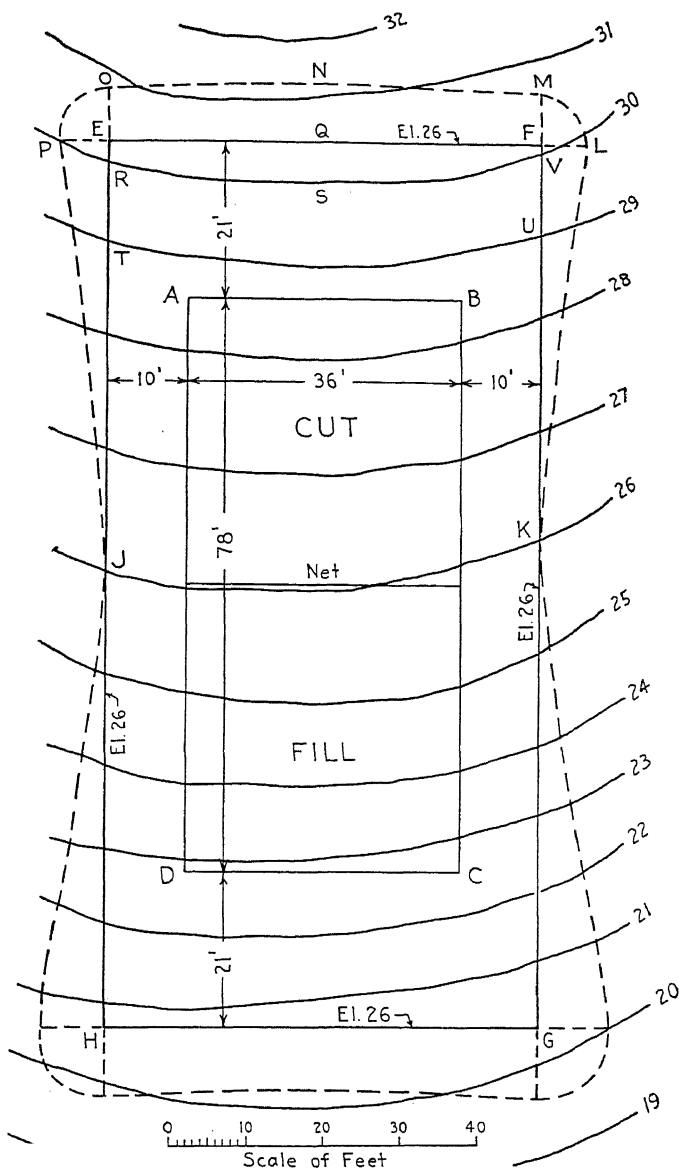


FIG. 185. GRADING FOR A TENNIS COURT.

EQSR. Similarly, the volume of *QFVS* = $\frac{4.5 + 4.1 + 4.0 + 4.0}{4}$

× area *QFVS*. The volume of *VUTR* = $\frac{4.0 + 3.0 + 3.0 + 4.0}{4}$

× area *VUTRS*. The areas are found by use of the planimeter (Art. 312). The total cut in that area is obtained by applying this same method to the entire area *EFKJ*.

There remains the calculation of the quantity of earth in the sloping parts of the excavation; these slopes are planes running up to the natural ground. The volume *JEP* is a pyramid; at *J* and *P* there is no cut, but at *E* there is $30.3 - 26.0 = 4.3$ ft. cut. The vertical plane at *PE* is a triangle; its altitude is at *E* = 4.3; its base is *PE* = cut at *P* times the rate of slope, i.e., $(30.0 - 26)1.5 = 6.0$; its area is $\frac{6.0 \times 4.3}{2} = 12.9$ s.f. This is the area of the base of the pyramid whose height is *EJ* = 58 ft. scaled from the plan; its volume is $\frac{12.9 \times 58}{3} = 249$ c.f.

The volume *EPO* is an approximate quarter cone, the altitude of which is the cut at *E*, and the base varies in radius from *PE* to *EO*. The volume of such a solid may be conveniently computed by assuming it to be a solid of revolution; i.e., the volume is "generated" by rotating the average cross-sectional area from the vertical plane *PE* to the vertical plane *EO* about a vertical axis at *E*. The base of the triangular cross-section of revolution is commonly assumed to be the average of the distances *PE* and *EO*; the average area of revolution is therefore one-half the cut at *E* times the average of *PE* and *EO*, i.e., $\frac{4.3}{2} \left(\frac{6.0 + 7.2}{2} \right) = 14.2$ s.f. By the *theorem of Pappus*, *volume of revolution* = *area* × *path of the center of gravity*. The latter is one-third of the average of *PE* and *EO* times $\frac{\pi}{2}$ (90° in radians). The volume of *PEO*, then, equals $14.2 \times \frac{1}{3} \left(\frac{6.0 + 7.2}{2} \right) \frac{\pi}{2} = 49.0$ c.f.

The quantity *OEQFMN* is a triangular prism of varying cross-section. If the line *MNO* is practically straight, we may compute

the area of this prism of cut on vertical planes at EO and at FM , average them and multiply by 56 ft.; the quantity of cut between EO and FM on the slope is then obtained. But if at N the triangle representing the right section of the prism $EOMF$ is not an average of the area at EO and FM , an additional vertical area at QN should be interpolated, its area should be determined, and the volumes of $EONQ$ and $QFMN$ should be computed and added to give a more accurate value of the volume $EOMF$.

The above directions have shown explicitly how to compute the cut in the slopes from J through E to F outside of the boundary of the level field. All other computations of slope cuts or fills are similar.

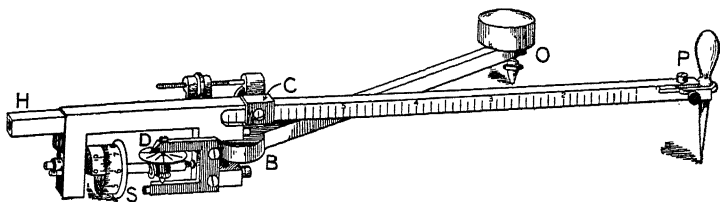


FIG. 186. POLAR PLANIMETER.

312. The Planimeter.* One of the most common ways of obtaining the areas of irregular plane figures is by means of an instrument called the *planimeter*, Fig. 186. Its frame touches the paper at only three points: the anchor point O , the tracing point P , and the circumference of a small wheel S which is free to revolve. On the rim of this wheel is a scale and beside it is a vernier which is used in reading the scale. Disk D records the number of full revolutions of wheel S . The length of the arm PH can be regulated by using the clamp and slow motion at C , thus setting it at the proper reading on a scale which is marked on the arm, so that a unit on the wheel scale will represent any desired unit area on the plan, such as a square inch or a square centimeter.

In using the instrument the anchor point is set at some convenient position on the drawing **outside** of the area to be measured,

* The theory underlying the Amsler Polar Planimeter will be found in *Elementary Surveying* by Breed and Hosmer, Vol. I, Appendix A.

and the tracing point is then run around the perimeter of the area to be determined. The reading on the wheel is recorded when the tracer is at the starting point. The tracer, in passing around the perimeter, should be kept as **closely as possible** on the boundary line and it should return **exactly** to the starting point. Then the scale is again read, and the difference between the two readings is the area that has been traced out, expressed in some unit depending on the length of the arm. The result can be transposed easily into the unit of the scale of the map.

Usually the settings for the scale on the arm for various units of area are furnished by the maker. It is safer, however, to test this setting by running the instrument around a known area, such as 4 square inches drawn on the plan and determining the interval passed over by the wheel by making several tests and by setting the anchor point at different positions. This interval divided by 4 will be the value of one square inch of plan area and this is equivalent to a certain number of square feet of surface, depending upon the scale of the map. If, for example, the plan has a scale of 20 ft. = 1 in., then 1 sq. in. on the plan is 400 sq. ft. on the ground. It is important that the sides of the trial square should be laid off so that they agree with the present scale of the map which, owing to swelling or shrinking of the paper, is frequently not quite the same as when it was first drawn.

By the use of a polar planimeter a result is easily obtained which is not in error more than one per cent, except for very small areas.

PROBLEMS

1. Compute the Cut in Fig. 178 from Sta. 5 + 40 to 7 + 50 by End-Area Method. Results to nearest cu. yd.

2. Compute the excavation in the borrow-pit shown in Fig. 179 down to Elev. 86.0 for the part lying between lines *A* and *B* and rows 0 to 5. Result to nearest cu. yd.

3. Compute the fill in *JRTUCB* in Fig. 184, assuming scale of Fig. 184 is 1 in. = 20 ft. Result to nearest cu. yd.

4. Compute the volume of fill in Fig. 185 up to Elev. 26.0. Result to nearest cu. yd.

5. STA.	GRADE ELEV.	Grade line is straight between Stas. 24 and 30.
24	66.40	H.I. = 69.20 for parts (a), (b) and (c).
30	70.78	

(a) Slope stakes are to be set at Sta. 27 + 60. What is "rod-reading for grade" for this station (to nearest $\frac{1}{16}$ ft)?

(b) If rod-reading on surface at center line at Sta. $27 + 60$ is 1.8, what is amount of cut or fill at this point to $\frac{1}{10}$ ft? Be sure to state whether it is *cut* or *fill*.

(c) Find "rod-reading for grade" at Sta. $29 + 00$. Show a check on the answer to this part from the answer to part (a).

(d) Use simple cross-sections as in Art. 301. Make up a reasonable set of *Cross-Section Notes* from Sta. 24 to 30, assuming the following conditions. Grade line is as given above. Ground slopes down transversely from right to left. The line is in cut from Sta. 24 to 26 inclusive, and in fill from Sta. 27 to 30. Line passes from cut to fill between Sta. 26 and 27, the intermediate sections being taken at $26 + 40$, $26 + 75$ and $26 + 90$. Section is a level section at Sta. 24; 3-level sections at Sta. 25, 26 and 27; a 5-level section at Sta. 28, and irregular sections at Sta. 29 and 30. Write the complete notes in proper form including grade elevations, surface elevations at center line, and cross-sections for all full stations and for the three intermediate sections taken in passing from cut to fill. Bases are 24 ft. in cut, 18 ft. in fill, side slopes $1\frac{1}{2}$ to 1 (hor. to vert.).

6. A highway is to have a base width of 40 ft. and side slopes of 2 to 1. At Sta. $10 + 00$ the elevation of the proposed grade is 162.40 and the height of instrument is 158.62.

(a) Find the rod-reading for grade at Sta. $10 + 00$.

(b) If the rod-reading on the surface of the ground at the center line at Sta. $10 + 00$ is 2.6, what is cut or fill at that station? Indicate whether cut or fill.

(c) In locating the position of the left-hand slope stake a trial reading has been taken 32 ft. from the center line. If this reading is 4.2 and the ground is level in the vicinity of the reading, should the rod be moved toward or away from the center line and how much to locate the proper position of the stake? What cut or fill should be recorded on the stake?

CHAPTER XIV

MISCELLANEOUS SURVEYING PROBLEMS

313. TRIANGULATION. The principle of triangulation is used in surveying large areas and in determining inaccessible distances, such as across a river. In a triangulation system, one line is measured directly (base-line); all the other lengths are derived by measuring the angles of all the triangles and calculating the sides by trigonometry. The precision of the results may be great or small according to the methods employed.

In a single triangle ABC a base-line AB is measured; the angles A and B are measured, and the remaining sides BC and AC are calculated. Whenever possible, the angle C should also be measured as a check. The base AB should be so chosen that it can be easily measured to the desired precision and also so chosen that no angle in the triangle will be less than 30° ; the latter precaution will insure a good intersection of the lines and will tend to produce what is known as a strong location of the points.

EXAMPLE

Length of $AB = (1) 229.36; (2) 229.34$ ft.

Tape, 100.005 ft. long at 68° F. Angles: $A = 89^\circ 10' 15''$, $B = 44^\circ 51' 10''$, $C = 45^\circ 58' 25''$. Find AC .

$$AC = \text{base} \times \frac{\sin B}{\sin C}.$$

BASE	ANGLES	BALANCED ANGLES	LOGS
229.36 ft.	$A \quad ' 10' 15''$	89-10-19	$\log AB = 2.36051$
229.34	$B \quad ' 51' 10''$	44-51-13	$\log \sin B = \underline{9.}$
Mean 229.35	$C \quad 45' 58' 25''$	<u>45-58-28</u>	2.20889
Tape Corr. +.01		180-00-00	$\log \sin C = \underline{9.85675}$
229.36 ft. = AB	error 10"		$\log AC = 2.35214$
			$AC = 224.98$

The tape measurement of the base-line should be corrected for temperature and standard length; the tension in the tape should be applied by means of a spring balance; and the tape should be used under the conditions for which it was standardized with respect to tension and support (Chap. II). If the tape is not level, the difference in elevation of the ends of the tape for each tape-length should be obtained by leveling, and the horizontal distance computed (Art. 18).

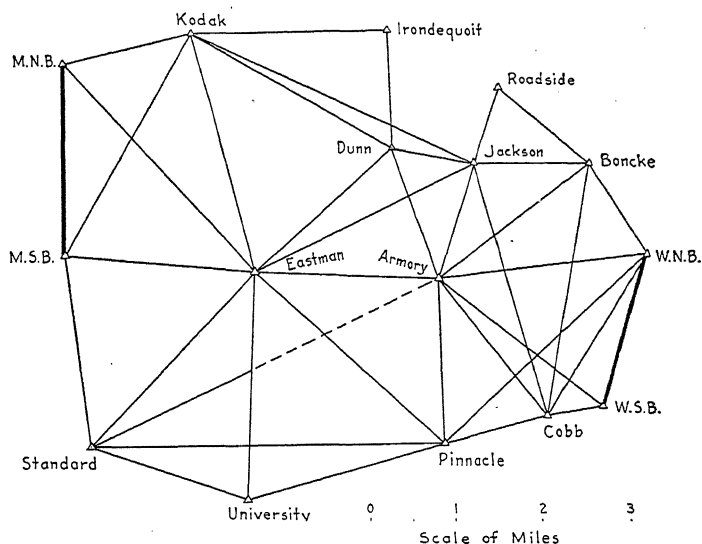


FIG. 187. TRIANGULATION NET FOR CITY OF ROCHESTER, N. Y., AND VICINITY.

Triangulation is used to establish control points for such surveys as those of state boundary lines and city areas.* In these surveys a network of triangles and quadrilaterals is laid out covering the entire section, and the points, if possible, are so selected that a side of certain triangles can be measured as well as the original base-line, thus giving a check on the network, as shown in Fig. 187.

* See *Elementary Surveying* by Breed and Hosmer, Vol. I, Arts. 313-19, and *Higher Surveying* by Breed and Hosmer, Vol. II, Chap. I.

314. Triangulation for Bridge Location. When a wide body of water is to be spanned by a bridge, it is necessary to determine by triangulation the precise center-line length of the structure, so that the span, or spans, will fit exactly on the abutments. In Fig. 188, AC is the center line of a bridge and AB is a base-line measured on one bank of a river. By establishing a fourth point on the opposite bank at D , a quadrilateral is formed in which all the angles 1 to 8 inclusive are measured; a second base-line CD is also measured. These measurements provide more data than are necessary for computing AC , but by making independent solutions, errors will be detected and a more precise value for

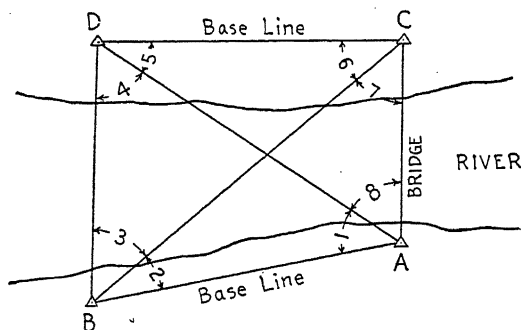


FIG. 188. TRIANGULATION ACROSS A RIVER.

AC can be determined than would be otherwise possible. For example, CD may be computed in four ways from base AB . These four values of CD and the direct measurement of CD will all differ, but the amount of variation will make it possible to estimate the precision attained in the required distance AC . Even if some of the angles cannot be measured, the use of a quadrilateral is an excellent way to determine the distance AC . By applying the method of least squares it is possible to so adjust the quadrilateral that the small discrepancies are distributed and the best theoretical value of AC is determined.*

The precautions mentioned in Chap. II should be taken in measuring AB and CD ; the measurements should be taken four

* See Higher Surveying by Breed and Hosmer, Vol. II, Appendix B.

times with a standardized steel tape (or an Invar tape), preferably on a cloudy day of fairly uniform temperature. Each angle should be measured 6 times in one direction and 6 times in the opposite direction with a 10'' transit. (Art. 59.)

315. Leveling Across a River. Two levels are used, one on each shore. Simultaneous rod-readings are taken on a known B.M. on one shore and on an unknown B.M. on the other shore. If only one level is used, the effect of curvature and refraction will have to be determined and applied (Art. 100); but if two levels are used, as described above, the effect of both curvature and refraction is eliminated. The levels should be set up near the B.Ms. To avoid the effect of irregular refraction near the water surface, particularly on a hot day, the instruments should be set at least 10 ft. above the water surface; when possible, sights should be made on a cloudy day. Immediately after a reading is taken on a rod across the river, another reading is also taken on the B.M. that is close to the level. The reading across the river is taken on a target that is fastened to a self-reading rod; the reading on the nearby B.M. is taken by reading the rod directly. It will be noticed that this series of readings is the same as that which would be procured in the peg adjustment of the level. (See Art. 117.) The difference in elevation is also obtained in the same manner as in the peg adjustment method. This difference in elevation is applied to the known elevation of the B.M. on one bank to obtain the elevation of the B.M. on the other bank.

316. Measuring Horizontal Angles with a Tape. In Fig. 189 the horizontal angle CAB is required from measurements made with a tape. Lay off $AE = 100$ ft. and set stake E carefully by eye on line AB ; similarly, set stake D on line AC . Measure DE . Since triangle ADE is an isosceles triangle, a perpendicular from A to DE will bisect the base DE at F and divide AED into

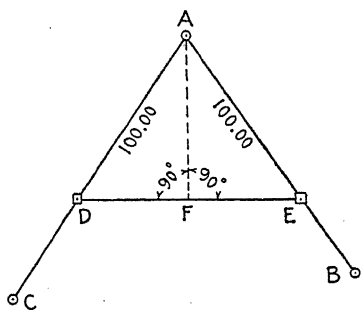


FIG. 189.

would be turned off and the swing offset from B measured; also the swing offset BE' from B to EA would be measured. The computations would be reduced to the solution of the right triangle ABE' .

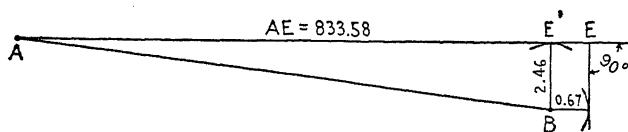


FIG. 191.

318. Obstacles on Line. When an obstacle, such as a building, large rock, or pond lies on the transit line, special methods must be used to prolong the line "through" the obstacle. One common method is to offset the transit line, as illustrated in Fig. 192, where it is required to produce the line AB beyond the house. Point B is set on line and as close to the house as is practicable. The instrument is then set up at B and a right angle ABF laid off with the transit. BF is made any convenient distance which will bring the auxiliary line outside the building. Similarly, point E is set opposite point A , and sometimes a second point E'

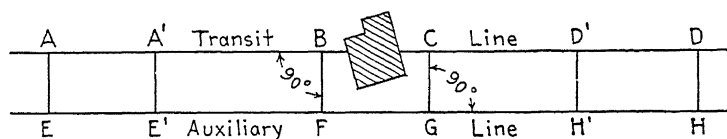


FIG. 192.

opposite A' , which is **exactly** on the transit line. These points E and E' need not be set by means of a transit set up at A and at A' unless AE is quite long.

The instrument is then set up at F and backsighted on E , the sight is checked on E' , the telescope is inverted, and points G , H' and H are set on line. Leaving the telescope inverted, another backsight is taken on E , and the process is repeated, as described in Art. 64. The transit is then moved to point G , a right angle turned off, and point C set on the right angle line, the distance GC being made equal to BF . Similarly, set D' and D .

Then by setting up at C , sighting ahead on D ($DH = GC$), and checking on point D' ($D'H' = GC$), the transit line is again run forward in its original location. The distance FG is measured carefully and is used as the length for BC . Thus it is apparent why it is so necessary that the lines BF and GC shall be laid off at right angles by means of the transit. The other offsets AE , $A'E'$, DH and $D'H'$ are not in any way connected with the measurement along the line; they simply define the direction of the line. The offsets $A'E'$ and $D'H'$ are not absolutely necessary, but they serve as desirable checks on the work, and in first-class surveying they should not be omitted. The offsets AE and DH should be taken as far back from the obstacle as is practicable.

319. Another method of measuring around a small obstacle is shown in Fig. 193, where the line AB runs through a tree, and

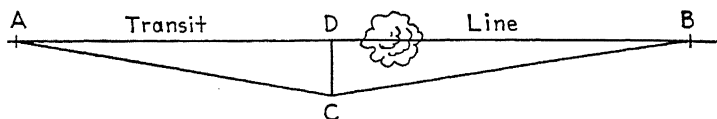


FIG. 193.

points A , D and B have been set on line. DC is made some convenient short distance, judged by eye to be at right angles to the transit line. Then AC and CB are measured. The distance

$$AB = AC - \frac{DC^2}{2 AC} + CB - \frac{DC^2}{2 CB}$$

When DC is taken as some whole number of feet, the computation of the above is extremely simple.

A line may be prolonged across a pond or river by measuring a base-line on one shore and triangulating for the distance, as explained in Art. 314.

320. Inaccessible Distances. Sometimes the distance between two points and the direction of the line connecting them are desired, but it may be inconvenient or impossible to measure the line between them or to occupy the points with the transit. In Fig. 194 the points A and B are inaccessible, and it is desired

to obtain the distance AB and the angle that AB makes with the transit line. From the point D the distance DC and the angles BDA and ADC are measured; similarly, at C the angles ACB and BCD are measured. AB can then be calculated as follows: in the triangle CBD , compute CB ; in triangle ACD , compute AC ; and in the triangle ACB , calculate AB , the inaccessible distance. In the triangle ACB , angle ABC can be computed; this, together with the measured angle BCD , will give the difference in direction between AB and CD . It was not necessary that DC should have been measured as one straight line in the

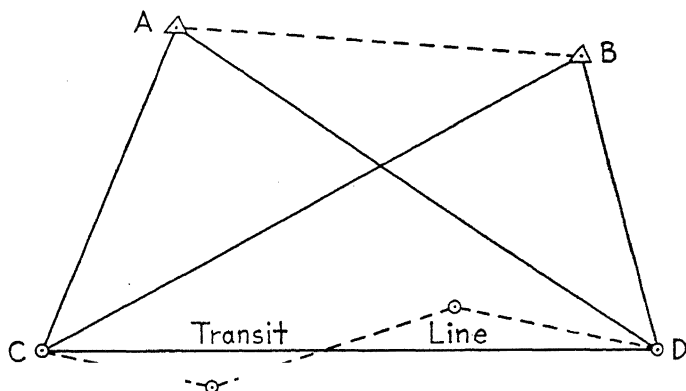


FIG. 194.

traverse; the traverse might have been run as indicated by the dashed lines. If it had been, the distance CD and the necessary angles could be computed; the remainder of the computation would be the same as before.

Another problem of the same type is one in which A and B (Fig. 194) are accessible points and C and D are inaccessible, but the distance DC is known. To find the distance AB , assume that AB is unity. Then by using measured angles at A and B and the same process as that described in the preceding problem, the length of CD can be found. This establishes a ratio between the lengths of the lines AB and CD , and since the actual length of CD is known, the distance AB can be computed.

321. Measuring the Height of an Inaccessible Object. In Fig. 195 it is desired to determine the height OF of the tower.

A base-line is first laid out on the ground (represented by the two stakes) in such a position that the point O may be seen with a transit from both ends of the ground base. With the transit at A , measure the horizontal angle DAB , the vertical angle DAO , and the vertical distance GD , which is the distance from the base of the tower to point D where the horizontal cross-hair intersects the tower when the telescope is horizontal; $GD = EF$.

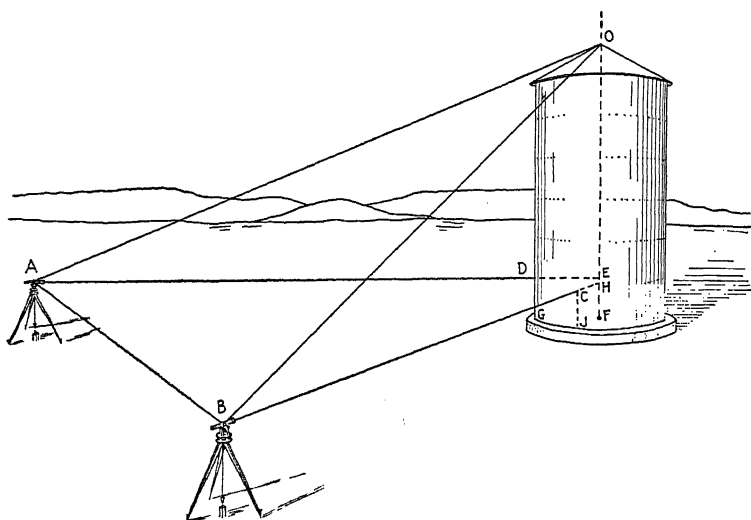


FIG. 195. HEIGHT OF WATER TANK.

With the transit at B , measure the horizontal angle ABC , the vertical angle CBO , and the vertical distance $CJ = HF$.

Note that points E and H are in the same vertical line, and that the horizontal projection of AEB and of AHB is the same triangle. Solve the triangle for AE and BH . In the triangle OAE , solve for OE ; and in the triangle OBH , solve for OH . Then the height $OF = OE + DG = OH + CJ$.

322. Party Wall Survey. A party wall is one that is built along the division line between two properties to serve the purpose of supporting the framework of the buildings on both sides

of the line. A party wall survey usually requires running a survey line inside of each building, and tying the face of the wall to each of these traverse lines. These lines should form part of a closed traverse which is run along the front and the rear of the building and through the windows or doorways of each building on each side of the party wall. From these data the position of the center line of the wall can be computed; this is usually the property line.

The problem of surveys within existing buildings and the interpretation of party wall agreements are too complicated for treatment in this text. They will be found completely treated in *Elementary Surveying* by Breed and Hosmer, Vol. I, Seventh Ed., Chap. X.

323. Locating Objects from a Traverse. When locating objects from a traverse the measurements should be such as will permit of rapid and accurate plotting. The surveyor should constantly keep in mind how the object which is being located in the field is to be plotted in the office.

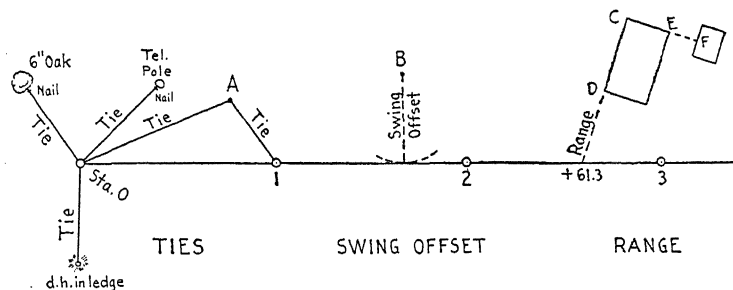


FIG. 196. TIES.

324. TIES. In surveying, the word *tie* means a direct horizontal measurement between two points. In Fig. 196 Sta. 0 is referenced to three objects by ties, and point A is located with respect to Stas. 0 and 1 by two ties.

325. SWING OFFSETS. A *swing offset* is the perpendicular distance from a point to a line, as at B, Fig. 196. With its zero end at the point B, the tape, **always being held horizontally**, is swung through a small arc, as indicated. A transit set at Sta. 0 is

sighted along the traverse, and the shortest reading obtainable on the tape is the perpendicular distance desired.

326. RANGE LINES. A *range line* is a line produced to intersect a traverse line, or some other line. In Fig. 196 the side of the building is produced until it intersects the transit line (at Sta. 2 + 61.3). The rodman holds a range pole or plumb-line on the traverse line and shifts its position until it lines in with the side of the house CD . At the same time the instrumentman sets the pole or bob on the transit line. The plus station is then measured with the tape. Another example of a range is the line EF , which is CE produced.

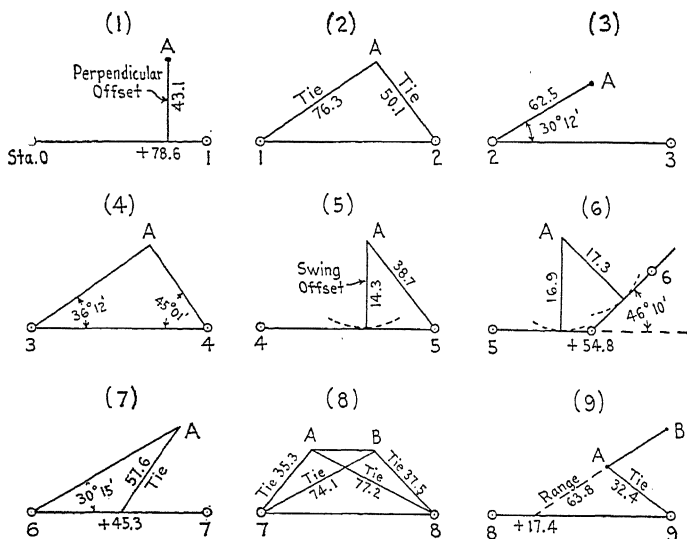


FIG. 197. METHODS OF LOCATING POINTS AND LINES.

327. METHODS OF LOCATING A POINT AND A LINE. A point may be located from a traverse by any of the following methods:

- (1) Rectangular coordinates, i.e., by its station and perpendicular offset;
- (2) Two ties from known points;
- (3) Angle and distance from a known point;
- (4) Angle at each of two known points;

- (5) A swing offset and tie;
- (6) Two swing offsets;
- (7) Angle from one point and distance from another point.

A line may be defined by

- (8) Locating two points on the line;
- (9) Locating one point on the line, and the direction of the line by an angle or range.

Each of the above methods is illustrated in Fig. 197. To avoid confusion the traverse is purposely broken between stations. In Case (1) a trial perpendicular is erected near *A* and adjusted to the true position by measuring the offset from *A* to

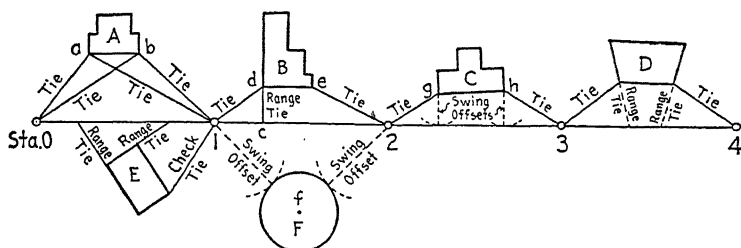


FIG. 198. BUILDING LOCATIONS.

the trial perpendicular. The method is similar to that used to locate *B* in Fig. 191.

328. Building Locations. The foregoing methods are applied to the locations shown in Fig. 198. It is assumed that the dimensions of the buildings have been measured and that the angles at the corners are right angles unless otherwise indicated.

Building *A* is located and checked by ties from 0 and 1. The intersections of the ties at *a* and at *b* are near 90° . This method should only be used where the intersection angles are between 60° and 120° . When plotting, the distance *ab* is scaled for a check.

Building *B* is obviously too close to the traverse line to obtain favorable intersections for locating it by ties. Since the left side is long, a range is taken, point *c* is located on the traverse, and the distances 1 *d*, *dc* and *c* 1 are measured; *ae* is measured for a check. In plotting, lay off 1 *c* to scale. Locate *d* by swinging an arc with radius 1 *d* and intersecting it with distance

cd. Draw a line through *c* and *d*. The left side of the house is along the line. Plot the house and check by scaling $2e$.

Building *C* is located by swing offsets. It is too close to the line to be located by ties, and the sides are so short that ranges would be unsatisfactory. To plot this building, place the scale at right angles to the traverse line by estimation with zero of the scale on the line. Scale and lay off the swing-offset distances, and through these points draw lines parallel to the traverse line. With the scale, find the point where distance $2g$ intersects the parallel line through *g*. This locates one corner. Locate the other corner *h* in a similar manner. Scale *gh* as a check.

Building *F*, a circular tank, is located by swing offsets from Stas. 1 and 2. The radius is computed from the measured circumference. This value added to the swing-offset distances will give the distances from Stas. 1 and 2 to point *f*, the center of the tank. Point *f* is plotted from these distances. The radius is set off on the compass, and the perimeter of the tank is drawn with *f* as a center.

329. SOUNDINGS. Determining the shape of the bottom of a river, harbor, or lake is a common problem for the surveyor. Such surveys are made to procure data for navigation charts, to determine where dredging should be done and to establish a basis for computing the quantity of dredged material. Subaqueous surveys are usually controlled by survey stations previously defined on land by triangulation and traverse. To these land points all of the underwater survey is referred. The depth of water is usually measured from a boat by a rod or sounding line, and the positions of these soundings are located with respect to the land points. Because the shape of the bottom cannot be seen, it is customary to take soundings at much closer intervals than levels are taken in determining the shape of the ground out in the open.

The points on shore to which the hydrographic survey is referred should be so chosen that they will be in clear view from the water surface. Such prominent objects as church spires, windmills, lighthouses, flag-poles, and the like are serviceable, provided they are close enough to the shore to come within the boundaries of the sheet when plotted on the map. The shore-

line and some of the adjacent topography are usually desired; these are obtained as explained in Chap. X.

330. METHODS OF LOCATING SOUNDINGS.* Soundings are usually located by one of the following methods:

(1) The boat is rowed on a range at a uniform rate of speed, and the soundings are located by time intervals.

(2) The boat is rowed on a range line, and the positions of the soundings are "cut in" by a transit angle taken on shore or by an angle taken with a sextant from the boat.

(3) The boat may or may not be rowed on any definite range; its position is located by angles taken simultaneously by two transits on shore, or by angles taken simultaneously to shore points from the boat by means of two sextants.

(4) The positions of the soundings are located by the stadia method.

(5) The positions of the soundings are defined by the intersections of fixed ranges.

(6) A wire or line is stretched across a stream from shore to shore, and soundings are taken at different points along this wire and located by measured distances from one end of it.

In this text only the two most common methods, namely, (2) and (3), will be further explained.

331. Locating Soundings by Range and Angle from Shore. In Fig. 199 *ABC* is part of the shore traverse which may be connected to a more extensive traverse involving considerable land area; or it may be merely a limited shore traverse run out as a control of the soundings to be taken, and solely for the purpose of relating those soundings to the shore-line.

In Fig. 199 the sounding at *F* is located by range line and one angle, at *G* it is located by two angles from shore, and at *H* by two angles taken from a boat.

Between *A* and *B* five lining-poles are set at equal distances apart. Points *D* and *E* are set at 90° to line *AB* and far enough away to furnish a good range sight while the boat is being rowed out from shore. The spacing of the range markers along *DE* is the same as along *AB*. In Fig. 199 the boat is proceeding along

* See Higher Surveying by Breed and Hosmer, Vol. II, Chap. X, for complete treatment of this subject.

taken. As the boat is being rowed along its course, the sounding rod, or line, is followed by the instrumentmen. At a signal from the boat (say, by means of a white flag), each transitman quickly perfects his sighting on the bow of the sounding boat and clamps the upper plate. At the same moment the leadsman (who stands in the bow) takes the sounding. The two angles, one from each end of the base-line, accurately locate the boat's bow, the position of which is usually plotted graphically. For example, in Fig. 199 an angle is measured simultaneously at A and at C to the boat at G that is being rowed on range 9. In this two-angle method it is necessary to row the boat on the approximate range lines, in order to be sure to cover the area properly.

333. Locating Soundings by Two Angles from Boat. At the instant the sounding is taken, two angles, such as α and β (Fig. 199), are measured from the boat at H with sextants.

To plot the position of the boat requires solution of the three-point problem. This is most conveniently accomplished by setting off angles α and β on a three-armed protractor. The protractor is so placed on the map that each of its three arms passes through B , S and C respectively. The center of the protractor is then at the boat position, H .

An improvised three-armed protractor can be made for any individual pair of angles by plotting them on tracing cloth, α on one side and β on the other side of a line. The tracing is then used as though it were a three-armed protractor.

334. Sounding Equipment. In addition to the usual equipment for a transit party, a boat, a sounding-pole or lead-line, a gage for reading the change in water level, and lining-poles which are to be placed on shore are required. Sometimes sextants, signals, and buoys are used.

A round-bottom boat is to be preferred to a flat-bottom boat because it can be kept more readily on a range line when wind and current prevail. In calm water a flat-bottom skiff is wholly suitable. Since the leadsman stands in the bow when taking soundings, the boat should be fairly stiff.

335. Sounding-Poles and Lead-Lines. *Sounding-poles* are made similar to ordinary self-reading leveling rods. They are usually from 10 to 20 ft. long, 1 to 2 in. square in section, and

with an iron or lead shoe of sufficient area to prevent the pole from sinking into the mud or sand. The shoe is sometimes provided with a cup-shaped cavity in the bottom, which enables samples of material to be collected. This cavity should be smeared with some such material as tallow or soap, so that the soil will adhere to the metal shoe. The pole is graduated to feet and tenths, and it will be found convenient in reading the sounding to have these graduations on opposite faces.

336. The *Lead-line* consists of a long chain or of a line of hemp or cotton; a metal weight is attached at the end. Chains are hard on the leadsman's hands; but lines shrink and stretch.

If a line is used, it should be stretched before it is graduated, so that it will not change much in length with use. To stretch the line, wind it tightly around a circular post, fasten both ends securely, and then wet it thoroughly. When it dries out, repeat the operation three or four times. Wet the line thoroughly again, and, with the zero-point at the bottom of the weight, mark the foot-marks by cloth or leather tags. Make every fifth foot-mark a distinctive color. Lead-lines should be frequently tested for length, and, if in error, the soundings should be corrected in the office at the same time that the soundings are corrected for water stage. The lines should be soaked in water an hour or two before the soundings are taken.

When a chain is used for the lead-line, the foot-marks should be of leather or cloth; metal tags are hard on the leadsman's hands.

The weights used on lead-lines vary from 3 to 30 pounds, depending upon the depth of water and the speed of current. A window weight makes a suitable weight for depths up to about 40 ft. If samples of the bottom are required, the weight should have a cup-shaped bottom in which to collect samples.

337. **Shore Signals.** Temporary shore signals or range markers may be constructed of $2'' \times 4''$ joist for the posts and braces, and of $\frac{7}{8}'' \times 2''$ scantling for the cross pieces. These signals are usually covered with white or colored cotton cloth, the color depending upon the background against which it must be seen.

A lining-pole shod with a long, somewhat slender, steel shoe makes an excellent marker for shore signals in cases where the

soundings are to extend not more than a quarter to half a mile from shore.

A gage for measuring the water level should be fastened, usually in a vertical position, to a wharf or structure, preferably where it can be read by means of a field-glass from any part of the area to be sounded. Its zero-point should be connected by leveling to established B.Ms. The gage usually consists of a white board about 10'' wide, with a scale of feet and tenths painted in black. As the soundings progress, the gage is read at such close intervals that every sounding, the time of which has been taken, can be referred to mean low water or to any other desired datum.

338. ORGANIZATION OF SOUNDING PARTY. The organization of the sounding party will depend upon the method used in locating the soundings. In any case there will be in the boat the **chief of party**, the **recorder**, the **leadsman**, the **oarsman**, and possibly a **signalman**. This will comprise the entire complement of the boat party provided the soundings are to be located by angles taken from shore, in which case the transitman and assistants (if needed) will remain on land. If, however, the angles are to be taken from the boat, the instrumentmen who will use the sextants will also be in the boat. When the work is to be done in tidal water, a **gage reader** may also be needed.

The *recorder* writes in his note-book the depths, as they are called off by the leadsman, and the watch times when the soundings are made. He also notifies the leadsman by calling out "Sound" about 5 seconds before each sounding is desired.

FIELD EXERCISES

1. Measure a base-line AB on one side of an obstacle such as a river, ravine or pond. Measure the length forward and backward using spring balance to obtain proper tension. Record temperature and apply all necessary corrections to the length of tape. Choose two well-defined points on the opposite side of the obstacle, such as C and D , and measure the angles to C and D from A and B . If possible also measure the angles at C and D and check the closure of the angles. Compute the distance CD by two separate methods and compare results.

2. (a) To prolong a straight line AB across a river. Place a point D on the same bank of the river as B and about as far up the river as the width of the river. Set up at B , backsight on A and produce the line across the river, and set C on line on the other bank. Check the position of C by double reversal of the instrument.

Measure the angle CBD and the distance BD . Move the instrument to D and measure the angle CDB . Cross the river to C and measure angle BCD . Adjust the angles and compute distance BC .

(b) If the river is about 600 ft. wide and the base-line is correct to the nearest $\frac{1}{10}$ ft., how precisely should the angles be measured to obtain the distance BC to nearest $\frac{1}{10}$ ft.?

3. Find the difference in elevation of two points, one on each side of a river, lake or ravine 1000 or more feet wide. Determine in the field the amount of the curvature and refraction correction for the distance sighted.

4. (a) Set three points A , B and C about 200 ft. apart and arranged so that the angle ABC is near 60° . Determine the angle ABC using only a 100-ft. tape and measuring distances to nearest .01 ft.

(b) Set up a transit over B and measure the angle ABC repeating it four times. Compare the mean angle so obtained with that computed from the tape measurements.

(c) If all fieldwork was performed with equal care, what allowable difference should there be in the results of the two methods for determining the angle?

5. Using the transit and tape determine the height of an object such as a steeple, water tank, silo or flag-pole. Make two independent determinations of the height.

6. Run a transit line in front of a group of buildings of irregular shape, establishing station points at every 50 ft. Locate the buildings from this transit line by ties, ranges and offsets. Also measure the dimensions of the buildings to nearest $\frac{1}{10}$ ft. Plot the buildings on a scale of 1 in. = 40 ft. and apply all "check" dimensions. Examine plot for obvious errors or inconsistencies.

7. Establish ground control points for making a hydrographic survey of the bottom of a small pond or a portion of a lake or river. Find the relative position of these points by measurement and triangulation. Establish range lines for covering the area. Take soundings from a boat and locate their position employing each of the following methods for a part of the work (1) intersection of range with angle from one shore point, (2) intersection by angles from two shore points, (3) two angles with the sextant from the boat to three shore points. Plot the survey. Note the relative amount of work required to plot the soundings taken by each of the above methods. Examine the depths for unreasonable values. Draw a contour map of the bottom of the body of water using a contour interval of 2 ft.

PROBLEMS

1. Referring to Fig. 190, compute the length AB and the angle BAC to $\frac{1}{2}$ min.

2. From a corner post at A a random line was run in a westerly direction toward another corner B . The perpendicular offset to B in a northerly direction from the random line at X was 2.64 ft. The distance AX on the random line was 1052.40 ft. Another random line was run southerly from A toward another corner C . The perpendicular offset to C in an easterly direction from a point Y on the random line was 1.98 ft., and the distance AY was 1292.50 ft. The angle at A between random lines was $87^\circ 49' 20''$.

(a) Find the angle (to 10 sec.) between the property lines AB and AC .

(b) Find the length of the property lines AB and AC .

3. A line is being measured along the side of a straight highway pavement. Between transit points A and C a river intervenes, crossed by a narrow bridge, and a point B is chosen off line on the floor of the bridge. AB measures 538.21 ft. and BC , 762.18 ft. By means of a rod, the perpendicular distance from B to the transit line AC measures 7.5 ft.

(a) Compute distance AC to nearest 0.01 ft.

(b) If the offset at B were 1 ft. instead of 7.5 ft., what would be the tape distances AB and BC consistent with the computed value of AC ?

4. Referring to Fig. 189, suppose that the point D was carelessly placed .02 ft. outside of the line AC and that E was placed .02 ft. outside of the line AB . What would be the error in the computed angle CAB ? (See Art. 316.)

5. Referring to Fig. 195, $AB = 80$ ft., angles $BAD = 60^\circ 38'$, $CBA = 58^\circ 02'$, $DAO = 30^\circ 18'$, $CBO = 31^\circ 06'$, $GD = 4.8$ ft., $JC = 2.1$ ft. Find OF , the height (to 0.1 ft.) of the tower, by two different solutions.

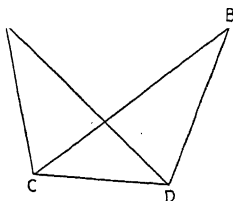


FIG. 200.

6. Referring to Fig. 200, find the bearing of line AB , no point on the line being accessible. The bearings of the other sides are $CA = N 15^\circ 34' W$, $CB = N 48^\circ 41' E$, $CD = N 88^\circ 12' E$; $DC = S 88^\circ 12' W$, $DA = N 52^\circ 06' W$, $DB = N 14^\circ 51' E$. (For the solution of this problem assume $CD =$ unity.)

7. Referring to Fig. 200, CD is a base-line 1052.61 ft. long. The measured angles are $ACB = 62^\circ 24' 15''$, $BCD = 43^\circ 31' 30''$, $ADC = 40^\circ 44' 40''$, $ADB = 71^\circ 36' 50''$. Compute the lengths of the sides AC , AB , BD , and the angles CAB and ABD .

CHAPTER XV

PLOTTING AND FINISHING MAPS

339. For drawing plans and maps, the following equipment is required: a set of drawing instruments, a straightedge, an engineer's scale, a protractor, 45° and 60° - 30° triangles, French (irregular) curves, pencils of various hardness, pen holder and several pen points, and waterproof black ink. In addition to the above the following equipment is usually provided in engineering offices: a pantograph, beam compass, proportional dividers, circular curve templates, flexible curve, and spline.

340. **Engineer's Scale.** These scales are of triangular cross-section, thus providing 6 different scales on one rule; or are flat with both edges beveled two ways, thus providing for 4 scales; or they have only one bevel on an edge thus providing for 2 scales.

Surveyors' plans are usually plotted to such scales as 10, 20, 30, 40, 50, 60, 80, 100, 200 feet to an inch. The engineer's scale is therefore divided on its several faces into 10, 20, 30, 40, etc. parts to the inch, instead of eighths, sixteenths, etc., as in the architect's scales. If the scale of a plan is 1 in. = 40 ft., the scale in which the inch is divided into 40 parts is used; one division on this scale corresponds to 1 foot. If the scale had been 1 in. = 400 ft., then one division would correspond to 10 ft. For closer scaling, fractional parts of a division are estimated.

341. **Drawing Papers.** The paper on which surveyors' plans and maps are plotted ranges from inexpensive manila detail paper to expensive white paper mounted on muslin. The cheaper grades are used for plotting when the drawings are temporary or when copies are to be made (traced); they are usually executed in pencil. Since permanent drawings are executed in ink, the paper should be of such texture as to take ink well even after erasures have been made.

Plans and maps are often traced on paper or tracing cloth. The former is a thin transparent paper, and the copy may be made in ink or pencil; it is less durable than cloth and will not stand repeated erasures. Tracing cloth is a fine grade of linen, treated to make it transparent. One side is glazed, the other side is dull. Either side will take ink, but only the dull side will take pencil lines; the dull side is almost always used. Before any ink is applied to tracing cloth, the entire surface should be sprinkled and rubbed with talcum powder or with chalk dust.

Erasures of ink lines on tracing cloth may be made with an ordinary soft pencil eraser after the body of the ink of the line has been gently shaved off with a knife edge or razor blade; do not use an ink eraser.

If tracing cloth is sharply folded or if water gets on the cloth, the blemish cannot be eradicated.

342. Profile and Cross-Section Papers. For plotting profiles and cross-sections, specially ruled paper and tracing cloth are available; the rulings are printed in colored ink (commonly green or orange). Profile paper usually comes in rolls 20 in. wide; cross-section paper comes either in rolls or sheets.

A common type of profile paper has heavy vertical rulings spaced $2\frac{1}{2}$ in. apart with lighter rulings at $\frac{1}{4}$ in. intervals. There are heavy horizontal rulings at $2\frac{1}{2}$ in., intermediate weight rulings at $\frac{1}{4}$ in., and light weight rulings $\frac{1}{16}$ in. apart.

The commoner types of ruling on cross-section paper have vertical and horizontal heavy rulings 1 in. apart with lighter rulings between them at intervals of $\frac{1}{16}$, $\frac{1}{10}$, $\frac{1}{8}$ and $\frac{1}{4}$ in.

So-called profile-plan paper is available; the upper part (for the plan) is blank and the lower part (for the profile) is ruled.

343. Reproducing Drawings. Transparent drawings may be readily reproduced by the *blueprint* or by the *black-and-white print* process. The former results in a print with white lines on a blue background (blueprints); the latter process results in a print with black lines on a white background. Blueprints are subject to distortions in scale in the washing and drying process; black-and-white prints are produced by a "dry" process which gives a nearly exact reproduction of the scale of the original tracing.

Opaque drawings may be reproduced by the photostat, the "Kodalith" or the photo-offset process. A photostat is a photographic reproduction of the original, which may be to the same scale as the original, or to a larger or smaller scale, as desired. The first print is a negative with white lines on a black background. A photostat of this negative gives a "positive," which has black lines on a white background. A photostat of a blueprint produces a positive print directly. The photostat process is useful for obtaining exact duplicate copies of all kinds of surveying and engineering data, such as field-notes, computations, deeds, plans, etc. Photostat prints, however, are subject to distortions in scale during the washing and drying process.

If 20 prints or more are required, the Kodalith process will prove advantageous. In this process a transparent negative is produced photographically. This negative can be of any desired scale. The Kodalith negative is printed by the usual blueprint process, producing prints with blue lines on a white background.

For 100 or more prints the photo-offset (sometimes called "planograph" or "lithograph") process is economical. It produces to any desired scale prints that have black ink lines on a white background. Such prints are frequently greatly reduced from the original for inclusion in reports.

344. What Should Appear on Drawings. Aside from the delineation of the survey data, whether it be a topographic map, plan, profile, or cross-section, there are several important things that should always appear on the drawing.

All drawings should contain a title. The title should state whether the drawing is a plan, a map, a profile, or a cross-section, what the survey is and to whom it belongs; the location, the scale, the date, the name and address of the engineer. The drawing should also show the direction of the meridian; it usually has notes relating to datum of levels, etc., and a legend for interpreting any unusual symbols used for representing the various features.

In addition to the above, property plans should contain the names of the owners of abutting property, the length and bearing (true or magnetic) of all sides, and a notation of how the corners are marked, i.e., by monument, pipe, or stake. The finished drawing for the survey described in Chaps. VI and VII is shown in Fig. 201.

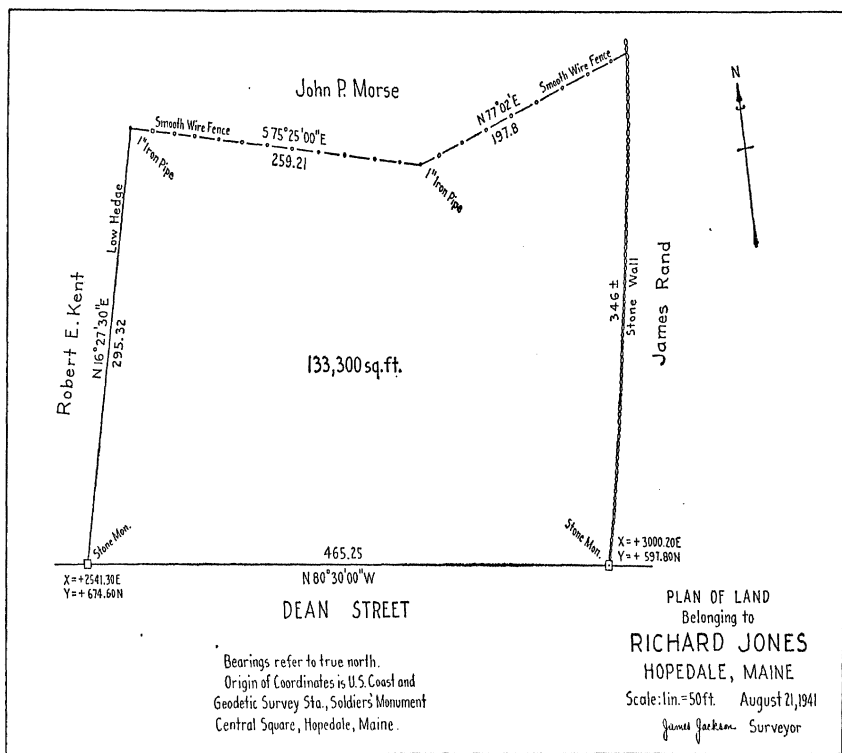


FIG. 201. FINISHED PLAN OF LAND SURVEY.
(Scale of reduced map is 125 ft. to an inch.)

345. METHODS OF PLOTTING. Plotting survey data requires great care and good eyesight. Maps are often used in studies for construction projects; on them proposed structures are drawn to scale. Obviously these maps should be accurately drawn so that none of the scaled measurements will be in error.

The plotting of maps may be divided into two parts, (1) the control, and (2) the detail. The field control may have been a traverse from which measurements were made to locate the detail (buildings, roads, etc.). Where large areas are concerned the control may be triangulation or a combination of triangulation and traverse, as explained in Chap. VI.

Although there are several ways of plotting a traverse, the three common methods are: scale and protractor, tangents, and coordinates.

Before starting the plot on the drawing paper, it is advisable to make a rough plot of the traverse in order to know its shape so that it may be placed on the drawing paper properly. This plot may be made with a scale and protractor (described below), and from it the position of the first line may be so chosen on the drawing paper that the plan will lie on the paper.

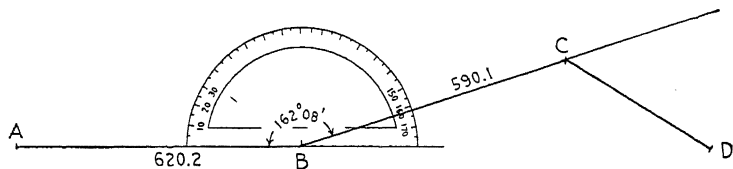


FIG. 202. PLOTING TRAVERSE WITH PROTRACTOR.

346. Plotting Traverses by Scale and Protractor. Fig. 202 shows a traverse $ABCD$ that is to be plotted with a scale and protractor. The line AB is first plotted to scale on the drawing paper. The protractor is then centered on B , and the 0° line on the protractor is placed along the line BA . The angle $162^\circ 08'$ is found on the protractor, and a pencil dot is made on the drawing paper opposite that angle on the protractor. The protractor is then removed, and a line is drawn from B through the dot. The distance 590.1 ft. is scaled off along the line BC to locate C . The protractor is then centered at C with the 0° line on CB and

the operation at B is repeated to locate D . Always extend the line through the angle point, so that both the 0° and 180° points on the protractor will fit the line before an angle is plotted.

The smaller protractors and those printed on cardboard read directly to degrees or half-degrees. More precise protractors, made of metal, may be obtained that read directly to minutes by means of a vernier. If the cheaper protractors are used it is evident that in laying off an angle it is necessary to estimate the minutes on the periphery of the protractor. The error due to such estimation is magnified when the scaled length of line is greater than the radius of the protractor. Referring to Fig. 202 if BC is twice the radius of the protractor, then any error in estimating the angle will be doubled at C . It is clear that if the length of BC is less than the radius of the protractor the error in estimating the angle will be reduced correspondingly.

While the method of plotting by scale and protractor is rapid, it is not so accurate as the tangent or the coordinate method.

347. Plotting a Traverse by Tangent Method. In this method a table of natural tangents is used. The position of AB (Fig. 203) is located and plotted on the drawing paper as explained in Art. 345. Using a straightedge AB is extended and BE is made exactly 10 in. EF is drawn perpendicular to BE and on that side of BE on which the angle ($180^\circ - 162^\circ 08' = 17^\circ 52'$) is to be laid off. To erect EF exactly at right angles to BE use the triangle as explained in Art. 349. $EG = BE \tan CBE = 10(0.3223) = 3.22$ in. Along EF lay off exactly 3.22 in. and mark G . Along BG scale $BC = 590.1$ ft., which locates point C . In similar manner prolong BC and lay off 10 in. locating point H ; there erect a perpendicular and lay off $HJ = 10 \tan 49^\circ 32' = 11.72$ in. Along CJ scale $CD = 472.8$ ft.

It will be observed that the above method should give accurate results, if the drafting is carefully done; the method is especially applicable to route traverses where the angles (deflection angles) are comparatively small. In case any line extended to 10 in. should run off the drawing paper a 5-in. line could be used, but the longer line is always preferable.

Every fifth traverse line should be checked by extending it to the line AB produced, and then measuring the angle it makes

with AB . The angle it makes with AB on the map is found by laying off 10 in. from the vertex of the angle, erecting a perpendicular and scaling the length to the line in question. This length divided by 10 is the tangent of the angle. The tables will give the value of the angle. This angle should be the algebraic sum of the deflection angles used in plotting the last 4 courses.

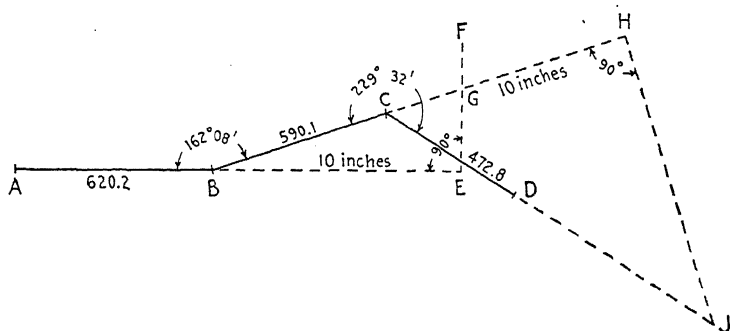


FIG. 203. PLOTTING BY TANGENT METHOD.

348. Plotting a Traverse by Coordinate Method. In plotting by this method the coordinates of all stations are referred to a pair of rectangular coordinate axes; i.e., the X and Y coordinates of each station are computed from the balanced latitudes and balanced departures, as explained in Art. 147. Although the stations may be referred to any pair of coordinate axes, it is far more convenient to refer the coordinates of stations to that pair of coordinate axes in which the Y axis (North and South) passes through the most *westerly* station of the traverse and the X axis (East and West) passes through the most *southerly* station of the traverse. Then all coordinates will be positive, or $+$; the Y of the most southerly station will be zero, and the X coordinate of the most westerly station will be zero. When this system is used the Y coordinates are called the "total latitudes," and the X coordinates the "total departures." To illustrate the method of computing the total latitudes and total departures, use is made of the survey in Art. 120. The balanced latitudes and balanced departures for that survey are computed in Fig. 78; the table below shows the computation of the total latitudes and total departures.

STA.	BAL. LATs.	BAL. DEPS.	COORDINATES FOR PLOTting	
			TOTAL LATs.	TOTAL DEPS
A	+ 76.80	- 458.90	0	458.90
B	+ 283.25	+ 83.66	76.80	0
C	- 79.48	+ 246.83	360.05	83.66
D	+ 41.54	+ 182.32	280.57	330.49
E	- 322.11	- 53.91	322.11	512.81

It will be observed in Fig. 204 that the maximum total latitude and the maximum total departure are the dimensions of a rectangle inside of which the traverse may be inscribed. The first step then is to lay out to scale on the drawing paper a rectangle having the above dimensions. Since the size of the rectangle is known, it may be placed on the drawing paper in any desired position.

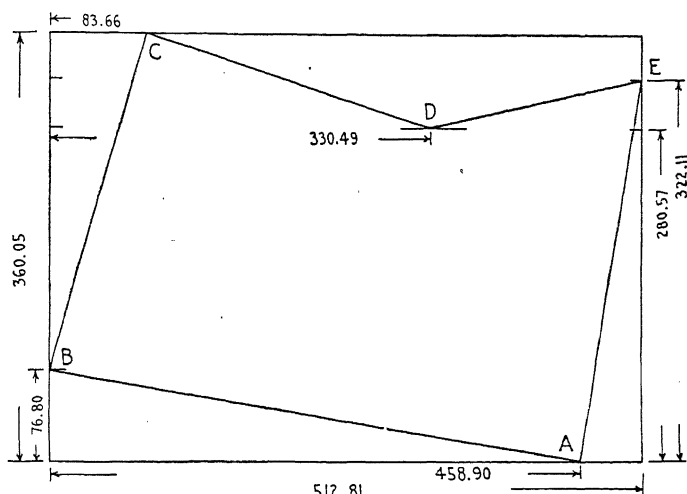


FIG. 204

349. In constructing the rectangle, use is made of the straight-edge, triangles and scale. It is important that the rectangle be constructed accurately. Frequently the material of which triangles are made shrinks unevenly, so that the 90° is not accurate, but perpendicularity is assured, if the triangle is used as indicated in Fig. 205. To erect a perpendicular to AB at A , place the straightedge along AB , and the triangle on the straightedge at A in position 1. Draw a short line, or mark a point at the side

of the triangle at m . Reverse the triangle to position 2; mark point at n . If m and n do not coincide, a line connecting A with a point midway between m and n will be perpendicular to AB at A .

When the rectangle has been drawn, it should be checked, since the accuracy of all subsequent plotting will depend on the accuracy with which it was constructed. The opposite sides of the rectangle should be equal, and **the two diagonals should be equal; this condition should be tested before continuing with the plot.** The total latitudes are then scaled off on both easterly and westerly boundaries of the rectangle, and lines are drawn with a

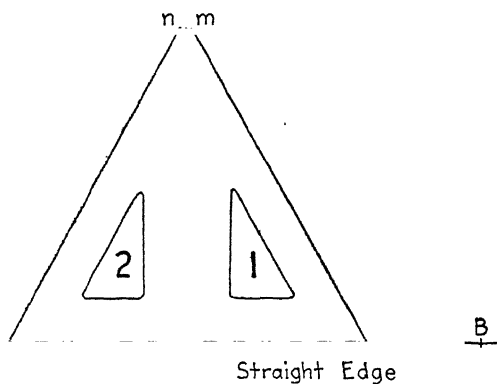


FIG. 205. ERECTING PERPENDICULAR WITH TRIANGLE.

straightedge, connecting the corresponding points. Along these lines the total departures are plotted, starting from the westerly side of the rectangle; thus each angle point is located. As a check on the plotting, the scaled distances between stations should check the measured distances. If they do not, the work should be gone over until the error is found.

Plotting by rectangular coordinates locates each angle point **independently** (the other methods do not), and is the most accurate of all the methods employed.

In plotting traverses, it is good practice to indicate each angle point by a fine hole made in the paper with a pricker (needle point), and surrounded by a small pencil circle. The pricker mark should be as small in diameter as possible and should not

extend through the paper. In marking points in this manner the exact point is preserved, so that scaling can be readily made from it.

350. Plotting Details. The details on the map are plotted from the plotted traverse line just as the details of a survey are located from the traverse line in the field. Where details are located by angle and distance, the protractor and scale are used. The use of the protractor is permissible, for the lines are usually short compared with the radius of the protractor and, as has been pointed out, any error caused in estimating the angle on the protractor will be reduced, when this condition exists, in locating the point. Any slight error in locating a detail does not affect the rest of the plot, whereas an error in plotting the traverse will have an effect on the rest of the drawing. The plotting of buildings, fences, etc. from field data is explained in Chap. XIV.

351. FINISHED DRAWINGS. In addition to the map proper certain information should appear on all maps, as explained in Art. 344. The execution of these additions, as well as their location, will tax the ingenuity of the draftsman. Not only should each addition be correct in fact, but its size should be proportionate to the rest of the map and its position should be such as to give a balanced appearance to the map as a whole. An accurate map can be spoiled by poor lettering, and give the impression that the map is inaccurate.

For permanence, maps are finished in ink. Some maps are drawn entirely in black, and some, in colors. Titles, lettering, notes, and the meridian are executed in black; relief, i.e., contours, hachures, cuts and fills, in burnt sienna; bodies of water such as lakes, streams, and marshes, in Prussian blue; and grass and trees in green. If the traverse is to be shown on the finished map, it is drawn in scarlet vermillion.

352. Topographic Conventional Signs. Conventional signs or symbols are used to represent the form of the earth's surface, as well as the physical features, such as roads, buildings, rivers, woods, cultivation, etc. A chart of standard symbols, adopted by the United States Board of Surveys and Maps, is published and is for sale by the United States Geological Survey. As each map is made for a definite purpose, only those symbols that will give the desired information are shown on the map. Military

maps should show all features that are important in military operations; on a street map only the lines defining the streets should be prominently represented. On account of the wide divergence in the scale of maps the size of the symbols will vary somewhat. On very small-scale maps, no attempt is made to

Good Motor Road		Boundary Lines	
Poor or Private Road			
Road, Small Scale Map		Telegraph or Telephone Line	
Trail		Power-transmission Line	
Railroads			
Single Track		Bridges	
Double Track			
In Street		Dam	
Railroad Crossings		Streams	
Tunnel		Buildings	
Earth Slopes		Triangulation Station	
Fences		Transit Station	
Any kind		Stadia Station	
Wire		Permanent Bench Mark (and Elevation)	B M 1320
Hedge		Intermediate Bench Mark (and Elevation)	1320
Stone			

FIG. 206. CONVENTIONAL SIGNS.

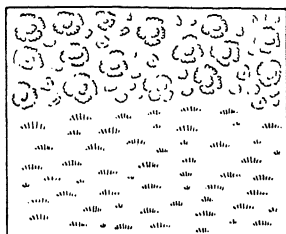
draw the object to scale; and the object is represented as being much larger than if drawn to actual size. On large-scale maps the objects may be drawn to scale. Many of the symbols are executed freehand. As the appearance of the map is largely dependent on the execution of the symbols, it is important that great care should be exercised in making them.

Some of the more common symbols are shown in Figs. 206-7.

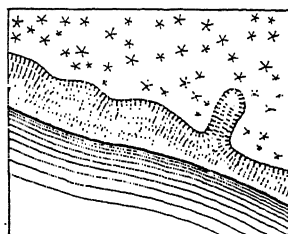
353. Lettering. One of the most important features that adds to the general appearance of a map is good lettering. Poor lettering indicates a poor map. The art of good lettering may be acquired by study of standard forms and by practice.

Several styles of lettering are shown in Fig. 208.

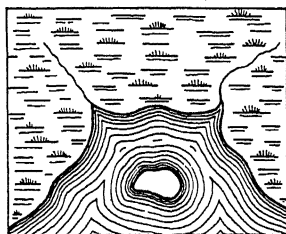
In general, Roman or Gothic letters are used in titles on land maps; Reinhardt's lettering is used for the details.



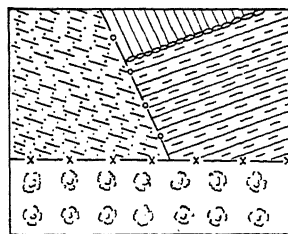
Deciduous Trees and Grass



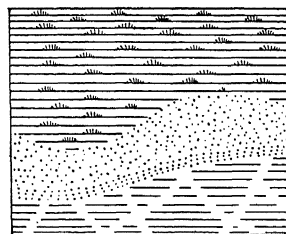
Evergreen Trees, Bluffs, Water-lining



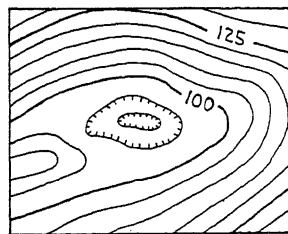
Fresh-water Marsh and Water-lining



Cultivated Land and Orchard



Salt Marsh, Sand and Tidal Flats



Contours and Depression
Contours

FIG. 207. CONVENTIONAL SIGNS.

Plans should be lettered to read from the bottom. Fig. 210 illustrates the direction of the letters on lines of various slopes.

Much lettering is done freehand. However, several devices are on the market (lettering templates) for lettering mechanically. These come in several styles and in a number of different sizes.

ROMAN

A B C D E F G H I J K L M N O P Q R S T U

V W X Y Z &

a b c d e f g h i j k l m n o p q r s t u v w x y z

Reinhardt's Style

A B C D E F G H I J K L M N O P Q

R S T U V W X Y Z &

a b c d e f g h i j k l m n o p q r s t u v w x y z

1 2 3 4 5 6 7 8 9 0 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{5}{32}$ $2\frac{3}{16}$ $5\frac{1}{8}$ 1 2 3 4 5 6 7 8 9 0 $\frac{1}{8}$ $9\frac{3}{4}$ $\frac{7}{8}$

A B C D E F G H I J K L M N O P Q R S

T U V W X Y Z &

a b c d e f g h i j k l m n o p q r s t u v w x y z

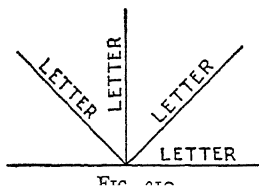
FIG. 208. LETTER PLATE.

The style of lettering obtained with one type of lettering guide is shown in Fig. 209.

ABCDEFGHIJKLMNOPQRSTUVWXYZ
 abcdefghijklmnopqrstuvwxyz
 1234567890&

FIG. 209.

354. **Titles.** In general the title on a map should be in the lower right-hand part of the sheet. It should be so placed that it will give a balanced appearance to the map. The title itself should also have a balanced appearance, and its size should conform to the size of the map. Each line of lettering should be centered, and the spacing between lines should be such that the lettering will not appear crowded or too far apart. The size of the letters chosen should be so weighted that the most important part of the title will strike the eye first.



Figs. 201 and 211 show the arrangement of individual titles.

UNITED STATES - EAST COAST
 MAINE
 WINTER HARBOR

Scale $\frac{1}{20000}$

FIG. 211. SAMPLE TITLE.

355. **Meridians.** The meridian should be shown on all maps. Sometimes only the magnetic meridian is shown, and sometimes only the true. When a single meridian is shown, it should be marked to indicate whether it is true or magnetic. When possible both should be shown and the declination indicated. It is common practice to indicate the true meridian with a full

arrow head and the magnetic meridian with a half arrow head, as shown in Fig. 212. The meridian should be placed where it will not cross any lines of the map, and it should be of sufficient length so that it may be referred to lines on the map and their bearings obtained by means of a protractor.

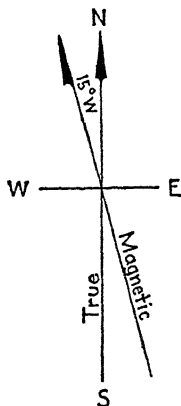


FIG. 212. NEEDLE.

356. Scales. The scale of the map should appear on all drawings. Scales are usually placed under the title or in one of the lower corners. In addition to stating the scale, as so many feet or miles to one inch, or as a representative fraction, i.e., $\frac{1}{5000}$ which means one unit on the map equals 5000 units of the same kind on the ground, it is customary to show also a graphical scale. Examples of the latter are shown in Fig. 213. In using the lower scale the distance desired between two

points on the map is set off on a pair of dividers. One point of the dividers is set on an even 100 ft. division so that the other point on the dividers will fall on that part of the scale to the left of the 0. Then the distance in feet is read directly to 10 ft. (on the above scale) and by estimation to the nearest foot.

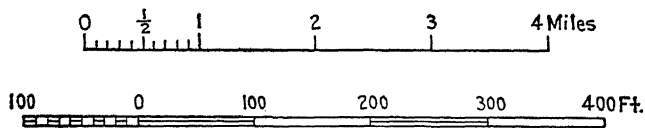


FIG. 213. GRAPHICAL SCALES.

357. Border Lines. It adds to the appearance of a drawing to have a suitable border line. This may be a single heavy line or a double line, consisting of one heavy line and one light line drawn close together. The weight of the border lines should not be so heavy as to detract the attention from the map. Outside the border line there should be a margin of about $\frac{3}{4}$ in. for small maps and of from 1 in. to $1\frac{1}{4}$ in. for large maps.

TABLES

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389
1	0432	0475	0518	0561	0604	0647	0689	0732	0775	0817
2	0860	0903	0945	0988	1030	1072	1115	1157	1199	1242
3	1284	1326	1368	1410	1452	1494	1536	1578	1620	1662
4	1703	1745	1787	1828	1870	1912	1953	1995	2036	2078
5	2119	2160	2202	2243	2284	2325	2366	2407	2449	2490
6	2531	2572	2612	2653	2694	2735	2776	2816	2857	2898
7	2938	2979	3019	3060	3100	3141	3181	3222	3262	3302
8	3342	3383	3423	3463	3503	3543	3583	3623	3663	3703
9	3743	3782	3822	3862	3902	3941	3981	4021	4060	4100
110	04139	04179	04218	04258	04297	04336	04376	04415	04454	04493
1	4532	4571	4610	4650	4689	4727	4766	4805	4844	4883
2	4922	4961	4999	5038	5077	5115	5154	5192	5231	5269
3	5308	5346	5385	5423	5461	5500	5538	5576	5614	5652
4	5690	5729	5767	5805	5843	5881	5918	5956	5994	6032
5	6070	6108	6145	6183	6221	6258	6296	6333	6371	6408
6	6446	6483	6521	6558	6595	6633	6670	6707	6744	6781
7	6819	6856	6893	6930	6967	7004	7041	7078	7115	7151
8	7188	7225	7262	7298	7335	7372	7408	7445	7482	7518
9	7555	7591	7628	7664	7700	7737	7773	7809	7846	7882
120	07918	07954	07990	08027	08063	08099	08135	08171	08207	08243
1	8279	8314	8350	8386	8422	8458	8493	8529	8565	8600
2	8636	8672	8707	8743	8778	8814	8849	8884	8920	8955
3	8991	9026	9061	9096	9132	9167	9202	9237	9272	9307
4	9342	9377	9412	9447	9482	9517	9552	9587	9621	9656
5	9691	9726	9760	9795	9830	9864	9899	9934	9968	10003
6	10037	10072	10106	10140	10175	10209	10243	10278	10312	10346
7	0380	0415	0449	0483	0517	0551	0585	0619	0653	0687
8	0721	0755	0789	0823	0857	0890	0924	0958	0992	1025
9	1059	1093	1126	1160	1193	1227	1261	1294	1327	1361
130	11394	11428	11461	11494	11528	11561	11594	11628	11661	11694
1	1727	1760	1793	1826	1860	1893	1926	1959	1992	2024
2	2057	2090	2123	2156	2189	2222	2254	2287	2320	2352
3	2385	2418	2450	2483	2516	2548	2581	2613	2646	2678
4	2710	2743	2775	2808	2840	2872	2905	2937	2969	3001
5	3033	3066	3098	3130	3162	3194	3226	3258	3290	3322
6	3354	3386	3418	3450	3481	3513	3545	3577	3609	3640
7	3672	3704	3735	3767	3799	3830	3862	3893	3925	3956
8	3988	4019	4051	4082	4114	4145	4176	4208	4239	4270
9	4301	4333	4364	4395	4426	4457	4489	4520	4551	4582
140	14613	14644	14675	14706	14737	14768	14799	14829	14860	14891
1	4922	4953	4983	5014	5045	5076	5106	5137	5168	5198
2	5229	5259	5290	5320	5351	5381	5412	5442	5473	5503
3	5534	5564	5594	5625	5655	5685	5715	5746	5776	5806
4	5836	5866	5897	5927	5957	5987	6017	6047	6077	6107
5	6137	6167	6197	6227	6256	6286	6316	6346	6376	6406
6	6435	6465	6495	6524	6554	6584	6613	6643	6673	6702
7	6732	6761	6791	6820	6850	6879	6909	6938	6967	6997
8	7026	7056	7085	7114	7143	7173	7202	7231	7260	7289
9	7319	7348	7377	7406	7435	7464	7493	7522	7551	7580
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869
1	7898	7926	7955	7984	8013	8041	8070	8099	8127	8156
2	8184	8213	8241	8270	8298	8327	8355	8384	8412	8441
3	8469	8498	8526	8554	8583	8611	8639	8667	8696	8724
4	8752	8780	8808	8837	8865	8893	8921	8949	8977	9005
5	9033	9061	9089	9117	9145	9173	9201	9229	9257	9285
6	9312	9340	9368	9396	9424	9451	9479	9507	9535	9562
7	9590	9618	9645	9673	9700	9728	9756	9783	9811	9838
8	9866	9893	9921	9948	9976	20003	20030	20058	20085	20112
9	20140	20167	20194	20222	20249	0276	0303	0330	0358	0385
160	20412	20439	20466	20493	20520	20548	20575	20602	20629	20656
1	0683	0710	0737	0763	0790	0817	0844	0871	0898	0925
2	0952	0978	1005	1032	1059	1085	1112	1139	1165	1192
3	1219	1245	1272	1299	1325	1352	1378	1405	1431	1458
4	1484	1511	1537	1564	1590	1617	1643	1669	1696	1722
5	1748	1775	1801	1827	1854	1880	1906	1932	1958	1985
6	2011	2037	2063	2089	2115	2141	2167	2194	2220	2246
7	2272	2298	2324	2350	2376	2401	2427	2453	2479	2505
8	2531	2557	2583	2608	2634	2660	2686	2712	2737	2763
9	2789	2814	2840	2866	2891	2917	2943	2968	2994	3019
170	23045	23070	23096	23121	23147	23172	23198	23223	23249	23274
1	3300	3325	3350	3376	3401	3426	3452	3477	3502	3528
2	3553	3578	3603	3629	3654	3679	3704	3729	3754	3779
3	3805	3830	3855	3880	3905	3930	3955	3980	4005	4030
4	4055	4080	4105	4130	4155	4180	4204	4229	4254	4279
5	4304	4329	4353	4378	4403	4428	4452	4477	4502	4527
6	4551	4576	4601	4625	4650	4674	4699	4724	4748	4773
7	4797	4822	4846	4871	4895	4920	4944	4969	4993	5018
8	5042	5066	5091	5115	5139	5164	5188	5212	5237	5261
9	5285	5310	5334	5358	5382	5406	5431	5455	5479	5503
180	25527	25551	25575	25600	25624	25648	25672	25696	25720	25744
1	5768	5792	5816	5840	5864	5888	5912	5935	5959	5983
2	6007	6031	6055	6079	6102	6126	6150	6174	6198	6221
3	6245	6269	6293	6316	6340	6364	6387	6411	6435	6458
4	6482	6505	6529	6553	6576	6600	6623	6647	6670	6694
5	6717	6741	6764	6788	6811	6834	6858	6881	6905	6928
6	6951	6975	6998	7021	7045	7068	7091	7114	7138	7161
7	7184	7207	7231	7254	7277	7300	7323	7346	7370	7393
8	7416	7439	7462	7485	7508	7531	7554	7577	7600	7623
9	7646	7669	7692	7715	7738	7761	7784	7807	7830	7852
190	27875	27898	27921	27944	27967	27989	28012	28035	28058	28081
1	8103	8126	8149	8171	8194	8217	8240	8262	8285	8307
2	8330	8353	8375	8398	8421	8443	8466	8488	8511	8533
3	8556	8578	8601	8623	8646	8668	8691	8713	8735	8758
4	8780	8803	8825	8847	8870	8892	8914	8937	8959	8981
5	9003	9026	9048	9070	9092	9115	9137	9159	9181	9203
6	9226	9248	9270	9292	9314	9336	9358	9380	9403	9425
7	9447	9469	9491	9513	9535	9557	9579	9601	9623	9645
8	9667	9688	9710	9732	9754	9776	9798	9820	9842	9863
9	9885	9907	9929	9951	9973	9994	30016	30038	30060	30081
200	30103	30125	30146	30168	30190	30211	30233	30255	30276	30298

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
200	30103	30125	30146	30168	30190	30211	30233	30255	30276	30298
1	0320	0341	0363	0384	0406	0428	0449	0471	0492	0514
2	0535	0557	0578	0600	0621	0643	0664	0685	0707	0728
3	0750	0771	0792	0814	0835	0856	0878	0899	0920	0942
4	0963	0984	1006	1027	1048	1069	1091	1112	1133	1154
5	1175	1197	1218	1239	1260	1281	1302	1323	1345	1366
6	1387	1408	1429	1450	1471	1492	1513	1534	1555	1576
7	1597	1618	1639	1660	1681	1702	1723	1744	1765	1785
8	1806	1827	1848	1869	1890	1911	1931	1952	1973	1994
9	2015	2035	2056	2077	2098	2118	2139	2160	2181	2201
210	32222	32243	32263	32284	32305	32325	32346	32366	32387	32408
1	2428	2449	2469	2490	2510	2531	2552	2572	2593	2613
2	2634	2654	2675	2695	2715	2736	2756	2777	2797	2818
3	2838	2858	2879	2899	2919	2940	2960	2980	3001	3021
4	3041	3062	3082	3102	3122	3143	3163	3183	3203	3224
5	3244	3264	3284	3304	3325	3345	3365	3385	3405	3425
6	3445	3465	3486	3506	3526	3546	3566	3586	3606	3626
7	3646	3666	3686	3706	3726	3746	3766	3786	3806	3826
8	3846	3866	3885	3905	3925	3945	3965	3985	4005	4025
9	4044	4064	4084	4104	4124	4143	4163	4183	4203	4223
220	34242	34262	34282	34301	34321	34341	34361	34380	34400	34420
1	4439	4459	4479	4498	4518	4537	4557	4577	4596	4616
2	4635	4655	4674	4694	4713	4733	4753	4772	4792	4811
3	4830	4850	4869	4889	4908	4928	4947	4967	4986	5005
4	5025	5044	5064	5083	5102	5122	5141	5160	5180	5199
5	5218	5238	5257	5276	5295	5315	5334	5353	5372	5392
6	5411	5430	5449	5468	5488	5507	5526	5545	5564	5583
7	5603	5622	5641	5660	5679	5698	5717	5736	5755	5774
8	5793	5813	5832	5851	5870	5889	5908	5927	5946	5965
9	5984	6003	6021	6040	6059	6078	6097	6116	6135	6154
230	36173	36192	36211	36229	36248	36267	36286	36305	36324	36342
1	6361	6380	6399	6418	6436	6455	6474	6493	6511	6530
2	6549	6568	6586	6605	6624	6642	6661	6680	6698	6717
3	6736	6754	6773	6791	6810	6829	6847	6866	6884	6903
4	6922	6940	6959	6977	6996	7014	7033	7051	7070	7088
5	7107	7125	7144	7162	7181	7199	7218	7236	7254	7273
6	7291	7310	7328	7346	7365	7383	7401	7420	7438	7457
7	7475	7493	7511	7530	7548	7566	7585	7603	7621	7639
8	7658	7676	7694	7712	7731	7749	7767	7785	7803	7822
9	7840	7858	7876	7894	7912	7931	7949	7967	7985	8003
240	38021	38039	38057	38075	38093	38112	38130	38148	38166	38184
1	8202	8220	8238	8256	8274	8292	8310	8328	8346	8364
2	8382	8399	8417	8435	8453	8471	8489	8507	8525	8543
3	8561	8578	8596	8614	8632	8650	8668	8686	8703	8721
4	8739	8757	8775	8792	8810	8828	8846	8863	8881	8899
5	8917	8934	8952	8970	8987	9005	9023	9041	9058	9076
6	9094	9111	9129	9146	9164	9182	9199	9217	9235	9252
7	9270	9287	9305	9322	9340	9358	9375	9393	9410	9428
8	9445	9463	9480	9498	9515	9533	9550	9568	9585	9602
9	9620	9637	9655	9672	9690	9707	9724	9742	9759	9777
250	39794	39811	39829	39846	39863	39881	39898	39915	39933	39950

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
250	39794	39811	39829	39846	39863	39881	39898	39915	39933	39950
1	9967	9985	40002	40019	40037	40054	40071	40088	40106	40123
2	40140	40157	0175	0192	0209	0226	0243	0261	0278	0295
3	0312	0329	0346	0364	0381	0398	0415	0432	0449	0466
4	0483	0500	0518	0535	0552	0569	0586	0603	0620	0637
5	0654	0671	0688	0705	0722	0739	0756	0773	0790	0807
6	0824	0841	0858	0875	0892	0909	0926	0943	0960	0976
7	0993	1010	1027	1044	1061	1078	1095	1111	1128	1145
8	1162	1179	1196	1212	1229	1246	1263	1280	1296	1313
9	1330	1347	1363	1380	1397	1414	1430	1447	1464	1481
260	41497	41514	41531	41547	41564	41581	41597	41614	41631	41647
1	1664	1681	1697	1714	1731	1747	1764	1780	1797	1814
2	1830	1847	1863	1880	1896	1913	1929	1946	1963	1979
3	1996	2012	2029	2045	2062	2078	2095	2111	2127	2144
4	2160	2177	2193	2210	2226	2243	2259	2275	2292	2308
5	2325	2341	2357	2374	2390	2406	2423	2439	2455	2472
6	2488	2504	2521	2537	2553	2570	2586	2602	2619	2635
7	2651	2667	2684	2700	2716	2732	2749	2765	2781	2797
8	2813	2830	2846	2862	2878	2894	2911	2927	2943	2959
9	2975	2991	3008	3024	3040	3056	3072	3088	3104	3120
270	43136	43152	43169	43185	43201	43217	43233	43249	43265	43281
1	3297	3313	3329	3345	3361	3377	3393	3409	3425	3441
2	3457	3473	3489	3505	3521	3537	3553	3569	3584	3600
3	3616	3632	3648	3664	3680	3696	3712	3727	3743	3759
4	3775	3791	3807	3823	3838	3854	3870	3886	3902	3917
5	3933	3949	3965	3981	3996	4012	4028	4044	4059	4075
6	4091	4107	4122	4138	4154	4170	4185	4201	4217	4232
7	4248	4264	4279	4295	4311	4326	4342	4358	4373	4389
8	4404	4420	4436	4451	4467	4483	4498	4514	4529	4545
9	4560	4576	4592	4607	4623	4638	4654	4669	4685	4700
280	44716	44731	44747	44762	44778	44793	44809	44824	44840	44855
1	4871	4886	4902	4917	4932	4948	4963	4979	4994	5010
2	5025	5040	5056	5071	5086	5102	5117	5133	5148	5163
3	5179	5194	5209	5225	5240	5255	5271	5286	5301	5317
4	5332	5347	5362	5378	5393	5408	5423	5439	5454	5469
5	5484	5500	5515	5530	5545	5561	5576	5591	5606	5621
6	5637	5652	5667	5682	5697	5712	5728	5743	5758	5773
7	5788	5803	5818	5834	5849	5864	5879	5894	5909	5924
8	5939	5954	5969	5984	6000	6015	6030	6045	6060	6075
9	6090	6105	6120	6135	6150	6165	6180	6195	6210	6225
290	46240	46255	46270	46285	46300	46315	46330	46345	46359	46374
1	6389	6404	6419	6434	6449	6464	6479	6494	6509	6523
2	6538	6553	6568	6583	6598	6613	6627	6642	6657	6672
3	6687	6702	6716	6731	6746	6761	6776	6790	6805	6820
4	6835	6850	6864	6879	6894	6909	6923	6938	6953	6967
5	6982	6997	7012	7026	7041	7056	7070	7085	7100	7114
6	7129	7144	7159	7173	7188	7202	7217	7232	7246	7261
7	7276	7290	7305	7319	7334	7349	7363	7378	7392	7407
8	7422	7436	7451	7465	7480	7494	7509	7524	7538	7553
9	7567	7582	7596	7611	7625	7640	7654	7669	7683	7698
300	47712	47727	47741	47756	47770	47784	47799	47813	47828	47842

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
300	47712	47727	47741	47756	47770	47784	47799	47813	47828	47842
1	7857	7871	7885	7900	7914	7929	7943	7958	7972	7986
2	8001	8015	8029	8044	8058	8073	8087	8101	8116	8130
3	8144	8159	8173	8187	8202	8216	8230	8244	8259	8273
4	8287	8302	8316	8330	8344	8359	8373	8387	8401	8416
5	8430	8444	8458	8473	8487	8501	8515	8530	8544	8558
6	8572	8586	8601	8615	8629	8643	8657	8671	8686	8700
7	8714	8728	8742	8756	8770	8785	8799	8813	8827	8841
8	8855	8869	8883	8897	8911	8926	8940	8954	8968	8982
9	8996	9010	9024	9038	9052	9066	9080	9094	9108	9122
310	49136	49150	49164	49178	49192	49206	49220	49234	49248	49262
1	9276	9290	9304	9318	9332	9346	9360	9374	9388	9402
2	9415	9429	9443	9457	9471	9485	9499	9513	9527	9541
3	9554	9568	9582	9596	9610	9624	9638	9651	9665	9679
4	9693	9707	9721	9734	9748	9762	9776	9790	9803	9817
5	9831	9845	9859	9872	9886	9900	9914	9927	9941	9955
6	9969	9982	9996	50010	50024	50037	50051	50065	50079	50092
7	50106	50120	50133	0147	0161	0174	0188	0202	0215	0229
8	0243	0256	0270	0284	0297	0311	0325	0338	0352	0365
9	0379	0393	0406	0420	0433	0447	0461	0474	0488	0501
320	50515	50529	50542	50556	50569	50583	50596	50610	50623	50637
1	0651	0664	0678	0691	0705	0718	0732	0745	0759	0772
2	0786	0799	0813	0826	0840	0853	0866	0880	0893	0907
3	0920	0934	0947	0961	0974	0987	1001	1014	1028	1041
4	1055	1068	1081	1095	1108	1121	1135	1148	1162	1175
5	1188	1202	1215	1228	1242	1255	1268	1282	1295	1308
6	1322	1335	1348	1362	1375	1388	1402	1415	1428	1441
7	1455	1468	1481	1495	1508	1521	1534	1548	1561	1574
8	1587	1601	1614	1627	1640	1654	1667	1680	1693	1706
9	1720	1733	1746	1759	1772	1786	1799	1812	1825	1838
330	51851	51865	51878	51891	51904	51917	51930	51943	51957	51970
1	1983	1996	2009	2022	2035	2048	2061	2075	2088	2101
2	2114	2127	2140	2153	2166	2179	2192	2205	2218	2231
3	2244	2257	2270	2284	2297	2310	2323	2336	2349	2362
4	2375	2388	2401	2414	2427	2440	2453	2466	2479	2492
5	2504	2517	2530	2543	2556	2569	2582	2595	2608	2621
6	2634	2647	2660	2673	2686	2699	2711	2724	2737	2750
7	2763	2776	2789	2802	2815	2827	2840	2853	2866	2879
8	2892	2905	2917	2930	2943	2956	2969	2982	2994	3007
9	3020	3033	3046	3058	3071	3084	3097	3110	3122	3135
340	53148	53161	53173	53186	53199	53212	53224	53237	53250	53263
1	3275	3288	3301	3314	3326	3339	3352	3364	3377	3390
2	3403	3415	3428	3441	3453	3466	3479	3491	3504	3517
3	3529	3542	3555	3567	3580	3593	3605	3618	3631	3643
4	3656	3668	3681	3694	3706	3719	3732	3744	3757	3769
5	3782	3794	3807	3820	3832	3845	3857	3870	3882	3895
6	3908	3920	3933	3945	3958	3970	3983	3995	4008	4020
7	4033	4045	4058	4070	4083	4095	4108	4120	4133	4145
8	4158	4170	4183	4195	4208	4220	4233	4245	4258	4270
9	4283	4295	4307	4320	4332	4345	4357	4370	4382	4394
350	54407	54419	54432	54444	54456	54469	54481	54494	54506	54518

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
350	54407	54419	54432	54444	54456	54469	54481	54494	54506	54518
1	4531	4543	4555	4568	4580	4593	4605	4617	4630	4642
2	4654	4667	4679	4691	4704	4716	4728	4741	4753	4765
3	4777	4790	4802	4814	4827	4839	4851	4864	4876	4888
4	4900	4913	4925	4937	4949	4962	4974	4986	4998	5011
5	5023	5035	5047	5060	5072	5084	5096	5108	5121	5133
6	5145	5157	5169	5182	5194	5206	5218	5230	5242	5255
7	5267	5279	5291	5303	5315	5328	5340	5352	5364	5376
8	5388	5400	5413	5425	5437	5449	5461	5473	5485	5497
9	5509	5522	5534	5546	5558	5570	5582	5594	5606	5618
360	55630	55642	55654	55666	55678	55691	55703	55715	55727	55739
1	5751	5763	5775	5787	5799	5811	5823	5835	5847	5859
2	5871	5883	5895	5907	5919	5931	5943	5955	5967	5979
3	5991	6003	6015	6027	6038	6050	6062	6074	6086	6098
4	6110	6122	6134	6146	6158	6170	6182	6194	6205	6217
5	6229	6241	6253	6265	6277	6289	6301	6312	6324	6336
6	6348	6360	6372	6384	6396	6407	6419	6431	6443	6455
7	6467	6478	6490	6502	6514	6526	6538	6549	6561	6573
8	6585	6597	6608	6620	6632	6644	6656	6667	6679	6691
9	6703	6714	6726	6738	6750	6761	6773	6785	6797	6808
370	56820	56832	56844	56855	56867	56879	56891	56902	56914	56926
1	6937	6949	6961	6972	6984	6996	7008	7019	7031	7043
2	7054	7066	7078	7089	7101	7113	7124	7136	7148	7159
3	7171	7183	7194	7206	7217	7229	7241	7252	7264	7276
4	7287	7299	7310	7322	7334	7345	7357	7368	7380	7392
5	7403	7415	7426	7438	7449	7461	7473	7484	7496	7507
6	7519	7530	7542	7553	7565	7576	7588	7600	7611	7623
7	7634	7646	7657	7669	7680	7692	7703	7715	7726	7738
8	7749	7761	7772	7784	7795	7807	7818	7830	7841	7852
9	7864	7875	7887	7898	7910	7921	7933	7944	7955	7967
380	57978	57990	58001	58013	58024	58035	58047	58058	58070	58081
1	8092	8104	8115	8127	8138	8149	8161	8172	8184	8195
2	8206	8218	8229	8240	8252	8263	8274	8286	8297	8309
3	8320	8331	8343	8354	8365	8377	8388	8399	8410	8422
4	8433	8444	8456	8467	8478	8490	8501	8512	8524	8535
5	8546	8557	8569	8580	8591	8602	8614	8625	8636	8647
6	8659	8670	8681	8692	8704	8715	8726	8737	8749	8760
7	8771	8782	8794	8805	8816	8827	8838	8850	8861	8872
8	8883	8894	8906	8917	8928	8939	8950	8961	8973	8984
9	8995	9006	9017	9028	9040	9051	9062	9073	9084	9095
390	59106	59118	59129	59140	59151	59162	59173	59184	59195	59207
1	9218	9229	9240	9251	9262	9273	9284	9295	9306	9318
2	9329	9340	9351	9362	9373	9384	9395	9406	9417	9428
3	9439	9450	9461	9472	9483	9494	9506	9517	9528	9539
4	9550	9561	9572	9583	9594	9605	9616	9627	9638	9649
5	9660	9671	9682	9693	9704	9715	9726	9737	9748	9759
6	9770	9780	9791	9802	9813	9824	9835	9846	9857	9868
7	9879	9890	9901	9912	9923	9934	9945	9956	9966	9977
8	9988	9999	60010	60021	60032	60043	60054	60065	60076	60086
9	60097	60108	0119	0130	0141	0152	0163	0173	0184	0195
400	60206	60217	60228	60239	60249	60260	60271	60282	60293	60304

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
400	60206	60217	60228	60239	60249	60260	60271	60282	60293	60304
1	0314	0325	0336	0347	0358	0369	0379	0390	0401	0412
2	0423	0433	0444	0455	0466	0477	0487	0498	0509	0520
3	0531	0541	0552	0563	0574	0584	0595	0606	0617	0627
4	0638	0649	0660	0670	0681	0692	0703	0713	0724	0735
5	0746	0756	0767	0778	0788	0799	0810	0821	0831	0842
6	0853	0863	0874	0885	0895	0906	0917	0927	0938	0949
7	0959	0970	0981	0991	1002	1013	1023	1034	1045	1055
8	1066	1077	1087	1098	1109	1119	1130	1140	1151	1162
9	1172	1183	1194	1204	1215	1225	1236	1247	1257	1268
410	61278	61289	61300	61310	61321	61331	61342	61352	61363	61374
1	1384	1395	1405	1416	1426	1437	1448	1458	1469	1479
2	1490	1500	1511	1521	1532	1542	1553	1563	1574	1584
3	1595	1606	1616	1627	1637	1648	1658	1669	1679	1690
4	1700	1711	1721	1731	1742	1752	1763	1773	1784	1794
5	1805	1815	1826	1836	1847	1857	1868	1878	1888	1899
6	1909	1920	1930	1941	1951	1962	1972	1982	1993	2003
7	2014	2024	2034	2045	2055	2066	2076	2086	2097	2107
8	2118	2128	2138	2149	2159	2170	2180	2190	2201	2211
9	2221	2232	2242	2252	2263	2273	2284	2294	2304	2315
420	62325	62335	62346	62356	62366	62377	62387	62397	62408	62418
1	2428	2439	2449	2459	2469	2480	2490	2500	2511	2521
2	2531	2542	2552	2562	2572	2583	2593	2603	2613	2624
3	2634	2644	2655	2665	2675	2685	2696	2706	2716	2726
4	2737	2747	2757	2767	2778	2788	2798	2808	2818	2829
5	2839	2849	2859	2870	2880	2890	2900	2910	2921	2931
6	2941	2951	2961	2972	2982	2992	3002	3012	3022	3033
7	3043	3053	3063	3073	3083	3094	3104	3114	3124	3134
8	3144	3155	3165	3175	3185	3195	3205	3215	3225	3236
9	3246	3256	3266	3276	3286	3296	3306	3317	3327	3337
430	63347	63357	63367	63377	63387	63397	63407	63417	63428	63438
1	3448	3458	3468	3478	3488	3498	3508	3518	3528	3538
2	3548	3558	3568	3579	3589	3599	3609	3619	3629	3639
3	3649	3659	3669	3679	3689	3699	3709	3719	3729	3739
4	3749	3759	3769	3779	3789	3799	3809	3819	3829	3839
5	3849	3859	3869	3879	3889	3899	3909	3919	3929	3939
6	3949	3959	3969	3979	3988	3998	4008	4018	4028	4038
7	4048	4058	4068	4078	4088	4098	4108	4118	4128	4137
8	4147	4157	4167	4177	4187	4197	4207	4217	4227	4237
9	4246	4256	4266	4276	4286	4296	4306	4316	4326	4335
440	64345	64355	64365	64375	64385	64395	64404	64414	64424	64434
1	4444	4454	4464	4473	4483	4493	4503	4513	4523	4532
2	4542	4552	4562	4572	4582	4591	4601	4611	4621	4631
3	4640	4650	4660	4670	4680	4689	4699	4709	4719	4729
4	4738	4748	4758	4768	4777	4787	4797	4807	4816	4826
5	4836	4846	4856	4865	4875	4885	4895	4904	4914	4924
6	4933	4943	4953	4963	4972	4982	4992	5002	5011	5021
7	5031	5040	5050	5060	5070	5079	5089	5099	5108	5118
8	5128	5137	5147	5157	5167	5176	5186	5196	5205	5215
9	5225	5234	5244	5254	5263	5273	5283	5292	5302	5312
450	65321	65331	65341	65350	65360	65369	65379	65389	65398	65408

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
450	65321	65331	65341	65350	65360	65369	65379	65389	65398	65408
1	5418	5427	5437	5447	5456	5466	5475	5485	5495	5504
2	5514	5523	5533	5543	5552	5562	5571	5581	5591	5600
3	5610	5619	5629	5639	5648	5658	5667	5677	5686	5696
4	5706	5715	5725	5734	5744	5753	5763	5772	5782	5792
5	5801	5811	5820	5830	5839	5849	5858	5868	5877	5887
6	5896	5906	5916	5925	5935	5944	5954	5963	5973	5982
7	5992	6001	6011	6020	6030	6039	6049	6058	6068	6077
8	6087	6096	6106	6115	6124	6134	6143	6153	6162	6172
9	6181	6191	6200	6210	6219	6229	6238	6247	6257	6266
460	66276	66285	66295	66304	66314	66323	66332	66342	66351	66361
1	6370	6380	6389	6398	6408	6417	6427	6436	6445	6455
2	6464	6474	6483	6492	6502	6511	6521	6530	6539	6549
3	6558	6567	6577	6586	6596	6605	6614	6624	6633	6642
4	6652	6661	6671	6680	6689	6699	6708	6717	6727	6736
5	6745	6755	6764	6773	6783	6792	6801	6811	6820	6829
6	6839	6848	6857	6867	6876	6885	6894	6904	6913	6922
7	6932	6941	6950	6960	6969	6978	6987	6997	7006	7015
8	7025	7034	7043	7052	7062	7071	7080	7089	7099	7108
9	7117	7127	7136	7145	7154	7164	7173	7182	7191	7201
470	67210	67219	67228	67237	67247	67256	67265	67274	67284	67293
1	7302	7311	7321	7330	7339	7348	7357	7367	7376	7385
2	7394	7403	7413	7422	7431	7440	7449	7459	7468	7477
3	7486	7495	7504	7514	7523	7532	7541	7550	7560	7569
4	7578	7587	7596	7605	7614	7624	7633	7642	7651	7660
5	7669	7679	7688	7697	7706	7715	7724	7733	7742	7752
6	7761	7770	7779	7788	7797	7806	7815	7825	7834	7843
7	7852	7861	7870	7879	7888	7897	7906	7916	7925	7934
8	7943	7952	7961	7970	7979	7988	7997	8006	8015	8024
9	8034	8043	8052	8061	8070	8079	8088	8097	8106	8115
480	68124	68133	68142	68151	68160	68169	68178	68187	68196	68205
1	8215	8224	8233	8242	8251	8260	8269	8278	8287	8296
2	8305	8314	8323	8332	8341	8350	8359	8368	8377	8386
3	8395	8404	8413	8422	8431	8440	8449	8458	8467	8476
4	8485	8494	8502	8511	8520	8529	8538	8547	8556	8565
5	8574	8583	8592	8601	8610	8619	8628	8637	8646	8655
6	8664	8673	8681	8690	8699	8708	8717	8726	8735	8744
7	8753	8762	8771	8780	8789	8797	8806	8815	8824	8833
8	8842	8851	8860	8869	8878	8886	8895	8904	8913	8922
9	8931	8940	8949	8958	8966	8975	8984	8993	9002	9011
490	69020	69028	69037	69046	69055	69064	69073	69082	69090	69099
1	9108	9117	9126	9135	9144	9152	9161	9170	9179	9188
2	9197	9205	9214	9223	9232	9241	9249	9258	9267	9276
3	9285	9294	9302	9311	9320	9329	9338	9346	9355	9364
4	9373	9381	9390	9399	9408	9417	9425	9434	9443	9452
5	9461	9469	9478	9487	9496	9504	9513	9522	9531	9539
6	9548	9557	9566	9574	9583	9592	9601	9609	9618	9627
7	9636	9644	9653	9662	9671	9679	9688	9697	9705	9714
8	9723	9732	9740	9749	9758	9767	9775	9784	9793	9801
9	9810	9819	9827	9836	9845	9854	9862	9871	9880	9888
500	69897	69906	69914	69923	69932	69940	69949	69958	69966	69975

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
500	69897	69906	69914	69923	69932	69940	69949	69958	69966	69975
1	9984	9992	70001	70010	70018	70027	70036	70044	70053	70062
2	70070	70079	0088	0096	0105	0114	0122	0131	0140	0148
3	0157	0165	0174	0183	0191	0200	0209	0217	0226	0234
4	0243	0252	0260	0269	0278	0286	0295	0303	0312	0321
5	0329	0338	0346	0355	0364	0372	0381	0389	0398	0406
6	0415	0424	0432	0441	0449	0458	0467	0475	0484	0492
7	0501	0509	0518	0526	0535	0544	0552	0561	0569	0578
8	0586	0595	0603	0612	0621	0629	0638	0646	0655	0663
9	0672	0680	0689	0697	0706	0714	0723	0731	0740	0749
510	70757	70766	70774	70783	70791	70800	70808	70817	70825	70834
1	0842	0851	0859	0868	0876	0885	0893	0902	0910	0919
2	0927	0935	0944	0952	0961	0969	0978	0986	0995	1003
3	1012	1020	1029	1037	1046	1054	1063	1071	1079	1088
4	1096	1105	1113	1122	1130	1139	1147	1155	1164	1172
5	1181	1189	1198	1206	1214	1223	1231	1240	1248	1257
6	1265	1273	1282	1290	1299	1307	1315	1324	1332	1341
7	1349	1357	1366	1374	1383	1391	1399	1408	1416	1425
8	1433	1441	1450	1458	1466	1475	1483	1492	1500	1508
9	1517	1525	1533	1542	1550	1559	1567	1575	1584	1592
520	71600	71609	71617	71625	71634	71642	71650	71659	71667	71675
1	1684	1692	1700	1709	1717	1725	1734	1742	1750	1759
2	1767	1775	1784	1792	1800	1809	1817	1825	1834	1842
3	1850	1858	1867	1875	1883	1892	1900	1908	1917	1925
4	1933	1941	1950	1958	1966	1975	1983	1991	1999	2008
5	2016	2024	2032	2041	2049	2057	2066	2074	2082	2090
6	2099	2107	2115	2123	2132	2140	2148	2156	2165	2173
7	2181	2189	2198	2206	2214	2222	2230	2239	2247	2255
8	2263	2272	2280	2288	2296	2304	2313	2321	2329	2337
9	2346	2354	2362	2370	2378	2387	2395	2403	2411	2419
530	72428	72436	72444	72452	72460	72469	72477	72485	72493	72501
1	2509	2518	2526	2534	2542	2550	2558	2567	2575	2583
2	2591	2599	2607	2616	2624	2632	2640	2648	2656	2665
3	2673	2681	2689	2697	2705	2713	2722	2730	2738	2746
4	2754	2762	2770	2779	2787	2795	2803	2811	2819	2827
5	2835	2843	2852	2860	2868	2876	2884	2892	2900	2908
6	2916	2925	2933	2941	2949	2957	2965	2973	2981	2989
7	2997	3006	3014	3022	3030	3038	3046	3054	3062	3070
8	3078	3086	3094	3102	3111	3119	3127	3135	3143	3151
9	3159	3167	3175	3183	3191	3199	3207	3215	3223	3231
540	73239	73247	73255	73263	73272	73280	73288	73296	73304	73312
1	3320	3328	3336	3344	3352	3360	3368	3376	3384	3392
2	3400	3408	3416	3424	3432	3440	3448	3456	3464	3472
3	3480	3488	3496	3504	3512	3520	3528	3536	3544	3552
4	3560	3568	3576	3584	3592	3600	3608	3616	3624	3632
5	3640	3648	3656	3664	3672	3679	3687	3695	3703	3711
6	3719	3727	3735	3743	3751	3759	3767	3775	3783	3791
7	3799	3807	3815	3823	3830	3838	3846	3854	3862	3870
8	3878	3886	3894	3902	3910	3918	3926	3933	3941	3949
9	3957	3965	3973	3981	3989	3997	4005	4013	4020	4028
550	74036	74044	74052	74060	74068	74076	74084	74092	74099	74107

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
550	74036	74044	74052	74060	74068	74076	74084	74092	74099	74107
1	4115	4123	4131	4139	4147	4155	4162	4170	4178	4186
2	4194	4202	4210	4218	4225	4233	4241	4249	4257	4265
3	4273	4280	4288	4296	4304	4312	4320	4327	4335	4343
4	4351	4359	4367	4374	4382	4390	4398	4406	4414	4421
5	4429	4437	4445	4453	4461	4468	4476	4484	4492	4500
6	4507	4515	4523	4531	4539	4547	4554	4562	4570	4578
7	4586	4593	4601	4609	4617	4624	4632	4640	4648	4656
8	4663	4671	4679	4687	4695	4702	4710	4718	4726	4733
9	4741	4749	4757	4764	4772	4780	4788	4796	4803	4811
560	74819	74827	74834	74842	74850	74858	74865	74873	74881	74889
1	4896	4904	4912	4920	4927	4935	4943	4950	4958	4966
2	4974	4981	4989	4997	5005	5012	5020	5028	5035	5043
3	5051	5059	5066	5074	5082	5089	5097	5105	5113	5120
4	5128	5136	5143	5151	5159	5166	5174	5182	5189	5197
5	5205	5213	5220	5228	5236	5243	5251	5259	5266	5274
6	5282	5289	5297	5305	5312	5320	5328	5335	5343	5351
7	5358	5366	5374	5381	5389	5397	5404	5412	5420	5427
8	5435	5442	5450	5458	5465	5473	5481	5488	5496	5504
9	5511	5519	5526	5534	5542	5549	5557	5565	5572	5580
570	75587	75595	75603	75610	75618	75626	75633	75641	75648	75656
1	5664	5671	5679	5686	5694	5702	5709	5717	5724	5732
2	5740	5747	5755	5762	5770	5778	5785	5793	5800	5808
3	5815	5823	5831	5838	5846	5853	5861	5868	5876	5884
4	5891	5899	5906	5914	5921	5929	5937	5944	5952	5959
5	5967	5974	5982	5989	5997	6005	6012	6020	6027	6035
6	6042	6050	6057	6065	6072	6080	6087	6095	6103	6110
7	6118	6125	6133	6140	6148	6155	6163	6170	6178	6185
8	6193	6200	6208	6215	6223	6230	6238	6245	6253	6260
9	6268	6275	6283	6290	6298	6305	6313	6320	6328	6335
580	76343	76350	76358	76365	76373	76380	76388	76395	76403	76410
1	6418	6425	6433	6440	6448	6455	6462	6470	6477	6485
2	6492	6500	6507	6515	6522	6530	6537	6545	6552	6559
3	6567	6574	6582	6589	6597	6604	6612	6619	6626	6634
4	6641	6649	6656	6664	6671	6678	6686	6693	6701	6708
5	6716	6723	6730	6738	6745	6753	6760	6768	6775	6782
6	6790	6797	6805	6812	6819	6827	6834	6842	6849	6856
7	6864	6871	6879	6886	6893	6901	6908	6916	6923	6930
8	6938	6945	6953	6960	6967	6975	6982	6989	6997	7004
9	7012	7019	7026	7034	7041	7048	7056	7063	7070	7078
590	77085	77093	77100	77107	77115	77122	77129	77137	77144	77151
1	7159	7166	7173	7181	7188	7195	7203	7210	7217	7225
2	7232	7240	7247	7254	7262	7269	7276	7283	7291	7298
3	7305	7313	7320	7327	7335	7342	7349	7357	7364	7371
4	7379	7386	7393	7401	7408	7415	7422	7430	7437	7444
5	7452	7459	7466	7474	7481	7488	7495	7503	7510	7517
6	7525	7532	7539	7546	7554	7561	7568	7576	7583	7590
7	7597	7605	7612	7619	7627	7634	7641	7648	7656	7663
8	7670	7677	7685	7692	7699	7706	7714	7721	7728	7735
9	7743	7750	7757	7764	7772	7779	7786	7793	7801	7808
600	77815	77822	77830	77837	77844	77851	77859	77866	77873	77880

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
600	77815	77822	77830	77837	77844	77851	77859	77866	77873	77880
1	7887	7895	7902	7909	7916	7924	7931	7938	7945	7952
2	7960	7967	7974	7981	7988	7996	8003	8010	8017	8025
3	8032	8039	8046	8053	8061	8068	8075	8082	8089	8097
4	8104	8111	8118	8125	8132	8140	8147	8154	8161	8168
5	8176	8183	8190	8197	8204	8211	8219	8226	8233	8240
6	8247	8254	8262	8269	8276	8283	8290	8297	8305	8312
7	8319	8326	8333	8340	8347	8355	8362	8369	8376	8383
8	8390	8398	8405	8412	8419	8426	8433	8440	8447	8455
9	8462	8469	8476	8483	8490	8497	8504	8512	8519	8526
610	78533	78540	78547	78554	78561	78569	78576	78583	78590	78597
1	8604	8611	8618	8625	8633	8640	8647	8654	8661	8668
2	8675	8682	8689	8696	8704	8711	8718	8725	8732	8739
3	8746	8753	8760	8767	8774	8781	8789	8796	8803	8810
4	8817	8824	8831	8838	8845	8852	8859	8866	8873	8880
5	8888	8895	8902	8909	8916	8923	8930	8937	8944	8951
6	8958	8965	8972	8979	8986	8993	9000	9007	9014	9021
7	9029	9036	9043	9050	9057	9064	9071	9078	9085	9092
8	9099	9106	9113	9120	9127	9134	9141	9148	9155	9162
9	9169	9176	9183	9190	9197	9204	9211	9218	9225	9232
620	79239	79246	79253	79260	79267	79274	79281	79288	79295	79302
1	9309	9316	9323	9330	9337	9344	9351	9358	9365	9372
2	9379	9386	9393	9400	9407	9414	9421	9428	9435	9442
3	9449	9456	9463	9470	9477	9484	9491	9498	9505	9511
4	9518	9525	9532	9539	9546	9553	9560	9567	9574	9581
5	9588	9595	9602	9609	9616	9623	9630	9637	9644	9650
6	9657	9664	9671	9678	9685	9692	9699	9706	9713	9720
7	9727	9734	9741	9748	9754	9761	9768	9775	9782	9789
8	9796	9803	9810	9817	9824	9831	9837	9844	9851	9858
9	9865	9872	9879	9886	9893	9900	9906	9913	9920	9927
630	79934	79941	79948	79955	79962	79969	79975	79982	79989	79996
1	80003	80010	80017	80024	80030	80037	80044	80051	80058	80065
2	0072	0079	0085	0092	0099	0106	0113	0120	0127	0134
3	0140	0147	0154	0161	0168	0175	0182	0188	0195	0202
4	0209	0216	0223	0229	0236	0243	0250	0257	0264	0271
5	0277	0284	0291	0298	0305	0312	0318	0325	0332	0339
6	0346	0353	0359	0366	0373	0380	0387	0393	0400	0407
7	0414	0421	0428	0434	0441	0448	0455	0462	0468	0475
8	0482	0489	0496	0502	0509	0516	0523	0530	0536	0543
9	0550	0557	0564	0570	0577	0584	0591	0598	0604	0611
640	80618	80625	80632	80638	80645	80652	80659	80665	80672	80679
1	0686	0693	0699	0706	0713	0720	0726	0733	0740	0747
2	0754	0760	0767	0774	0781	0787	0794	0801	0808	0814
3	0821	0828	0835	0841	0848	0855	0862	0868	0875	0882
4	0889	0895	0902	0909	0916	0922	0929	0936	0943	0949
5	0956	0963	0969	0976	0983	0990	0996	1003	1010	1017
6	1023	1030	1037	1043	1050	1057	1064	1070	1077	1084
7	1090	1097	1104	1111	1117	1124	1131	1137	1144	1151
8	1158	1164	1171	1178	1184	1191	1198	1204	1211	1218
9	1224	1231	1238	1245	1251	1258	1265	1271	1278	1285
650	81291	81298	81305	81311	81318	81325	81331	81338	81345	81351

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
650	81291	81298	81305	81311	81318	81325	81331	81338	81345	81351
1	1358	1365	1371	1378	1385	1391	1398	1405	1411	1418
2	1425	1431	1438	1445	1451	1458	1465	1471	1478	1485
3	1491	1498	1505	1511	1518	1525	1531	1538	1544	1551
4	1558	1564	1571	1578	1584	1591	1598	1604	1611	1617
5	1624	1631	1637	1644	1651	1657	1664	1671	1677	1684
6	1690	1697	1704	1710	1717	1723	1730	1737	1743	1750
7	1757	1763	1770	1776	1783	1790	1796	1803	1809	1816
8	1823	1829	1836	1842	1849	1856	1862	1869	1875	1882
9	1889	1895	1902	1908	1915	1921	1928	1935	1941	1948
660	81954	81961	81968	81974	81981	81987	81994	82000	82007	82014
1	2020	2027	2033	2040	2046	2053	2060	2066	2073	2079
2	2086	2092	2099	2105	2112	2119	2125	2132	2138	2145
3	2151	2158	2164	2171	2178	2184	2191	2197	2204	2210
4	2217	2223	2230	2236	2243	2249	2256	2263	2269	2276
5	2282	2289	2295	2302	2308	2315	2321	2328	2334	2341
6	2347	2354	2360	2367	2373	2380	2387	2393	2400	2406
7	2413	2419	2426	2432	2439	2445	2452	2458	2465	2471
8	2478	2484	2491	2497	2504	2510	2517	2523	2530	2536
9	2543	2549	2556	2562	2569	2575	2582	2588	2595	2601
670	82607	82614	82620	82627	82633	82640	82646	82653	82659	82666
1	2672	2679	2685	2692	2698	2705	2711	2718	2724	2730
2	2737	2743	2750	2756	2763	2769	2776	2782	2789	2795
3	2802	2808	2814	2821	2827	2834	2840	2847	2853	2860
4	2866	2872	2879	2885	2892	2898	2905	2911	2918	2924
5	2930	2937	2943	2950	2956	2963	2969	2975	2982	2988
6	2995	3001	3008	3014	3020	3027	3033	3040	3046	3052
7	3059	3065	3072	3078	3085	3091	3097	3104	3110	3117
8	3123	3129	3136	3142	3149	3155	3161	3168	3174	3181
9	3187	3193	3200	3206	3213	3219	3225	3232	3238	3245
680	83251	83257	83264	83270	83276	83283	83289	83296	83302	83308
1	3315	3321	3327	3334	3340	3347	3353	3359	3366	3372
2	3378	3385	3391	3398	3404	3410	3417	3423	3429	3436
3	3442	3448	3455	3461	3467	3474	3480	3487	3493	3499
4	3506	3512	3518	3525	3531	3537	3544	3550	3556	3563
5	3569	3575	3582	3588	3594	3601	3607	3613	3620	3626
6	3632	3639	3645	3651	3658	3664	3670	3677	3683	3689
7	3696	3702	3708	3715	3721	3727	3734	3740	3746	3753
8	3759	3765	3771	3778	3784	3790	3797	3803	3809	3816
9	3822	3828	3835	3841	3847	3853	3860	3866	3872	3879
690	83885	83891	83897	83904	83910	83916	83923	83929	83935	83942
1	3948	3954	3960	3967	3973	3979	3985	3992	3998	4004
2	4011	4017	4023	4029	4036	4042	4048	4055	4061	4067
3	4073	4080	4086	4092	4098	4105	4111	4117	4123	4130
4	4136	4142	4148	4155	4161	4167	4173	4180	4186	4192
5	4198	4205	4211	4217	4223	4230	4236	4242	4248	4255
6	4261	4267	4273	4280	4286	4292	4298	4305	4311	4317
7	4323	4330	4336	4342	4348	4354	4361	4367	4373	4379
8	4386	4392	4398	4404	4410	4417	4423	4429	4435	4442
9	4448	4454	4460	4466	4473	4479	4485	4491	4497	4504
700	84510	84516	84522	84528	84535	84541	84547	84553	84559	84566

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
700	84510	84516	84522	84528	84535	84541	84547	84553	84559	84566
1	4572	4578	4584	4590	4597	4603	4609	4615	4621	4628
2	4634	4640	4646	4652	4658	4665	4671	4677	4683	4689
3	4696	4702	4708	4714	4720	4726	4733	4739	4745	4751
4	4757	4763	4770	4776	4782	4788	4794	4800	4807	4813
5	4819	4825	4831	4837	4844	4850	4856	4862	4868	4874
6	4880	4887	4893	4899	4905	4911	4917	4924	4930	4936
7	4942	4948	4954	4960	4967	4973	4979	4985	4991	4997
8	5003	5009	5016	5022	5028	5034	5040	5046	5052	5058
9	5065	5071	5077	5083	5089	5095	5101	5107	5114	5120
710	85126	85132	85138	85144	85150	85156	85163	85169	85175	85181
1	5187	5193	5199	5205	5211	5217	5224	5230	5236	5242
2	5248	5254	5260	5266	5272	5278	5285	5291	5297	5303
3	5309	5315	5321	5327	5333	5339	5345	5352	5358	5364
4	5370	5376	5382	5388	5394	5400	5406	5412	5418	5425
5	5431	5437	5443	5449	5455	5461	5467	5473	5479	5485
6	5491	5497	5503	5509	5516	5522	5528	5534	5540	5546
7	5552	5558	5564	5570	5576	5582	5588	5594	5600	5606
8	5612	5618	5625	5631	5637	5643	5649	5655	5661	5667
9	5673	5679	5685	5691	5697	5703	5709	5715	5721	5727
720	85733	85739	85745	85751	85757	85763	85769	85775	85781	85788
1	5794	5800	5806	5812	5818	5824	5830	5836	5842	5848
2	5854	5860	5866	5872	5878	5884	5890	5896	5902	5908
3	5914	5920	5926	5932	5938	5944	5950	5956	5962	5968
4	5974	5980	5986	5992	5998	6004	6010	6016	6022	6028
5	6034	6040	6046	6052	6058	6064	6070	6076	6082	6088
6	6094	6100	6106	6112	6118	6124	6130	6136	6141	6147
7	6153	6159	6165	6171	6177	6183	6189	6195	6201	6207
8	6213	6219	6225	6231	6237	6243	6249	6255	6261	6267
9	6273	6279	6285	6291	6297	6303	6308	6314	6320	6326
730	86332	86338	86344	86350	86356	86362	86368	86374	86380	86386
1	6392	6398	6404	6410	6415	6421	6427	6433	6439	6445
2	6451	6457	6463	6469	6475	6481	6487	6493	6499	6504
3	6510	6516	6522	6528	6534	6540	6546	6552	6558	6564
4	6570	6576	6581	6587	6593	6599	6605	6611	6617	6623
5	6629	6635	6641	6646	6652	6658	6664	6670	6676	6682
6	6688	6694	6700	6705	6711	6717	6723	6729	6735	6741
7	6747	6753	6759	6764	6770	6776	6782	6788	6794	6800
8	6806	6812	6817	6823	6829	6835	6841	6847	6853	6859
9	6864	6870	6876	6882	6888	6894	6900	6906	6911	6917
740	86923	86929	86935	86941	86947	86953	86958	86964	86970	86976
1	6982	6988	6994	6999	7005	7011	7017	7023	7029	7035
2	7040	7046	7052	7058	7064	7070	7075	7081	7087	7093
3	7099	7105	7111	7116	7122	7128	7134	7140	7146	7151
4	7157	7163	7169	7175	7181	7186	7192	7198	7204	7210
5	7216	7221	7227	7233	7239	7245	7251	7256	7262	7268
6	7274	7280	7286	7291	7297	7303	7309	7315	7320	7326
7	7332	7338	7344	7349	7355	7361	7367	7373	7379	7384
8	7390	7396	7402	7408	7413	7419	7425	7431	7437	7442
9	7448	7454	7460	7466	7471	7477	7483	7489	7495	7500
750	87506	87512	87518	87523	87529	87535	87541	87547	87552	87558

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
750	87506	87512	87518	87523	87529	87535	87541	87547	87552	87558
1	7564	7570	7576	7581	7587	7593	7599	7604	7610	7616
2	7622	7628	7633	7639	7645	7651	7656	7662	7668	7674
3	7679	7685	7691	7697	7703	7708	7714	7720	7726	7731
4	7737	7743	7749	7754	7760	7766	7772	7777	7783	7789
5	7795	7800	7806	7812	7818	7823	7829	7835	7841	7846
6	7852	7858	7864	7869	7875	7881	7887	7892	7898	7904
7	7910	7915	7921	7927	7933	7938	7944	7950	7955	7961
8	7967	7973	7978	7984	7990	7996	8001	8007	8013	8018
9	8024	8030	8036	8041	8047	8053	8058	8064	8070	8076
760	88081	88087	88093	88098	88104	88110	88116	88121	88127	88133
1	8138	8144	8150	8156	8161	8167	8173	8178	8184	8190
2	8195	8201	8207	8213	8218	8224	8230	8235	8241	8247
3	8252	8258	8264	8270	8275	8281	8287	8292	8298	8304
4	8309	8315	8321	8326	8332	8338	8343	8349	8355	8360
5	8366	8372	8377	8383	8389	8395	8400	8406	8412	8417
6	8423	8429	8434	8440	8446	8451	8457	8463	8468	8474
7	8480	8485	8491	8497	8502	8508	8513	8519	8525	8530
8	8536	8542	8547	8553	8559	8564	8570	8576	8581	8587
9	8593	8598	8604	8610	8615	8621	8627	8632	8638	8643
770	88649	88655	88660	88666	88672	88677	88683	88689	88694	88700
1	8705	8711	8717	8722	8728	8734	8739	8745	8750	8756
2	8762	8767	8773	8779	8784	8790	8795	8801	8807	8812
3	8818	8824	8829	8835	8840	8846	8852	8857	8863	8868
4	8874	8880	8885	8891	8897	8902	8908	8913	8919	8925
5	8930	8936	8941	8947	8953	8958	8964	8969	8975	8981
6	8986	8992	8997	9003	9009	9014	9020	9025	9031	9037
7	9042	9048	9053	9059	9064	9070	9076	9081	9087	9092
8	9098	9104	9109	9115	9120	9126	9131	9137	9143	9148
9	9154	9159	9165	9170	9176	9182	9187	9193	9198	9204
780	89209	89215	89221	89226	89232	89237	89243	89248	89254	89260
1	9265	9271	9276	9282	9287	9293	9298	9304	9310	9315
2	9321	9326	9332	9337	9343	9348	9354	9360	9365	9371
3	9376	9382	9387	9393	9398	9404	9409	9415	9421	9426
4	9432	9437	9443	9448	9454	9459	9465	9470	9476	9481
5	9487	9492	9498	9504	9509	9515	9520	9526	9531	9537
6	9542	9548	9553	9559	9564	9570	9575	9581	9586	9592
7	9597	9603	9609	9614	9620	9625	9631	9636	9642	9647
8	9653	9658	9664	9669	9675	9680	9686	9691	9697	9702
9	9708	9713	9719	9724	9730	9735	9741	9746	9752	9757
790	89763	89768	89774	89779	89785	89790	89796	89801	89807	89812
1	9818	9823	9829	9834	9840	9845	9851	9856	9862	9867
2	9873	9878	9883	9889	9894	9900	9905	9911	9916	9922
3	9927	9933	9938	9944	9949	9955	9960	9966	9971	9977
4	9982	9988	9993	9998	90004	90009	90015	90020	90026	90031
5	90037	90042	90048	90053	90059	90064	90069	90075	90080	90086
6	0091	0097	0102	0108	0113	0119	0124	0129	0135	0140
7	0146	0151	0157	0162	0168	0173	0179	0184	0189	0195
8	0200	0206	0211	0217	0222	0227	0233	0238	0244	0249
9	0255	0260	0266	0271	0276	0282	0287	0293	0298	0304
800	90309	90314	90320	90325	90331	90336	90342	90347	90352	90358

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
800	90309	90314	90320	90325	90331	90336	90342	90347	90352	90358
1	0363	0369	0374	0380	0385	0390	0396	0401	0407	0412
2	0417	0423	0428	0434	0439	0445	0450	0455	0461	0466
3	0472	0477	0482	0488	0493	0499	0504	0509	0515	0520
4	0526	0531	0536	0542	0547	0553	0558	0563	0569	0574
5	0580	0585	0590	0596	0601	0607	0612	0617	0623	0628
6	0634	0639	0644	0650	0655	0660	0666	0671	0677	0682
7	0687	0693	0698	0703	0709	0714	0720	0725	0730	0736
8	0741	0747	0752	0757	0763	0768	0773	0779	0784	0789
9	0795	0800	0806	0811	0816	0822	0827	0832	0838	0843
810	90849	90854	90859	90865	90870	90875	90881	90886	90891	90897
1	0902	0907	0913	0918	0924	0929	0934	0940	0945	0950
2	0956	0961	0966	0972	0977	0982	0988	0993	0998	1004
3	1009	1014	1020	1025	1030	1036	1041	1046	1052	1057
4	1062	1068	1073	1078	1084	1089	1094	1100	1105	1110
5	1116	1121	1126	1132	1137	1142	1148	1153	1158	1164
6	1169	1174	1180	1185	1190	1196	1201	1206	1212	1217
7	1222	1228	1233	1238	1243	1249	1254	1259	1265	1270
8	1275	1281	1286	1291	1297	1302	1307	1312	1318	1323
9	1328	1334	1339	1344	1350	1355	1360	1365	1371	1376
820	91381	91387	91392	91397	91403	91408	91413	91418	91424	91429
1	1434	1440	1445	1450	1455	1461	1466	1471	1477	1482
2	1487	1492	1498	1503	1508	1514	1519	1524	1529	1535
3	1540	1545	1551	1556	1561	1566	1572	1577	1582	1587
4	1593	1598	1603	1609	1614	1619	1624	1630	1635	1640
5	1645	1651	1656	1661	1666	1672	1677	1682	1687	1693
6	1698	1703	1709	1714	1719	1724	1730	1735	1740	1745
7	1751	1756	1761	1766	1772	1777	1782	1787	1793	1798
8	1803	1808	1814	1819	1824	1829	1834	1840	1845	1850
9	1855	1861	1866	1871	1876	1882	1887	1892	1897	1903
830	91908	91913	91918	91924	91929	91934	91939	91944	91950	91955
1	1960	1965	1971	1976	1981	1986	1991	1997	2002	2007
2	2012	2018	2023	2028	2033	2038	2044	2049	2054	2059
3	2065	2070	2075	2080	2085	2091	2096	2101	2106	2111
4	2117	2122	2127	2132	2137	2143	2148	2153	2158	2163
5	2169	2174	2179	2184	2189	2195	2200	2205	2210	2215
6	2221	2226	2231	2236	2241	2247	2252	2257	2262	2267
7	2273	2278	2283	2288	2293	2298	2304	2309	2314	2319
8	2324	2330	2335	2340	2345	2350	2355	2361	2366	2371
9	2376	2381	2387	2392	2397	2402	2407	2412	2418	2423
840	92428	92433	92438	92443	92449	92454	92459	92464	92469	92474
1	2480	2485	2490	2495	2500	2505	2511	2516	2521	2526
2	2531	2536	2542	2547	2552	2557	2562	2567	2572	2578
3	2583	2588	2593	2598	2603	2609	2614	2619	2624	2629
4	2634	2639	2645	2650	2655	2660	2665	2670	2675	2681
5	2686	2691	2696	2701	2706	2711	2716	2722	2727	2732
6	2737	2742	2747	2752	2758	2763	2768	2773	2778	2783
7	2788	2793	2799	2804	2809	2814	2819	2824	2829	2834
8	2840	2845	2850	2855	2860	2865	2870	2875	2881	2886
9	2891	2896	2901	2906	2911	2916	2921	2927	2932	2937
850	92942	92947	92952	92957	92962	92967	92973	92978	92983	92988

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
850	92942	92947	92952	92957	92962	92967	92973	92978	92983	92988
1	2993	2998	3003	3008	3013	3018	3024	3029	3034	3039
2	3044	3049	3054	3059	3064	3069	3075	3080	3085	3090
3	3095	3100	3105	3110	3115	3120	3125	3131	3136	3141
4	3146	3151	3156	3161	3166	3171	3176	3181	3186	3192
5	3197	3202	3207	3212	3217	3222	3227	3232	3237	3242
6	3247	3252	3258	3263	3268	3273	3278	3283	3288	3293
7	3298	3303	3308	3313	3318	3323	3328	3334	3339	3344
8	3349	3354	3359	3364	3369	3374	3379	3384	3389	3394
9	3399	3404	3409	3414	3420	3425	3430	3435	3440	3445
860	93450	93455	93460	93465	93470	93475	93480	93485	93490	93495
1	3500	3505	3510	3515	3520	3526	3531	3536	3541	3546
2	3551	3556	3561	3566	3571	3576	3581	3586	3591	3596
3	3601	3606	3611	3616	3621	3626	3631	3636	3641	3646
4	3651	3656	3661	3666	3671	3676	3682	3687	3692	3697
5	3702	3707	3712	3717	3722	3727	3732	3737	3742	3747
6	3752	3757	3762	3767	3772	3777	3782	3787	3792	3797
7	3802	3807	3812	3817	3822	3827	3832	3837	3842	3847
8	3852	3857	3862	3867	3872	3877	3882	3887	3892	3897
9	3902	3907	3912	3917	3922	3927	3932	3937	3942	3947
870	93952	93957	93962	93967	93972	93977	93982	93987	93992	93997
1	4002	4007	4012	4017	4022	4027	4032	4037	4042	4047
2	4052	4057	4062	4067	4072	4077	4082	4086	4091	4096
3	4101	4106	4111	4116	4121	4126	4131	4136	4141	4146
4	4151	4156	4161	4166	4171	4176	4181	4186	4191	4196
5	4201	4206	4211	4216	4221	4226	4231	4236	4240	4245
6	4250	4255	4260	4265	4270	4275	4280	4285	4290	4295
7	4300	4305	4310	4315	4320	4325	4330	4335	4340	4345
8	4349	4354	4359	4364	4369	4374	4379	4384	4389	4394
9	4399	4404	4409	4414	4419	4424	4429	4433	4438	4443
880	94448	94453	94458	94463	94468	94473	94478	94483	94488	94493
1	4498	4503	4507	4512	4517	4522	4527	4532	4537	4542
2	4547	4552	4557	4562	4567	4571	4576	4581	4586	4591
3	4596	4601	4606	4611	4616	4621	4626	4630	4635	4640
4	4645	4650	4655	4660	4665	4670	4675	4680	4685	4689
5	4694	4699	4704	4709	4714	4719	4724	4729	4734	4738
6	4743	4748	4753	4758	4763	4768	4773	4778	4783	4787
7	4792	4797	4802	4807	4812	4817	4822	4827	4832	4836
8	4841	4846	4851	4856	4861	4866	4871	4876	4880	4885
9	4890	4895	4900	4905	4910	4915	4919	4924	4929	4934
890	94939	94944	94949	94954	94959	94963	94968	94973	94978	94983
1	4988	4993	4998	5002	5007	5012	5017	5022	5027	5032
2	5036	5041	5046	5051	5056	5061	5066	5071	5075	5080
3	5085	5090	5095	5100	5105	5109	5114	5119	5124	5129
4	5134	5139	5143	5148	5153	5158	5163	5168	5173	5177
5	5182	5187	5192	5197	5202	5207	5211	5216	5221	5226
6	5231	5236	5240	5245	5250	5255	5260	5265	5270	5274
7	5279	5284	5289	5294	5299	5303	5308	5313	5318	5323
8	5328	5332	5337	5342	5347	5352	5357	5361	5366	5371
9	5376	5381	5386	5390	5395	5400	5405	5410	5415	5419
900	95424	95429	95434	95439	95444	95448	95453	95458	95463	95468

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
900	95424	95429	95434	95439	95444	95448	95453	95458	95463	95468
1	5472	5477	5482	5487	5492	5497	5501	5506	5511	5516
2	5521	5525	5530	5535	5540	5545	5550	5554	5559	5564
3	5569	5574	5578	5583	5588	5593	5598	5602	5607	5612
4	5617	5622	5626	5631	5636	5641	5646	5650	5655	5660
5	5665	5670	5674	5679	5684	5689	5694	5698	5703	5708
6	5713	5718	5722	5727	5732	5737	5742	5746	5751	5756
7	5761	5766	5770	5775	5780	5785	5789	5794	5799	5804
8	5809	5813	5818	5823	5828	5832	5837	5842	5847	5852
9	5856	5861	5866	5871	5875	5880	5885	5890	5895	5899
910	95904	95909	95914	95918	95923	95928	95933	95938	95942	95947
1	5952	5957	5961	5966	5971	5976	5980	5985	5990	5995
2	5999	6004	6009	6014	6019	6023	6028	6033	6038	6042
3	6047	6052	6057	6061	6066	6071	6076	6080	6085	6090
4	6095	6099	6104	6109	6114	6118	6123	6128	6133	6137
5	6142	6147	6152	6156	6161	6166	6171	6175	6180	6185
6	6190	6194	6199	6204	6209	6213	6218	6223	6227	6232
7	6237	6242	6246	6251	6256	6261	6265	6270	6275	6280
8	6284	6289	6294	6298	6303	6308	6313	6317	6322	6327
9	6332	6336	6341	6346	6350	6355	6360	6365	6369	6374
920	96379	96384	96388	96393	96398	96402	96407	96412	96417	96421
1	6426	6431	6435	6440	6445	6450	6454	6459	6464	6468
2	6473	6478	6483	6487	6492	6497	6501	6506	6511	6515
3	6520	6525	6530	6534	6539	6544	6548	6553	6558	6562
4	6567	6572	6577	6581	6586	6591	6595	6600	6605	6609
5	6614	6619	6624	6628	6633	6638	6642	6647	6652	6656
6	6661	6666	6670	6675	6680	6685	6689	6694	6699	6703
7	6708	6713	6717	6722	6727	6731	6736	6741	6745	6750
8	6755	6759	6764	6769	6774	6778	6783	6788	6792	6797
9	6802	6806	6811	6816	6820	6825	6830	6834	6839	6844
930	96848	96853	96858	96862	96867	96872	96876	96881	96886	96890
1	6895	6900	6904	6909	6914	6918	6923	6928	6932	6937
2	6942	6946	6951	6956	6960	6965	6970	6974	6979	6984
3	6988	6993	6997	7002	7007	7011	7016	7021	7025	7030
4	7035	7039	7044	7049	7053	7058	7063	7067	7072	7077
5	7081	7086	7090	7095	7100	7104	7109	7114	7118	7123
6	7128	7132	7137	7142	7146	7151	7155	7160	7165	7169
7	7174	7179	7183	7188	7192	7197	7202	7206	7211	7216
8	7220	7225	7230	7234	7239	7243	7248	7253	7257	7262
9	7267	7271	7276	7280	7285	7290	7294	7299	7304	7308
940	97313	97317	97322	97327	97331	97336	97340	97345	97350	97354
1	7359	7364	7368	7373	7377	7382	7387	7391	7396	7400
2	7405	7410	7414	7419	7424	7428	7433	7437	7442	7447
3	7451	7456	7460	7465	7470	7474	7479	7483	7488	7493
4	7497	7502	7506	7511	7516	7520	7525	7529	7534	7539
5	7543	7548	7552	7557	7562	7566	7571	7575	7580	7585
6	7589	7594	7598	7603	7607	7612	7617	7621	7626	7630
7	7635	7640	7644	7649	7653	7658	7663	7667	7672	7676
8	7681	7685	7690	7695	7699	7704	7708	7713	7717	7722
9	7727	7731	7736	7740	7745	7749	7754	7759	7763	7768
950	97772	97777	97782	97786	97791	97795	97800	97804	97809	97813

TABLE I.—LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
950	97772	97777	97782	97786	97791	97795	97800	97804	97809	97813
1	7818	7823	7827	7832	7836	7841	7845	7850	7855	7859
2	7864	7868	7873	7877	7882	7886	7891	7896	7900	7905
3	7909	7914	7918	7923	7928	7932	7937	7941	7946	7950
4	7955	7959	7964	7968	7973	7978	7982	7987	7991	7996
5	8000	8005	8009	8014	8019	8023	8028	8032	8037	8041
6	8046	8050	8055	8059	8064	8068	8073	8078	8082	8087
7	8091	8096	8100	8105	8109	8114	8118	8123	8127	8132
8	8137	8141	8146	8150	8155	8159	8164	8168	8173	8177
9	8182	8186	8191	8195	8200	8204	8209	8214	8218	8223
960	98227	98232	98236	98241	98245	98250	98254	98259	98263	98268
1	8272	8277	8281	8286	8290	8295	8299	8304	8308	8313
2	8318	8322	8327	8331	8336	8340	8345	8349	8354	8358
3	8363	8367	8372	8376	8381	8385	8390	8394	8399	8403
4	8408	8412	8417	8421	8426	8430	8435	8439	8444	8448
5	8453	8457	8462	8466	8471	8475	8480	8484	8489	8493
6	8498	8502	8507	8511	8516	8520	8525	8529	8534	8538
7	8543	8547	8552	8556	8561	8565	8570	8574	8579	8583
8	8588	8592	8597	8601	8605	8610	8614	8619	8623	8628
9	8632	8637	8641	8646	8650	8655	8659	8664	8668	8673
970	98677	98682	98686	98691	98695	98700	98704	98709	98713	98717
1	8722	8726	8731	8735	8740	8744	8749	8753	8758	8762
2	8767	8771	8776	8780	8784	8789	8793	8798	8802	8807
3	8811	8816	8820	8825	8829	8834	8838	8843	8847	8851
4	8856	8860	8865	8869	8874	8878	8883	8887	8892	8896
5	8900	8905	8909	8914	8918	8923	8927	8932	8936	8941
6	8945	8949	8954	8958	8963	8967	8972	8976	8981	8985
7	8989	8994	8998	9003	9007	9012	9016	9021	9025	9029
8	9034	9038	9043	9047	9052	9056	9061	9065	9069	9074
9	9078	9083	9087	9092	9096	9100	9105	9109	9114	9118
980	99123	99127	99131	99136	99140	99145	99149	99154	99158	99162
1	9167	9171	9176	9180	9185	9189	9193	9198	9202	9207
2	9211	9216	9220	9224	9229	9233	9238	9242	9247	9251
3	9255	9260	9264	9269	9273	9277	9282	9286	9291	9295
4	9300	9304	9308	9313	9317	9322	9326	9330	9335	9339
5	9344	9348	9352	9357	9361	9366	9370	9374	9379	9383
6	9388	9392	9396	9401	9405	9410	9414	9419	9423	9427
7	9432	9436	9441	9445	9449	9454	9458	9463	9467	9471
8	9476	9480	9484	9489	9493	9498	9502	9506	9511	9515
9	9520	9524	9528	9533	9537	9542	9546	9550	9555	9559
990	99564	99568	99572	99577	99581	99585	99590	99594	99599	99603
1	9607	9612	9616	9621	9625	9629	9634	9638	9642	9647
2	9651	9656	9660	9664	9669	9673	9677	9682	9686	9691
3	9695	9699	9704	9708	9712	9717	9721	9726	9730	9734
4	9739	9743	9747	9752	9756	9760	9765	9769	9774	9778
5	9782	9787	9791	9795	9800	9804	9808	9813	9817	9822
6	9826	9830	9835	9839	9843	9848	9852	9856	9861	9865
7	9870	9874	9878	9883	9887	9891	9896	9900	9904	9909
8	9913	9917	9922	9926	9930	9935	9939	9944	9948	9952
9	9957	9961	9965	9970	9974	9978	9983	9987	9991	9996
1000	00000	00004	00009	00013	00017	00022	00026	00030	00035	00039

USE OF TABLES FOR LOGARITHMIC FUNCTIONS OF SMALL ANGLES

Logarithmic Sines and Tangents of Small Angles. To obtain the log sin or log tan of an angle less than 1° with greater precision than is possible by direct interpolation from the 5-place tables in this text, add the log of the number of minutes and decimals to the log sin of one minute (6.463 7261) or, add the log of the number of seconds to the log sin of one second (4.685 5749).

EXAMPLE: — Required the log sin $0^\circ 31' 43''$ ($= \log \sin 31'.717$):

$$\begin{aligned}\log \sin 1' &= 6.46373 \\ \log 31'.717 &= \underline{1.50129} \\ \log \sin 0^\circ 31' 43'' &= 7.96502\end{aligned}$$

or, using seconds,

$$\begin{aligned}\log \sin 1'' &= 4.68557 \\ \log 1903 &= \underline{3.27944} \\ \log \sin 0^\circ 31' 43'' &= 7.96501\end{aligned}$$

This method is approximate but the error is only 1 or 2 units in the fifth place for an angle of 1° ; it is even less for smaller angles. If the log sin had been obtained by direct interpolation in the 5-place table the error would be 5 or 6 in the fifth place, as shown below.

$$\begin{aligned}\log \sin 0^\circ 31' &= 7.95508 \\ \log \sin 0^\circ 32' &= \underline{7.96887} \\ \text{difference} &= .01379\end{aligned}$$

$$\frac{43}{60} \times 1379 = 988$$

$$\log \sin 0^\circ 31' 43'' = 7.95508 + 988 = 7.96496$$

To obtain the angle corresponding to a log sin subtract 6.463 7261 (or 4.685 5749 for seconds) from the given log sin; the result is the log of the number of minutes (or seconds) in the angle. For example, the angle corresponding to 7.72059 is

found as follows:

$$\begin{aligned}\log \sin x &= 7.72059 \\ \log \sin 1' &= \underline{6.46373} \\ \log x' &= 1.25686 \\ x' &= 18.066 \\ &= 18' 04''\end{aligned}$$

or, using seconds,

$$\begin{aligned}\log \sin x &= 7.72059 \\ \log \sin 1'' &= \underline{4.68557} \\ \log x'' &= 3.03502 \\ x'' &= 1084'' \\ &= 18' 04''\end{aligned}$$

The rule illustrated above also applies to a log tangent.

If greater accuracy is required than can be obtained by the preceding method, or if the angle is greater than 1° , the log sin may be found from the relation

$$n' : n = \sin n' : \sin n$$

where n is the nearest angle whose sin is given in the table and n' is the angle whose log sin is desired. The log sin is then given by

$$\log \sin n' = \log \sin n + \log n' - \log n.$$

For $\log \sin 0^\circ 31' 43''$ this latter method gives

$$\begin{aligned}\log \sin 32' &= 7.96887 \\ \log 31.717 &= 1.50129 \\ \text{co-log } 32 &= \underline{8.49485} \\ \log \sin 0^\circ 31' 43'' &= 7.96501\end{aligned}$$

which is correct to five decimal places. In the above illustration the nearest angle whose sine was given in the table was $32'$, had the angle whose sin was desired been $31' 15''$, then the log sin of $31'$ would have been used for $\log \sin n$.

For angles greater than 5° direct interpolation in the tables is sufficiently accurate.

TABLE II.—LOGARITHMIC SINES AND COSINES.

/	0°		1°		2°		/
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	—∞	10.00000	8.24186	9.99993	8.54282	9.99974	60
1	6.46373	00000	24903	99993	54642	99973	59
2	76476	00000	25609	99993	54999	99973	58
3	94085	00000	26304	99993	55354	99972	57
4	7.06579	00000	26988	99992	55705	99972	56
5	16270	00000	27661	99992	56054	99971	55
6	24188	00000	28324	99992	56400	99971	54
7	30882	00000	28977	99992	56743	99970	53
8	36682	00000	29621	99992	57084	99970	52
9	41797	00000	30255	99991	57421	99969	51
10	7.46373	10.00000	8.30879	9.99991	8.57757	9.99969	50
11	50512	00000	31495	99991	58089	99968	49
12	54291	00000	32103	99990	58419	99968	48
13	57767	00000	32702	99990	58747	99967	47
14	60985	00000	33292	99990	59072	99967	46
15	63982	00000	33875	99990	59395	99967	45
16	66784	00000	34450	99989	59715	99966	44
17	69417	9.99999	35018	99989	60033	99966	43
18	71900	99999	35578	99989	60349	99965	42
19	74248	99999	36131	99989	60662	99964	41
20	7.76475	9.99999	8.36678	9.99988	8.60973	9.99964	40
21	78594	99999	37217	99988	61282	99963	39
22	80615	99999	37750	99988	61589	99963	38
23	82545	99999	38276	99987	61894	99962	37
24	84393	99999	38796	99987	62196	99962	36
25	86166	99999	39310	99987	62497	99961	35
26	87870	99999	39818	99986	62795	99961	34
27	89509	99999	40320	99986	63091	99960	33
28	91088	99999	40816	99986	63385	99960	32
29	92612	99998	41307	99985	63678	99959	31
30	7.94084	9.99998	8.41792	9.99985	8.63968	9.99959	30
31	95508	99998	42272	99985	64256	99958	29
32	96887	99998	42746	99984	64543	99958	28
33	98223	99998	43216	99984	64827	99957	27
34	99520	99998	43680	99984	65110	99956	26
35	8.00779	99998	44139	99983	65391	99956	25
36	02002	99998	44594	99983	65670	99955	24
37	03192	99997	45044	99983	65947	99955	23
38	04350	99997	45489	99982	66223	99954	22
39	05478	99997	45930	99982	66497	99954	21
40	8.06578	9.99997	8.46366	9.99982	8.66769	9.99953	20
41	07650	99997	46799	99981	67039	99952	19
42	08696	99997	47226	99981	67308	99952	18
43	09718	99997	47650	99981	67575	99951	17
44	10717	99996	48069	99980	67841	99951	16
45	11693	99996	48485	99980	68104	99950	15
46	12647	99996	48896	99979	68367	99949	14
47	13581	99996	49304	99979	68627	99949	13
48	14495	99996	49708	99979	68886	99948	12
49	15391	99996	50108	99978	69144	99948	11
50	8.16268	9.99995	8.50504	9.99978	8.69400	9.99947	10
51	17128	99995	50897	99977	69654	99946	9
52	17971	99995	51287	99977	69907	99946	8
53	18798	99995	51673	99977	70159	99945	7
54	19610	99995	52055	99976	70409	99944	6
55	20407	99994	52434	99976	70658	99944	5
56	21189	99994	52810	99975	70905	99943	4
57	21958	99994	53183	99975	71151	99942	3
58	22713	99994	53552	99974	71395	99942	2
59	23456	99994	53919	99974	71638	99941	1
60	24186	99993	54282	99974	71880	99940	0
/	Cosine		Cosine		Cosine		/
	89°		88°		87°		

TABLE II.—LOGARITHMIC SINES AND COSINES.

/	3°		4°		5°		/
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	8.71880	9.99940	8.84358	9.99894	8.94030	9.99834	60
1	72120	99940	84539	99893	94174	99833	59
2	72359	99939	84718	99892	94317	99832	58
3	72597	99938	84897	99891	94461	99831	57
4	72834	99938	85075	99891	94603	99830	56
5	73069	99937	85252	99890	94746	99829	55
6	73303	99936	85429	99889	94887	99828	54
7	73535	99936	85605	99888	95029	99827	53
8	73767	99935	85780	99887	95170	99825	52
9	73997	99934	85955	99886	95310	99824	51
10	8.74226	9.99934	8.86128	9.99885	8.95450	9.99823	50
11	74454	99933	86301	99884	95589	99822	49
12	74680	99932	86474	99883	95728	99821	48
13	74906	99932	86645	99882	95867	99820	47
14	75130	99931	86816	99881	96005	99819	46
15	75353	99930	86987	99880	96143	99817	45
16	75575	99929	87156	99879	96280	99816	44
17	75795	99929	87325	99879	96417	99815	43
18	76015	99928	87494	99878	96553	99814	42
19	76234	99927	87661	99877	96689	99813	41
20	8.76451	9.99926	8.87829	9.99876	8.96825	9.99812	40
21	76667	99926	87995	99875	96960	99810	39
22	76883	99925	88161	99874	97095	99809	38
23	77097	99924	88326	99873	97229	99808	37
24	77310	99923	88490	99872	97363	99807	36
25	77522	99923	88654	99871	97496	99806	35
26	77733	99922	88817	99870	97629	99804	34
27	77943	99921	88980	99869	97762	99803	33
28	78152	99920	89142	99868	97894	99802	32
29	78360	99920	89304	99867	98026	99801	31
30	8.78568	9.99919	8.89464	9.99866	8.98157	9.99800	30
31	78774	99918	89625	99865	98288	99798	29
32	78979	99917	89784	99864	98419	99797	28
33	79183	99917	89943	99863	98549	99796	27
34	79386	99916	90102	99862	98679	99795	26
35	79588	99915	90260	99861	98808	99793	25
36	79789	99914	90417	99860	98937	99792	24
37	79990	99913	90574	99859	99066	99791	23
38	80189	99913	90730	99858	99194	99790	22
39	80388	99912	90885	99857	99322	99788	21
40	8.80585	9.99911	8.91040	9.99856	8.99450	9.99787	20
41	80782	99910	91195	99855	99577	99786	19
42	80978	99909	91349	99854	99704	99785	18
43	81173	99909	91502	99853	99830	99783	17
44	81367	99908	91655	99852	99956	99782	16
45	81560	99907	91807	99851	9.00082	99781	15
46	81752	99906	91959	99850	00207	99780	14
47	81944	99905	92110	99848	00332	99778	13
48	82134	99904	92261	99847	00456	99777	12
49	82324	99904	92411	99846	00581	99776	11
50	8.82513	9.99903	8.92561	9.99845	9.00704	9.99775	10
51	82701	99902	92710	99844	00828	99773	9
52	82888	99901	92859	99843	00951	99772	8
53	83075	99900	93007	99842	01074	99771	7
54	83261	99899	93154	99841	01196	99769	6
55	83446	99898	93301	99840	01318	99768	5
56	83630	99898	93448	99839	01440	99767	4
57	83813	99897	93594	99838	01561	99765	3
58	83996	99896	93740	99837	01682	99764	2
59	84177	99895	93885	99836	01803	99763	1
60	84358	99894	94030	99834	01923	99761	0
/	Cosine		Cosine		Cosine		/
	86°		85°		84°		

TABLE II.—LOGARITHMIC SINES AND COSINES.

/	6°		7°		8°		/
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.01923	9.99761	9.08589	9.99675	9.14356	9.99575	60
1	02043	99760	08692	99674	14445	99574	59
2	02163	99759	08795	99672	14535	99572	58
3	02283	99757	08897	99670	14624	99570	57
4	02402	99756	08999	99669	14714	99568	56
5	02520	99755	09101	99667	14803	99566	55
6	02639	99753	09202	99666	14891	99565	54
7	02757	99752	09304	99664	14980	99563	53
8	02874	99751	09405	99663	15069	99561	52
9	02992	99749	09506	99661	15157	99559	51
10	9.03109	9.99748	9.09606	9.99659	9.15245	9.99557	50
11	03226	99747	09707	99658	15333	99556	49
12	03342	99745	09807	99656	15421	99554	48
13	03458	99744	09907	99655	15508	99552	47
14	03574	99742	10006	99653	15596	99550	46
15	03690	99741	10106	99651	15683	99548	45
16	03805	99740	10205	99650	15770	99546	44
17	03920	99738	10304	99648	15857	99545	43
18	04034	99737	10402	99647	15944	99543	42
19	04149	99736	10501	99645	16030	99541	41
20	9.04262	9.99734	9.10599	9.99643	9.16116	9.99539	40
21	04376	99733	10697	99642	16203	99537	39
22	04490	99731	10795	99640	16289	99535	38
23	04603	99730	10893	99638	16374	99533	37
24	04715	99728	10990	99637	16460	99532	36
25	04828	99727	11087	99635	16545	99530	35
26	04940	99726	11184	99633	16631	99528	34
27	05052	99724	11281	99632	16716	99526	33
28	05164	99723	11377	99630	16801	99524	32
29	05275	99721	11474	99629	16886	99522	31
30	9.05386	9.99720	9.11570	9.99627	9.16970	9.99520	30
31	05497	99718	11666	99625	17055	99518	29
32	05607	99717	11761	99624	17139	99517	28
33	05717	99716	11857	99622	17223	99515	27
34	05827	99714	11952	99620	17307	99513	26
35	05937	99713	12047	99618	17391	99511	25
36	06046	99711	12142	99617	17474	99509	24
37	06155	99710	12236	99615	17558	99507	23
38	06264	99708	12331	99613	17641	99505	22
39	06372	99707	12425	99612	17724	99503	21
40	9.06481	9.99705	9.12519	9.99610	9.17807	9.99501	20
41	06589	99704	12612	99608	17890	99499	19
42	06696	99702	12706	99607	17973	99497	18
43	06804	99701	12799	99605	18055	99495	17
44	06911	99699	12892	99603	18137	99494	16
45	07018	99698	12985	99601	18220	99492	15
46	07124	99696	13078	99600	18302	99490	14
47	07231	99695	13171	99598	18383	99488	13
48	07337	99693	13263	99596	18465	99486	12
49	07442	99692	13355	99595	18547	99484	11
50	9.07548	9.99690	9.13447	9.99593	9.18628	9.99482	10
51	07653	99689	13539	99591	18709	99480	9
52	07758	99687	13630	99589	18790	99478	8
53	07863	99686	13722	99588	18871	99476	7
54	07968	99684	13813	99586	18952	99474	6
55	08072	99683	13904	99584	19033	99472	5
56	08176	99681	13994	99582	19113	99470	4
57	08280	99680	14085	99581	19193	99468	3
58	08383	99678	14175	99579	19273	99466	2
59	08486	99677	14266	99577	19353	99464	1
60	08589	99675	14356	99575	19433	99462	0
/	83°		82°		81°		/
	Cosine	Sine	Cosine	Sine	Cosine	Sine	

TABLE II.—LOGARITHMIC SINES AND COSINES.

/	9°		10°		11°		/
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.19453	9.99462	9.23967	9.99335	9.28060	9.99195	60
1	19513	99460	24039	99333	28125	99192	59
2	19592	99458	24110	99331	28190	99190	58
3	19672	99456	24181	99328	28254	99187	57
4	19751	99454	24253	99326	28319	99185	56
5	19830	99452	24324	99324	28384	99182	55
6	19909	99450	24395	99322	28448	99180	54
7	19988	99448	24466	99319	28512	99177	53
8	20067	99446	24536	99317	28577	99175	52
9	20145	99444	24607	99315	28641	99172	51
10	9.20223	9.99442	9.24677	9.99313	9.28705	9.99170	50
11	20302	99440	24748	99310	28769	99167	49
12	20380	99438	24818	99308	28833	99165	48
13	20458	99436	24888	99306	28896	99162	47
14	20535	99434	24958	99304	28960	99160	46
15	20613	99432	25028	99301	29024	99157	45
16	20691	99429	25098	99299	29087	99155	44
17	20768	99427	25168	99297	29150	99152	43
18	20845	99425	25237	99294	29214	99150	42
19	20922	99423	25307	99292	29277	99147	41
20	9.20999	9.99421	9.25376	9.99290	9.29340	9.99145	40
21	21076	99419	25445	99288	29403	99142	39
22	21153	99417	25514	99285	29466	99140	38
23	21229	99415	25583	99283	29529	99137	37
24	21306	99413	25652	99281	29591	99135	36
25	21382	99411	25721	99278	29654	99132	35
26	21458	99409	25790	99276	29716	99130	34
27	21534	99407	25858	99274	29779	99127	33
28	21610	99404	25927	99271	29841	99124	32
29	21685	99402	25995	99269	29903	99122	31
30	9.21761	9.99400	9.26063	9.99267	9.29966	9.99119	30
31	21836	99398	26131	99264	30028	99117	29
32	21912	99396	26199	99262	30090	99114	28
33	21987	99394	26267	99260	30151	99112	27
34	22062	99392	26335	99257	30213	99109	26
35	22137	99390	26403	99255	30275	99106	25
36	22211	99388	26470	99252	30336	99104	24
37	22286	99385	26538	99250	30398	99101	23
38	22361	99383	26605	99248	30459	99099	22
39	22435	99381	26672	99245	30521	99096	21
40	9.22509	9.99379	9.26739	9.99243	9.30582	9.99093	20
41	22583	99377	26806	99241	30643	99091	19
42	22657	99375	26873	99238	30704	99088	18
43	22731	99372	26940	99236	30765	99086	17
44	22805	99370	27007	99233	30826	99083	16
45	22878	99368	27073	99231	30887	99080	15
46	22952	99366	27140	99229	30947	99078	14
47	23025	99364	27206	99226	31008	99075	13
48	23098	99362	27273	99224	31068	99072	12
49	23171	99359	27339	99221	31129	99070	11
50	9.23244	9.99357	9.27405	9.99219	9.31189	9.99067	10
51	23317	99355	27471	99217	31250	99064	9
52	23390	99353	27537	99214	31310	99062	8
53	23462	99351	27602	99212	31370	99059	7
54	23535	99348	27668	99209	31430	99056	6
55	23607	99346	27734	99207	31490	99054	5
56	23679	99344	27799	99204	31549	99051	4
57	23752	99342	27864	99202	31609	99048	3
58	23823	99340	27930	99200	31669	99046	2
59	23895	99337	27995	99197	31728	99043	1
60	23967	99335	28060	99195	31788	99040	0
/	80°		79°		78°		/
	Cosine	Sine	Cosine	Sine	Cosine	Sine	

TABLE II.—LOGARITHMIC SINES AND COSINES.

/	12°		13°		14°		/
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.31788	9.99040	9.35209	9.98872	9.38368	9.98690	60
1	31847	99038	35263	98869	38418	98687	59
2	31907	99035	35318	98867	38469	98684	58
3	31966	99032	35373	98864	38519	98681	57
4	32025	99030	35427	98861	38570	98678	56
5	32084	99027	35481	98858	38620	98675	55
6	32143	99024	35536	98855	38670	98671	54
7	32202	99022	35590	98852	38721	98668	53
8	32261	99019	35644	98849	38771	98665	52
9	32319	99016	35698	98846	38821	98662	51
10	9.32378	9.99013	9.35752	9.98843	9.38871	9.98659	50
11	32437	99011	35806	98840	38921	98656	49
12	32495	99008	35860	98837	38971	98652	48
13	32553	99005	35914	98834	39021	98649	47
14	32612	99002	35968	98831	39071	98646	46
15	32670	99000	36022	98828	39121	98643	45
16	32728	98997	36075	98825	39170	98640	44
17	32786	98994	36129	98822	39220	98636	43
18	32844	98991	36182	98819	39270	98633	42
19	32902	98989	36236	98816	39319	98630	41
20	9.32960	9.98986	9.36289	9.98813	9.39369	9.98627	40
21	33018	98983	36342	98810	39418	98623	39
22	33075	98980	36395	98807	39467	98620	38
23	33133	98978	36449	98804	39517	98617	37
24	33190	98975	36502	98801	39566	98614	36
25	33248	98972	36555	98798	39615	98610	35
26	33305	98969	36608	98795	39664	98607	34
27	33362	98967	36660	98792	39713	98604	33
28	33420	98964	36713	98789	39762	98601	32
29	33477	98961	36766	98786	39811	98597	31
30	9.33534	9.98958	9.36819	9.98783	9.39860	9.98594	30
31	33591	98955	36871	98780	39909	98591	29
32	33647	98953	36924	98777	39958	98588	28
33	33704	98950	36976	98774	40006	98584	27
34	33761	98947	37028	98771	40055	98581	26
35	33818	98944	37081	98768	40103	98578	25
36	33874	98941	37133	98765	40152	98574	24
37	33931	98938	37185	98762	40200	98571	23
38	33987	98936	37237	98759	40249	98568	22
39	34043	98933	37289	98756	40297	98565	21
40	9.34100	9.98930	9.37341	9.98753	9.40346	9.98561	20
41	34156	98927	37393	98750	40394	98558	19
42	34212	98924	37445	98746	40442	98555	18
43	34268	98921	37497	98743	40490	98551	17
44	34324	98919	37549	98740	40538	98548	16
45	34380	98916	37600	98737	40586	98545	15
46	34436	98913	37652	98734	40634	98541	14
47	34491	98910	37703	98731	40682	98538	13
48	34547	98907	37755	98728	40730	98535	12
49	34602	98904	37806	98725	40778	98531	11
50	9.34658	9.98901	9.37858	9.98722	9.40825	9.98528	10
51	34715	98898	37909	98719	40873	98525	9
52	34769	98896	37960	98715	40921	98521	8
53	34824	98893	38011	98712	40968	98518	7
54	34879	98890	38062	98709	41016	98515	6
55	34934	98887	38113	98706	41063	98511	5
56	34989	98884	38164	98703	41111	98508	4
57	35044	98881	38215	98700	41158	98505	3
58	35099	98878	38266	98697	41205	98501	2
59	35154	98875	38317	98694	41252	98498	1
60	35209	98872	38368	98690	41300	98494	0
/	77°		76°		75°		/
	Cosine	Sine	Cosine	Sine	Cosine	Sine	

TABLE II.—LOGARITHMIC SINES AND COSINES.

/	15°		16°		17°		/
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.41300	9.98494	9.44034	9.98284	9.46594	9.98060	60
1	41347	98491	44078	98281	46635	98056	59
2	41394	98488	44122	98277	46676	98052	58
3	41441	98484	44166	98273	46717	98048	57
4	41488	98481	44210	98270	46758	98044	56
5	41535	98477	44253	98266	46800	98040	55
6	41582	98474	44297	98262	46841	98036	54
7	41628	98471	44341	98259	46882	98032	53
8	41675	98467	44385	98255	46923	98029	52
9	41722	98464	44428	98251	46964	98025	51
10	9.41768	9.98460	9.44472	9.98248	9.47005	9.98021	50
11	41815	98457	44516	98244	47045	98017	49
12	41861	98453	44559	98240	47086	98013	48
13	41908	98450	44602	98237	47127	98009	47
14	41954	98447	44646	98233	47168	98005	46
15	42001	98443	44689	98229	47209	98001	45
16	42047	98440	44733	98226	47249	97997	44
17	42093	98436	44776	98222	47290	97993	43
18	42140	98433	44819	98218	47330	97989	42
19	42186	98429	44862	98215	47371	97986	41
20	9.42232	9.98426	9.44905	9.98211	9.47411	9.97982	40
21	42278	98422	44948	98207	47452	97978	39
22	42324	98419	44992	98204	47492	97974	38
23	42370	98415	45035	98200	47533	97970	37
24	42416	98412	45077	98196	47573	97966	36
25	42461	98409	45120	98192	47613	97962	35
26	42507	98405	45163	98189	47654	97958	34
27	42553	98402	45206	98185	47694	97954	33
28	42599	98398	45249	98181	47734	97950	32
29	42644	98395	45292	98177	47774	97946	31
30	9.42690	9.98391	9.45334	9.98174	9.47814	9.97942	30
31	42735	98388	45377	98170	47854	97938	29
32	42781	98384	45419	98166	47894	97934	28
33	42826	98381	45462	98162	47934	97930	27
34	42872	98377	45504	98159	47974	97926	26
35	42917	98373	45547	98155	48014	97922	25
36	42962	98370	45589	98151	48054	97918	24
37	43008	98366	45632	98147	48094	97914	23
38	43053	98363	45674	98144	48133	97910	22
39	43098	98359	45716	98140	48173	97906	21
40	9.43143	9.98356	9.45758	9.98136	9.48213	9.97902	20
41	43188	98352	45801	98132	48252	97898	19
42	43233	98349	45843	98129	48292	97894	18
43	43278	98345	45885	98125	48332	97890	17
44	43323	98342	45927	98121	48371	97886	16
45	43367	98338	45969	98117	48411	97882	15
46	43412	98334	46011	98113	48450	97878	14
47	43457	98331	46053	98110	48490	97874	13
48	43502	98327	46095	98106	48529	97870	12
49	43546	98324	46136	98102	48568	97866	11
50	9.43591	9.98320	9.46178	9.98098	9.48607	9.97861	10
51	43635	98317	46220	98094	48647	97857	9
52	43680	98313	46262	98090	48686	97853	8
53	43724	98309	46303	98087	48725	97849	7
54	43769	98306	46345	98083	48764	97845	6
55	43813	98302	46386	98079	48803	97841	5
56	43857	98299	46428	98075	48842	97837	4
57	43901	98295	46469	98071	48881	97833	3
58	43946	98291	46511	98067	48920	97829	2
59	43990	98288	46552	98063	48959	97825	1
60	44034	98284	46594	98060	48998	97821	0
/	74°		73°		72°		/
	Cosine	Sine	Cosine	Sine	Cosine	Sine	

TABLE II.—LOGARITHMIC SINES AND COSINES.

/	18°		19°		20°		/
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.48998	9.97821	9.51264	9.97567	9.53405	9.97299	60
1	49037	97817	51301	97563	53440	97294	59
2	49076	97812	51338	97558	53475	97289	58
3	49115	97808	51374	97554	53509	97285	57
4	49153	97804	51411	97550	53544	97280	56
5	49192	97800	51447	97545	53578	97276	55
6	49231	97796	51484	97541	53613	97271	54
7	49269	97792	51520	97536	53647	97266	53
8	49308	97788	51557	97532	53682	97262	52
9	49347	97784	51593	97528	53716	97257	51
10	9.49385	9.97779	9.51629	9.97523	9.53751	9.97252	50
11	49424	97775	51666	97519	53785	97248	49
12	49462	97771	51702	97515	53819	97243	48
13	49500	97767	51738	97510	53854	97238	47
14	49539	97763	51774	97506	53888	97234	46
15	49577	97759	51811	97501	53922	97229	45
16	49615	97754	51847	97497	53957	97224	44
17	49654	97750	51883	97492	53991	97220	43
18	49692	97746	51919	97488	54025	97215	42
19	49730	97742	51955	97484	54059	97210	41
20	9.49768	9.97738	9.51991	9.97479	9.54093	9.97206	40
21	49806	97734	52027	97475	54127	97201	39
22	49844	97729	52063	97470	54161	97196	38
23	49882	97725	52099	97466	54195	97192	37
24	49920	97721	52135	97461	54229	97187	36
25	49958	97717	52171	97457	54263	97182	35
26	49996	97713	52207	97453	54297	97178	34
27	50034	97708	52242	97448	54331	97173	33
28	50072	97704	52278	97444	54365	97168	32
29	50110	97700	52314	97439	54399	97163	31
30	9.50148	9.97696	9.52350	9.97435	9.54433	9.97159	30
31	50185	97691	52385	97430	54466	97154	29
32	50223	97687	52421	97426	54500	97149	28
33	50261	97683	52456	97421	54534	97145	27
34	50298	97679	52492	97417	54567	97140	26
35	50336	97674	52527	97412	54601	97135	25
36	50374	97670	52563	97408	54635	97130	24
37	50411	97666	52598	97403	54668	97126	23
38	50449	97662	52634	97399	54702	97121	22
39	50486	97657	52669	97394	54735	97116	21
40	9.50523	9.97653	9.52705	9.97390	9.54769	9.97111	20
41	50561	97649	52740	97385	54802	97107	19
42	50598	97645	52775	97381	54836	97102	18
43	50635	97640	52811	97376	54869	97097	17
44	50673	97636	52846	97372	54903	97092	16
45	50710	97632	52881	97367	54936	97087	15
46	50747	97628	52916	97363	54969	97083	14
47	50784	97623	52951	97358	55003	97078	13
48	50821	97619	52986	97353	55036	97073	12
49	50858	97615	53021	97349	55069	97068	11
50	9.50896	9.97610	9.53056	9.97344	9.55102	9.97063	10
51	50933	97606	53092	97340	55136	97059	9
52	50970	97602	53126	97335	55169	97054	8
53	51007	97597	53161	97331	55202	97049	7
54	51043	97593	53196	97326	55235	97044	6
55	51080	97589	53231	97322	55268	97039	5
56	51117	97584	53266	97317	55301	97035	4
57	51154	97580	53301	97312	55334	97030	3
58	51191	97576	53336	97308	55367	97025	2
59	51227	97571	53370	97303	55400	97020	1
60	51264	97567	53405	97299	55433	97015	0
/	71°		70°		69°		/
	Cosine	Sine	Cosine	Sine	Cosine	Sine	

TABLE II.—LOGARITHMIC SINES AND COSINES.

/	21°		22°		23°		/
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.55433	9.97015	9.57358	9.96717	9.59188	9.96403	60
1	55466	97010	57389	96711	59218	96397	59
2	55499	97005	57420	96706	59247	96392	58
3	55532	97001	57451	96701	59277	96387	57
4	55564	96996	57482	96696	59307	96381	56
5	55597	96991	57514	96691	59336	96376	55
6	55630	96986	57545	96686	59366	96370	54
7	55663	96981	57576	96681	59396	96365	53
8	55695	96976	57607	96676	59425	96360	52
9	55728	96971	57638	96670	59455	96354	51
10	9.55761	9.96966	9.57669	9.96665	9.59484	9.96349	50
11	55793	96962	57700	96660	59514	96343	49
12	55826	96957	57731	96655	59543	96338	48
13	55858	96952	57762	96650	59573	96333	47
14	55891	96947	57793	96645	59602	96327	46
15	55923	96942	57824	96640	59632	96322	45
16	55956	96937	57855	96634	59661	96316	44
17	55988	96932	57885	96629	59690	96311	43
18	56021	96927	57916	96624	59720	96305	42
19	56053	96922	57947	96619	59749	96300	41
20	9.56085	9.96917	9.57978	9.96614	9.59778	9.96294	40
21	56118	96912	58008	96608	59808	96289	39
22	56150	96907	58039	96603	59837	96284	38
23	56182	96903	58070	96598	59866	96278	37
24	56215	96898	58101	96593	59895	96273	36
25	56247	96893	58131	96588	59924	96267	35
26	56279	96888	58162	96582	59954	96262	34
27	56311	96883	58192	96577	59983	96256	33
28	56343	96878	58223	96572	60012	96251	32
29	56375	96873	58253	96567	60041	96245	31
30	9.56408	9.96868	9.58284	9.96562	9.60070	9.96240	30
31	56440	96863	58314	96556	60099	96234	29
32	56472	96858	58345	96551	60128	96229	28
33	56504	96853	58375	96546	60157	96223	27
34	56536	96848	58406	96541	60186	96218	26
35	56568	96843	58436	96535	60215	96212	25
36	56599	96838	58467	96530	60244	96207	24
37	56631	96833	58497	96525	60273	96201	23
38	56663	96828	58527	96520	60302	96196	22
39	56695	96823	58557	96514	60331	96190	21
40	9.56727	9.96818	9.58588	9.96509	9.60359	9.96185	20
41	56759	96813	58618	96504	60388	96179	19
42	56790	96808	58648	96498	60417	96174	18
43	56822	96803	58678	96493	60446	96168	17
44	56854	96798	58709	96488	60474	96162	16
45	56886	96793	58739	96483	60503	96157	15
46	56917	96788	58769	96477	60532	96151	14
47	56949	96783	58799	96472	60561	96146	13
48	56980	96778	58829	96467	60589	96140	12
49	57012	96772	58859	96461	60618	96135	11
50	9.57044	9.96767	9.58889	9.96456	9.60646	9.96129	10
51	57075	96762	58919	96451	60675	96123	9
52	57107	96757	58949	96445	60704	96118	8
53	57138	96752	58979	96440	60732	96112	7
54	57169	96747	59009	96435	60761	96107	6
55	57201	96742	59039	96429	60789	96101	5
56	57232	96737	59069	96424	60818	96095	4
57	57264	96732	59098	96419	60846	96090	3
58	57295	96727	59128	96413	60875	96084	2
59	57326	96722	59158	96408	60903	96079	1
60	57358	96717	59188	96403	60931	96073	0
/	68°		67°		66°		/
	Cosine	Sine	Cosine	Sine	Cosine	Sine	

TABLE II.—LOGARITHMIC SINES AND COSINES.

/	24°		25°		26°		/
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.60931	9.96073	9.62595	9.95728	9.64184	9.95366	60
1	60960	96067	62622	95722	64210	95360	59
2	60988	96062	62649	95716	64236	95354	58
3	61016	96056	62676	95710	64262	95348	57
4	61045	96050	62703	95704	64288	95341	56
5	61073	96045	62730	95698	64313	95335	55
6	61101	96039	62757	95692	64339	95329	54
7	61129	96034	62784	95686	64365	95323	53
8	61158	96028	62811	95680	64391	95317	52
9	61186	96022	62838	95674	64417	95310	51
10	9.61214	9.96017	9.62865	9.95668	9.64442	9.95304	50
11	61242	96011	62892	95663	64468	95298	49
12	61270	96005	62918	95657	64494	95292	48
13	61298	96000	62945	95651	64519	95286	47
14	61326	95994	62972	95645	64545	95279	46
15	61354	95988	62999	95639	64571	95273	45
16	61382	95982	63026	95633	64596	95267	44
17	61411	95977	63052	95627	64622	95261	43
18	61438	95971	63079	95621	64647	95254	42
19	61466	95965	63106	95615	64673	95248	41
20	9.61494	9.95960	9.63133	9.95609	9.64698	9.95242	40
21	61522	95954	63159	95603	64724	95236	39
22	61550	95948	63186	95597	64749	95229	38
23	61578	95942	63213	95591	64775	95223	37
24	61606	95937	63239	95585	64800	95217	36
25	61634	95931	63266	95579	64826	95211	35
26	61662	95925	63292	95573	64851	95204	34
27	61689	95920	63319	95567	64877	95198	33
28	61717	95914	63345	95561	64902	95192	32
29	61745	95908	63372	95555	64927	95185	31
30	9.61773	9.95902	9.63398	9.95549	9.64953	9.95179	30
31	61800	95897	63425	95543	64978	95173	29
32	61828	95891	63451	95537	65003	95167	28
33	61856	95885	63478	95531	65029	95160	27
34	61883	95879	63504	95525	65054	95154	26
35	61911	95873	63531	95519	65079	95148	25
36	61939	95868	63557	95513	65104	95141	24
37	61966	95862	63583	95507	65130	95135	23
38	61994	95856	63610	95500	65155	95129	22
39	62021	95850	63636	95494	65180	95122	21
40	9.62049	9.95844	9.63662	9.95488	9.65205	9.95116	20
41	62076	95839	63689	95482	65230	95110	19
42	62104	95833	63715	95476	65255	95103	18
43	62131	95827	63741	95470	65281	95097	17
44	62159	95821	63767	95464	65306	95090	16
45	62186	95815	63794	95458	65331	95084	15
46	62214	95810	63820	95452	65356	95078	14
47	62241	95804	63846	95446	65381	95071	13
48	62268	95798	63872	95440	65406	95065	12
49	62296	95792	63898	95434	65431	95059	11
50	9.62323	9.95786	9.63924	9.95427	9.65456	9.95052	10
51	62350	95780	63950	95421	65481	95046	9
52	62377	95775	63976	95415	65506	95039	8
53	62405	95769	64002	95409	65531	95033	7
54	62432	95763	64028	95403	65556	95027	6
55	62459	95757	64054	95397	65580	95020	5
56	62486	95751	64080	95391	65605	95014	4
57	62513	95745	64106	95384	65630	95007	3
58	62541	95739	64132	95378	65655	95001	2
59	62568	95733	64158	95372	65680	94995	1
60	62595	95728	64184	95366	65705	94988	0
/	65°		64°		63°		/
	Cosine	Sine	Cosine	Sine	Cosine	Sine	

TABLE II.—LOGARITHMIC SINES AND COSINES.

/	27°		28°		29°		/
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.65705	9.94988	9.67161	9.94593	9.68557	9.94182	60
1	65729	94982	67185	94587	68580	94175	59
2	65754	94975	67208	94580	68603	94168	58
3	65779	94969	67232	94573	68625	94161	57
4	65804	94962	67256	94567	68648	94154	56
5	65828	94956	67280	94560	68671	94147	55
6	65853	94949	67303	94553	68694	94140	54
7	65878	94943	67327	94546	68716	94133	53
8	65902	94936	67350	94540	68739	94126	52
9	65927	94930	67374	94533	68762	94119	51
10	9.65952	9.94923	9.67398	9.94526	9.68784	9.94112	50
11	65976	94917	67421	94519	68807	94105	49
12	66001	94911	67445	94513	68829	94098	48
13	66025	94904	67468	94506	68852	94090	47
14	66050	94898	67492	94499	68875	94083	46
15	66075	94891	67515	94492	68897	94076	45
16	66099	94885	67539	94485	68920	94069	44
17	66124	94878	67562	94479	68942	94062	43
18	66148	94871	67586	94472	68965	94055	42
19	66173	94865	67609	94465	68987	94048	41
20	9.66197	9.94858	9.67633	9.94458	9.69010	9.94041	40
21	66221	94852	67656	94451	69032	94034	39
22	66246	94845	67680	94445	69055	94027	38
23	66270	94839	67703	94438	69077	94020	37
24	66295	94832	67726	94431	69100	94012	36
25	66319	94826	67750	94424	69122	94005	35
26	66343	94819	67773	94417	69144	93998	34
27	66368	94813	67796	94410	69167	93991	33
28	66392	94806	67820	94404	69189	93984	32
29	66416	94799	67843	94397	69212	93977	31
30	9.66441	9.94793	9.67866	9.94390	9.69234	9.93970	30
31	66465	94786	67890	94383	69256	93963	29
32	66489	94780	67913	94376	69279	93955	28
33	66513	94773	67936	94369	69301	93948	27
34	66537	94767	67959	94362	69323	93941	26
35	66562	94760	67982	94355	69345	93934	25
36	66586	94753	68006	94349	69368	93927	24
37	66610	94747	68029	94342	69390	93920	23
38	66634	94740	68052	94335	69412	93912	22
39	66658	94734	68075	94328	69434	93905	21
40	9.66682	9.94727	9.68098	9.94321	9.69456	9.93898	20
41	66706	94720	68121	94314	69479	93891	19
42	66731	94714	68144	94307	69501	93884	18
43	66755	94707	68167	94300	69523	93876	17
44	66779	94700	68190	94293	69545	93869	16
45	66803	94694	68213	94286	69567	93862	15
46	66827	94687	68237	94279	69589	93855	14
47	66851	94680	68260	94273	69611	93847	13
48	66875	94674	68283	94266	69633	93840	12
49	66899	94667	68305	94259	69655	93833	11
50	9.66922	9.94660	9.68328	9.94252	9.69677	9.93826	10
51	66946	94654	68351	94245	69699	93819	9
52	66970	94647	68374	94238	69721	93811	8
53	66994	94640	68397	94231	69743	93804	7
54	67018	94634	68420	94224	69765	93797	6
55	67042	94627	68443	94217	69787	93789	5
56	67066	94620	68466	94210	69809	93782	4
57	67090	94614	68489	94203	69831	93775	3
58	67113	94607	68512	94196	69853	93768	2
59	67137	94600	68534	94189	69875	93760	1
60	67161	94593	68557	94182	69897	93753	0
/	Cosine		Cosine		Cosine		/
	62°		61°		60°		

TABLE II.—LOGARITHMIC SINES AND COSINES.

/	30°		31°		32°		/
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.69897	9.93753	9.71184	9.93307	9.72421	9.92842	60
1	69919	93746	71205	93299	72441	92834	59
2	69941	93738	71226	93291	72461	92826	58
3	69963	93731	71247	93284	72482	92818	57
4	69984	93724	71268	93276	72502	92810	56
5	70006	93717	71289	93269	72522	92803	55
6	70028	93709	71310	93261	72542	92796	54
7	70050	93702	71331	93253	72562	92787	53
8	70072	93695	71352	93246	72582	92779	52
9	70093	93687	71373	93238	72602	92771	51
10	9.70115	9.93680	9.71393	9.93230	9.72622	9.92763	50
11	70137	93673	71414	93223	72643	92755	49
12	70159	93665	71435	93215	72663	92747	48
13	70180	93658	71456	93207	72683	92739	47
14	70202	93650	71477	93200	72703	92731	46
15	70224	93643	71498	93192	72723	92723	45
16	70245	93636	71519	93184	72743	92715	44
17	70267	93628	71539	93177	72763	92707	43
18	70288	93621	71560	93169	72783	92699	42
19	70310	93614	71581	93161	72803	92691	41
20	9.70332	9.93606	9.71602	9.93154	9.72823	9.92683	40
21	70353	93599	71622	93146	72843	92675	39
22	70375	93591	71643	93138	72863	92667	38
23	70396	93584	71664	93131	72883	92659	37
24	70418	93577	71685	93123	72902	92651	36
25	70439	93569	71705	93115	72922	92643	35
26	70461	93562	71726	93108	72942	92635	34
27	70482	93554	71747	93100	72962	92627	33
28	70504	93547	71767	93092	72982	92619	32
29	70525	93539	71788	93084	73002	92611	31
30	9.70547	9.93532	9.71809	9.93077	9.73022	9.92603	30
31	70568	93525	71829	93069	73041	92595	29
32	70590	93517	71850	93061	73061	92587	28
33	70611	93510	71870	93053	73081	92579	27
34	70633	93502	71891	93046	73101	92571	26
35	70654	93495	71911	93038	73121	92563	25
36	70675	93487	71932	93030	73140	92555	24
37	70697	93480	71952	93022	73160	92546	23
38	70718	93472	71973	93014	73180	92538	22
39	70739	93465	71994	93007	73200	92530	21
40	9.70761	9.93457	9.72014	9.92999	9.73219	9.92522	20
41	70782	93450	72034	92991	73239	92514	19
42	70803	93442	72055	92983	73259	92506	18
43	70824	93435	72075	92976	73278	92498	17
44	70846	93427	72096	92968	73298	92490	16
45	70867	93420	72116	92960	73318	92482	15
46	70888	93412	72137	92952	73337	92473	14
47	70909	93405	72157	92944	73357	92465	13
48	70931	93397	72177	92936	73377	92457	12
49	70952	93390	72198	92929	73396	92449	11
50	9.70973	9.93382	9.72218	9.92921	9.73416	9.92441	10
51	70994	93375	72238	92913	73435	92433	9
52	71015	93367	72259	92905	73455	92425	8
53	71036	93360	72279	92897	73474	92416	7
54	71058	93352	72299	92889	73494	92408	6
55	71079	93344	72320	92881	73513	92400	5
56	71100	93337	72340	92874	73533	92392	4
57	71121	93329	72360	92866	73552	92384	3
58	71142	93322	72381	92858	73572	92376	2
59	71163	93314	72401	92850	73591	92367	1
60	71184	93307	72421	92842	73611	92359	0
/	59°		58°		57°		/
	Cosine	Sine	Cosine	Sine	Cosine	Sine	

TABLE II.—LOGARITHMIC SINES AND COSINES.

/	36°		37°		38°		/
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.76922	9.90796	9.77946	9.90235	9.78934	9.89653	60
1	76939	90787	77963	90225	78950	89643	59
2	76957	90777	77980	90216	78967	89633	58
3	76974	90768	77997	90206	78983	89624	57
4	76991	90759	78013	90197	78999	89614	56
5	77009	90750	78030	90187	79015	89604	55
6	77026	90741	78047	90178	79031	89594	54
7	77043	90731	78063	90168	79047	89584	53
8	77061	90722	78080	90159	79063	89574	52
9	77078	90713	78097	90149	79079	89564	51
10	9.77095	9.90704	9.78113	9.90139	9.79095	9.89554	50
11	77112	90694	78130	90130	79111	89544	49
12	77130	90685	78147	90120	79128	89534	48
13	77147	90676	78163	90111	79144	89524	47
14	77164	90667	78180	90101	79160	89514	46
15	77181	90657	78197	90091	79176	89504	45
16	77199	90648	78213	90082	79192	89495	44
17	77216	90639	78230	90072	79208	89485	43
18	77233	90630	78246	90063	79224	89475	42
19	77250	90620	78263	90053	79240	89465	41
20	9.77268	9.90611	9.78280	9.90043	9.79256	9.89455	40
21	77285	90602	78296	90034	79272	89445	39
22	77302	90592	78313	90024	79288	89435	38
23	77319	90583	78329	90014	79304	89425	37
24	77336	90574	78346	90005	79319	89415	36
25	77353	90565	78362	89995	79335	89405	35
26	77370	90555	78379	89985	79351	89395	34
27	77387	90546	78395	89976	79367	89385	33
28	77405	90537	78412	89966	79383	89375	32
29	77422	90527	78428	89956	79399	89364	31
30	9.77439	9.90518	9.78445	9.89947	9.79415	9.89354	30
31	77456	90509	78461	89937	79431	89344	29
32	77473	90499	78478	89927	79447	89334	28
33	77490	90490	78494	89918	79463	89324	27
34	77507	90480	78510	89908	79478	89314	26
35	77524	90471	78527	89898	79494	89304	25
36	77541	90462	78543	89888	79510	89294	24
37	77558	90452	78560	89879	79526	89284	23
38	77575	90443	78576	89869	79542	89274	22
39	77592	90434	78592	89859	79558	89264	21
40	9.77609	9.90424	9.78609	9.89849	9.79573	9.89254	20
41	77626	90415	78625	89840	79589	89244	19
42	77643	90405	78642	89830	79605	89233	18
43	77660	90396	78658	89820	79621	89223	17
44	77677	90386	78674	89810	79636	89213	16
45	77694	90377	78691	89801	79652	89203	15
46	77711	90368	78707	89791	79668	89193	14
47	77728	90358	78723	89781	79684	89183	13
48	77744	90349	78739	89771	79699	89173	12
49	77761	90339	78756	89761	79715	89162	11
50	9.77778	9.90330	9.78772	9.89752	9.79731	9.89152	10
51	77795	90320	78788	89742	79746	89142	9
52	77812	90311	78805	89732	79762	89132	8
53	77829	90301	78821	89722	79778	89122	7
54	77846	90292	78837	89712	79793	89112	6
55	77862	90282	78853	89702	79809	89101	5
56	77879	90273	78869	89693	79825	89091	4
57	77896	90263	78886	89683	79840	89081	3
58	77913	90254	78902	89673	79856	89071	2
59	77930	90244	78918	89663	79872	89060	1
60	77946	90235	78934	89653	79887	89050	0
/	53°		52°		51°		/
	Cosine	Sine	Cosine	Sine	Cosine	Sine	

TABLE II.—LOGARITHMIC SINES AND COSINES.

/	39°		40°		41°		/
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.79887	9.89050	9.80807	9.88425	9.81694	9.87778	60
1	79903	89040	80822	88415	81709	87767	59
2	79918	89030	80837	88404	81723	87756	58
3	79934	89020	80852	88394	81738	87745	57
4	79950	89009	80867	88383	81752	87734	56
5	79965	88999	80882	88372	81767	87723	55
6	79981	88989	80897	88362	81781	87712	54
7	79996	88978	80912	88351	81796	87701	53
8	80012	88968	80927	88340	81810	87690	52
9	80027	88958	80942	88330	81825	87679	51
10	9.80043	9.88948	9.80957	9.88319	9.81839	9.87668	50
11	80058	88937	80972	88308	81854	87657	49
12	80074	88927	80987	88298	81868	87646	48
13	80089	88917	81002	88287	81882	87635	47
14	80105	88906	81017	88276	81897	87624	46
15	80120	88896	81032	88266	81911	87613	45
16	80136	88886	81047	88255	81926	87601	44
17	80151	88875	81061	88244	81940	87590	43
18	80166	88865	81076	88234	81955	87579	42
19	80182	88855	81091	88223	81969	87568	41
20	9.80197	9.88844	9.81106	9.88212	9.81983	9.87557	40
21	80213	88834	81121	88201	81998	87546	39
22	80228	88824	81136	88191	82012	87535	38
23	80244	88813	81151	88180	82026	87524	37
24	80259	88803	81166	88169	82041	87513	36
25	80274	88793	81180	88158	82055	87501	35
26	80290	88782	81195	88148	82069	87490	34
27	80305	88772	81210	88137	82084	87479	33
28	80320	88761	81225	88126	82098	87468	32
29	80336	88751	81240	88115	82112	87457	31
30	9.80351	9.88741	9.81254	9.88105	9.82126	9.87446	30
31	80366	88730	81269	88094	82141	87434	29
32	80382	88720	81284	88083	82155	87423	28
33	80397	88709	81299	88072	82169	87412	27
34	80412	88699	81314	88061	82184	87401	26
35	80428	88688	81328	88051	82198	87390	25
36	80443	88678	81343	88040	82212	87378	24
37	80458	88668	81358	88029	82226	87367	23
38	80473	88657	81372	88018	82240	87356	22
39	80489	88647	81387	88007	82255	87345	21
40	9.80504	9.88636	9.81402	9.87996	9.82269	9.87334	20
41	80519	88626	81417	87985	82283	87322	19
42	80534	88615	81431	87975	82297	87311	18
43	80550	88605	81446	87964	82311	87300	17
44	80565	88594	81461	87953	82326	87288	16
45	80580	88584	81475	87942	82340	87277	15
46	80595	88573	81490	87931	82354	87266	14
47	80610	88563	81505	87920	82368	87255	13
48	80625	88552	81519	87909	82382	87243	12
49	80641	88542	81534	87898	82396	87232	11
50	9.80656	9.88531	9.81549	9.87887	9.82410	9.87221	10
51	80671	88521	81563	87877	82424	87209	9
52	80686	88510	81578	87866	82439	87198	8
53	80701	88499	81592	87855	82453	87187	7
54	80716	88489	81607	87844	82467	87175	6
55	80731	88478	81622	87833	82481	87164	5
56	80746	88468	81636	87822	82495	87153	4
57	80762	88457	81651	87811	82509	87141	3
58	80777	88447	81665	87800	82523	87130	2
59	80792	88436	81680	87789	82537	87119	1
60	80807	88425	81694	87778	82551	87107	0
/	50°		49°		48°		/
	Cosine	Sine	Cosine	Sine	Cosine	Sine	

TABLE II.—LOGARITHMIC SINES AND COSINES.

/	42°		43°		44°		/
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.82551	9.87107	9.83378	9.86413	9.84177	9.85693	60
1	82565	87096	83392	86401	84190	85681	59
2	82579	87085	83405	86389	84203	85669	58
3	82593	87073	83419	86377	84216	85657	57
4	82607	87062	83432	86366	84229	85645	56
5	82621	87050	83446	86354	84242	85632	55
6	82635	87039	83459	86342	84255	85620	54
7	82649	87028	83473	86330	84269	85608	53
8	82663	87016	83486	86318	84282	85596	52
9	82677	87005	83500	86306	84295	85583	51
10	9.82691	9.86993	9.83513	9.86295	9.84308	9.85571	50
11	82705	86982	83527	86283	84321	85559	49
12	82719	86970	83540	86271	84334	85547	48
13	82733	86959	83554	86259	84347	85534	47
14	82747	86947	83567	86247	84360	85522	46
15	82761	86936	83581	86235	84373	85510	45
16	82775	86924	83594	86223	84385	85497	44
17	82788	86913	83608	86211	84398	85485	43
18	82802	86902	83621	86200	84411	85473	42
19	82816	86890	83634	86188	84424	85460	41
20	9.82830	9.86879	9.83648	9.86176	9.84437	9.85448	40
21	82844	86867	83661	86164	84450	85436	39
22	82858	86855	83674	86152	84463	85423	38
23	82872	86844	83688	86140	84476	85411	37
24	82885	86832	83701	86128	84489	85399	36
25	82899	86821	83715	86116	84502	85386	35
26	82913	86809	83728	86104	84515	85374	34
27	82927	86798	83741	86092	84528	85361	33
28	82941	86786	83755	86080	84540	85349	32
29	82955	86775	83768	86068	84553	85337	31
30	9.82968	9.86763	9.83781	9.86056	9.84566	9.85324	30
31	82982	86752	83795	86044	84579	85312	29
32	82996	86740	83808	86032	84592	85299	28
33	83010	86728	83821	86020	84605	85287	27
34	83023	86717	83834	86008	84618	85274	26
35	83037	86705	83848	85996	84630	85262	25
36	83051	86694	83861	85984	84643	85250	24
37	83065	86682	83874	85972	84656	85237	23
38	83078	86670	83887	85960	84669	85225	22
39	83092	86659	83901	85948	84682	85212	21
40	9.83106	9.86647	9.83914	9.85936	9.84694	9.85200	20
41	83120	86635	83927	85924	84707	85187	19
42	83133	86624	83940	85912	84720	85175	18
43	83147	86612	83954	85900	84733	85162	17
44	83161	86600	83967	85888	84745	85150	16
45	83174	86589	83980	85876	84758	85137	15
46	83188	86577	83993	85864	84771	85125	14
47	83202	86565	84006	85851	84784	85112	13
48	83215	86554	84020	85839	84796	85100	12
49	83229	86542	84033	85827	84809	85087	11
50	9.83242	9.86530	9.84046	9.85815	9.84822	9.85074	10
51	83256	86518	84059	85803	84835	85062	9
52	83270	86507	84072	85791	84847	85049	8
53	83283	86495	84085	85779	84860	85037	7
54	83297	86483	84098	85766	84873	85024	6
55	83310	86472	84112	85754	84885	85012	5
56	83324	86460	84125	85742	84898	84999	4
57	83338	86448	84138	85730	84911	84986	3
58	83351	86436	84151	85718	84923	84974	2
59	83365	86425	84164	85706	84936	84961	1
60	83378	86413	84177	85693	84949	84949	0
/	47°		46°		45°		/
	Cosine	Sine	Cosine	Sine	Cosine	Sine	

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	0°		1°		2°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	—∞	∞	8.24192	11.75808	8.54308	11.45692	60
1	6.46373	13.53627	24910	75090	54669	45331	59
2	76476	23524	25616	74384	55027	44973	58
3	94085	05915	26312	73688	55382	44618	57
4	7.06579	12.93421	26996	73004	55734	44266	56
5	16270	83730	27669	72331	56083	43917	55
6	24188	75812	28332	71668	56429	43571	54
7	30882	69118	28986	71014	56773	43227	53
8	36682	63318	29629	70371	57114	42886	52
9	41797	58203	30263	69737	57452	42548	51
10	7.46373	12.53627	8.30888	11.69112	8.57788	11.42212	50
11	50512	49488	31505	68495	58121	41879	49
12	54291	45709	32112	67888	58451	41549	48
13	57767	42233	32711	67289	58779	41221	47
14	60986	39014	33302	66698	59105	40895	46
15	63982	36018	33886	66114	59428	40572	45
16	66785	33215	34461	65539	59749	40251	44
17	69418	30582	35029	64971	60068	39932	43
18	71900	28100	35590	64410	60384	39616	42
19	74248	25752	36143	63857	60698	39302	41
20	7.76476	12.23524	8.36689	11.63311	8.61009	11.38991	40
21	78595	21405	37229	62771	61319	38681	39
22	80615	19385	37762	62238	61626	38374	38
23	82546	17454	38289	61711	61931	38069	37
24	84394	15606	38809	61191	62234	37766	36
25	86167	13833	39323	60677	62535	37465	35
26	87871	12129	39832	60168	62834	37166	34
27	89510	10490	40334	59666	63131	36869	33
28	91089	08911	40830	59170	63426	36574	32
29	92613	07387	41321	58679	63718	36282	31
30	7.94086	12.05914	8.41807	11.58193	8.64009	11.35991	30
31	95510	04490	42287	57715	64298	35702	29
32	96889	03111	42762	57238	64585	35415	28
33	98225	01775	43232	56768	64870	35130	27
34	99522	00478	43696	56304	65154	34846	26
35	8.00781	11.99219	44156	55844	65435	34565	25
36	02004	97996	44611	55389	65715	34285	24
37	03194	96806	45061	54939	65993	34007	23
38	04353	95647	45507	54493	66269	33731	22
39	05481	94519	45948	54052	66543	33457	21
40	8.06581	11.93419	8.46385	11.53615	8.66816	11.33184	20
41	07653	92347	46817	53183	67087	32913	19
42	08700	91300	47245	52755	67356	32644	18
43	09722	90278	47669	52331	67624	32376	17
44	10720	89280	48089	51911	67890	32110	16
45	11696	88304	48505	51495	68154	31846	15
46	12651	87349	48917	51083	68417	31583	14
47	13585	86415	49325	50675	68678	31322	13
48	14500	85500	49729	50271	68938	31062	12
49	15395	84605	50130	49870	69196	30804	11
50	8.16273	11.83727	8.50527	11.49473	8.69453	11.30547	10
51	17133	82867	50920	49080	69708	30292	9
52	17976	82024	51310	48690	69962	30038	8
53	18804	81196	51696	48304	70214	29786	7
54	19616	80384	52079	47921	70465	29535	6
55	20413	79587	52459	47541	70714	29286	5
56	21195	78805	52835	47165	70962	29038	4
57	21964	78036	53208	46792	71208	28792	3
58	22720	77280	53578	46422	71453	28547	2
59	23462	76538	53945	46055	71697	28303	1
60	24192	75808	54308	45692	71940	28060	0
/	89°		88°		87°		/
	Cotan	Tan	Cotan	Tan	Cotan	Tan	

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	3°		4°		5°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	8.71940	11.28060	8.84464	11.15536	8.94195	11.05805	60
1	72181	27819	84646	15354	94340	05660	59
2	72420	27580	84826	15174	94485	05515	58
3	72659	27341	85006	14994	94630	05370	57
4	72896	27104	85185	14815	94773	05227	56
5	73132	26868	85363	14637	94917	05083	55
6	73366	26634	85540	14460	95060	04940	54
7	73600	26400	85717	14283	95202	04798	53
8	73832	26168	85893	14107	95344	04656	52
9	74063	25937	86069	13931	95486	04514	51
10	8.74292	11.25708	8.86243	11.13757	8.95627	11.04373	50
11	74521	25479	86417	13583	95767	04233	49
12	74748	25252	86591	13409	95908	04092	48
13	74974	25026	86763	13237	96047	03953	47
14	75199	24801	86935	13065	96187	03813	46
15	75423	24577	87106	12894	96325	03675	45
16	75645	24355	87277	12723	96464	03536	44
17	75867	24133	87447	12553	96602	03398	43
18	76087	23913	87616	12384	96739	03261	42
19	76306	23694	87785	12215	96877	03123	41
20	8.76525	11.23475	8.87953	11.12047	8.97013	11.02987	40
21	76742	23258	88120	11880	97150	02850	39
22	76958	23042	88287	11713	97285	02715	38
23	77173	22827	88453	11547	97421	02579	37
24	77387	22613	88618	11382	97556	02444	36
25	77600	22400	88783	11217	97691	02309	35
26	77811	22189	88948	11052	97825	02175	34
27	78022	21978	89111	10889	97959	02041	33
28	78232	21768	89274	10726	98092	01908	32
29	78441	21559	89437	10563	98225	01775	31
30	8.78649	11.21351	8.89598	11.10402	8.98358	11.01642	30
31	78855	21145	89760	10240	98490	01510	29
32	79061	20939	89920	10080	98622	01378	28
33	79266	20734	90080	09920	98753	01247	27
34	79470	20530	90240	09760	98884	01116	26
35	79673	20327	90399	09601	99015	00985	25
36	79875	20125	90557	09443	99145	00855	24
37	80076	19924	90715	09285	99275	00725	23
38	80277	19723	90872	09128	99405	00595	22
39	80476	19524	91029	08971	99534	00466	21
40	8.80674	11.19326	8.91185	11.08815	8.99662	11.00338	20
41	80872	19128	91340	08660	99791	00209	19
42	81068	18932	91495	08505	99919	00081	18
43	81264	18736	91650	08350	9.00046	10.99954	17
44	81459	18541	91803	08197	00174	99826	16
45	81653	18347	91957	08043	00301	99699	15
46	81846	18154	92110	07890	00427	99573	14
47	82038	17962	92262	07738	00553	99447	13
48	82230	17770	92414	07586	00679	99321	12
49	82420	17580	92565	07435	00805	99195	11
50	8.82610	11.17390	8.92716	11.07284	9.00930	10.99070	10
51	82799	17201	92866	07134	01055	98945	9
52	82987	17013	93016	06984	01179	98821	8
53	83175	16825	93165	06835	01303	98697	7
54	83361	16639	93313	06687	01427	98573	6
55	83547	16453	93462	06538	01550	98450	5
56	83732	16268	93609	06391	01673	98327	4
57	83916	16084	93756	06244	01796	98204	3
58	84100	15900	93903	06097	01918	98082	2
59	84282	15718	94049	05951	02040	97960	1
60	84464	15536	94195	05805	02162	97838	0
/	86°		85°		84°		/
	Cotan	Tan	Cotan	Tan	Cotan	Tan	

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	6°		7°		8°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.02162	10.97858	9.08914	10.91086	9.14780	10.85220	60
1	02283	97717	09019	90981	14872	85128	59
2	02404	97596	09123	90877	14963	85037	58
3	02525	97475	09227	90773	15054	84946	57
4	02645	97355	09330	90670	15145	84855	56
5	02766	97234	09434	90566	15236	84764	55
6	02885	97115	09537	90463	15327	84673	54
7	03005	96995	09640	90360	15417	84583	53
8	03124	96876	09742	90258	15508	84492	52
9	03242	96758	09845	90155	15598	84402	51
10	9.03361	10.96639	9.09947	10.90053	9.15688	10.84312	50
11	03479	96521	10049	89951	15777	84223	49
12	03597	96403	10150	89850	15867	84133	48
13	03714	96286	10252	89748	15956	84044	47
14	03832	96168	10353	89647	16046	83954	46
15	03948	96052	10454	89546	16135	83865	45
16	04065	95935	10555	89445	16224	83776	44
17	04181	95819	10656	89344	16312	83688	43
18	04297	95703	10756	89244	16401	83599	42
19	04413	95587	10856	89144	16489	83511	41
20	9.04528	10.95472	9.10956	10.89044	9.16577	10.83423	40
21	04643	95357	11056	88944	16665	83353	39
22	04758	95242	11155	88845	16753	83247	38
23	04873	95127	11254	88746	16841	83159	37
24	04987	95013	11353	88647	16928	83072	36
25	05101	94899	11452	88548	17016	82984	35
26	05214	94786	11551	88449	17103	82897	34
27	05328	94672	11649	88351	17190	82810	33
28	05441	94559	11747	88253	17277	82723	32
29	05553	94447	11845	88155	17363	82637	31
30	9.05666	10.94334	9.11943	10.88057	9.17450	10.82550	30
31	05778	94222	12040	87960	17536	82464	29
32	05890	94110	12138	87862	17622	82378	28
33	06002	93998	12235	87765	17708	82292	27
34	06113	93887	12332	87668	17794	82206	26
35	06224	93776	12428	87572	17880	82120	25
36	06335	93665	12525	87475	17965	82035	24
37	06445	93555	12621	87379	18051	81949	23
38	06556	93444	12717	87283	18136	81864	22
39	06666	93334	12813	87187	18221	81779	21
40	9.06775	10.93225	9.12909	10.87091	9.18306	10.81694	20
41	06885	93115	13004	86996	18391	81609	19
42	06994	93006	13099	86901	18475	81525	18
43	07103	92897	13194	86806	18560	81440	17
44	07211	92789	13289	86711	18644	81356	16
45	07320	92680	13384	86616	18728	81272	15
46	07428	92572	13478	86522	18812	81188	14
47	07536	92464	13573	86427	18896	81104	13
48	07643	92357	13667	86333	18979	81021	12
49	07751	92249	13761	86239	19063	80937	11
50	9.07858	10.92142	9.13854	10.86146	9.19146	10.80854	10
51	07964	92036	13948	86052	19229	80771	9
52	08071	91929	14041	85959	19312	80688	8
53	08177	91823	14134	85866	19395	80605	7
54	08283	91717	14227	85773	19478	80522	6
55	08389	91611	14320	85680	19561	80439	5
56	08495	91505	14412	85588	19643	80357	4
57	08600	91400	14504	85496	19725	80275	3
58	08705	91295	14597	85403	19807	80193	2
59	08810	91190	14688	85312	19889	80111	1
60	08914	91086	14780	85220	19971	80029	0
/	83°		82°		81°		/
	Cotan	Tan	Cotan	Tan	Cotan	Tan	

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	9°		10°		11°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.19971	10.80029	9.24632	10.75368	9.28865	10.71135	60
1	20053	79947	24706	75294	28933	71067	59
2	20154	79866	24779	75221	29000	71000	58
3	20216	79784	24853	75147	29067	70933	57
4	20297	79703	24926	75074	29134	70866	56
5	20378	79622	25000	75000	29201	70799	55
6	20459	79541	25073	74927	29268	70732	54
7	20540	79460	25146	74854	29335	70665	53
8	20621	79379	25219	74781	29402	70598	52
9	20701	79299	25292	74708	29468	70532	51
10	9.20782	10.79218	9.25365	10.74635	9.29535	10.70465	50
11	20862	79138	25437	74563	29601	70399	49
12	20942	79058	25510	74490	29668	70332	48
13	21022	78978	25582	74418	29734	70266	47
14	21102	78898	25655	74345	29800	70200	46
15	21182	78818	25727	74273	29866	70134	45
16	21261	78739	25799	74201	29932	70068	44
17	21341	78659	25871	74129	29998	70002	43
18	21420	78580	25943	74057	30064	69936	42
19	21499	78501	26015	73985	30130	69870	41
20	9.21578	10.78422	9.26086	10.73914	9.30195	10.69805	40
21	21657	78343	26158	73842	30261	69739	39
22	21736	78264	26229	73771	30326	69674	38
23	21814	78186	26301	73699	30391	69609	37
24	21893	78107	26372	73628	30457	69543	36
25	21971	78029	26443	73557	30522	69478	35
26	22049	77951	26514	73486	30587	69413	34
27	22127	77873	26585	73415	30652	69348	33
28	22205	77795	26655	73345	30717	69283	32
29	22283	77717	26726	73274	30782	69218	31
30	9.22361	10.77639	9.26797	10.73203	9.30846	10.69154	30
31	22438	77652	26867	73133	30911	69089	29
32	22516	77484	26937	73063	30975	69025	28
33	22593	77407	27008	72992	31040	68960	27
34	22670	77330	27078	72922	31104	68896	26
35	22747	77253	27148	72852	31168	68832	25
36	22824	77176	27218	72782	31233	68767	24
37	22901	77099	27288	72712	31297	68703	23
38	22977	77023	27357	72643	31361	68639	22
39	23054	76946	27427	72573	31425	68575	21
40	9.23130	10.76870	9.27496	10.72504	9.31489	10.68511	20
41	23206	76794	27566	72434	31552	68448	19
42	23283	76717	27635	72365	31616	68384	18
43	23359	76641	27704	72296	31679	68321	17
44	23435	76565	27773	72227	31743	68257	16
45	23510	76490	27842	72158	31806	68194	15
46	23586	76414	27911	72089	31870	68130	14
47	23661	76339	27980	72020	31933	68067	13
48	23737	76263	28049	71951	31996	68004	12
49	23812	76188	28117	71883	32059	67941	11
50	9.23887	10.76113	9.28186	10.71814	9.32122	10.67878	10
51	23962	76038	28254	71746	32185	67815	9
52	24037	75963	28323	71677	32248	67752	8
53	24112	75888	28391	71609	32311	67689	7
54	24186	75814	28459	71541	32373	67627	6
55	24261	75739	28527	71473	32436	67564	5
56	24335	75665	28595	71405	32498	67502	4
57	24410	75590	28662	71338	32561	67439	3
58	24484	75516	28730	71270	32623	67377	2
59	24558	75442	28798	71202	32685	67315	1
60	24632	75368	28865	71135	32747	67253	0
/	80°		79°		78°		/
	Cotan	Tan	Cotan	Tan	Cotan	Tan	

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	12°		13°		14°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.32747	10.67253	9.36336	10.63664	9.39677	10.60323	60
1	32810	67190	36394	63606	39731	60269	59
2	32872	67128	36452	63548	39785	60215	58
3	32933	67067	36509	63491	39838	60162	57
4	32995	67005	36566	63434	39892	60108	56
5	33057	66943	36624	63376	39945	60055	55
6	33119	66881	36681	63319	39999	60001	54
7	33180	66820	36738	63262	40052	59948	53
8	33242	66758	36795	63205	40106	59894	52
9	33303	66697	36852	63148	40159	59841	51
10	9.33365	10.66635	9.36909	10.63091	9.40212	10.59788	50
11	33426	66574	36966	63034	40266	59734	49
12	33487	66513	37023	62977	40319	59681	48
13	33548	66452	37080	62920	40372	59628	47
14	33609	66391	37137	62863	40425	59575	46
15	33670	66330	37193	62807	40478	59522	45
16	33731	66269	37250	62750	40531	59469	44
17	33792	66208	37306	62694	40584	59416	43
18	33853	66147	37363	62637	40636	59364	42
19	33913	66087	37419	62581	40689	59311	41
20	9.33974	10.66026	9.37476	10.62524	9.40742	10.59258	40
21	34034	65966	37532	62468	40795	59205	39
22	34095	65905	37588	62412	40847	59153	38
23	34155	65845	37644	62356	40900	59100	37
24	34215	65785	37700	62300	40952	59048	36
25	34276	65724	37756	62244	41005	58995	35
26	34336	65664	37812	62188	41057	58943	34
27	34396	65604	37868	62132	41109	58891	33
28	34456	65544	37924	62076	41161	58839	32
29	34516	65484	37980	62020	41214	58786	31
30	9.34576	10.65424	9.38035	10.61965	9.41266	10.58734	30
31	34635	65365	38091	61909	41318	58682	29
32	34695	65305	38147	61853	41370	58630	28
33	34755	65245	38202	61798	41422	58578	27
34	34814	65186	38257	61743	41474	58526	26
35	34874	65126	38313	61687	41526	58474	25
36	34933	65067	38368	61632	41578	58422	24
37	34992	65008	38423	61577	41629	58371	23
38	35051	64949	38479	61521	41681	58319	22
39	35111	64889	38534	61466	41733	58267	21
40	9.35170	10.64830	9.38589	10.61411	9.41784	10.58216	20
41	35229	64771	38644	61356	41836	58164	19
42	35288	64712	38699	61301	41887	58113	18
43	35347	64653	38754	61246	41939	58061	17
44	35405	64595	38808	61192	41990	58010	16
45	35464	64536	38863	61137	42041	57959	15
46	35523	64477	38918	61082	42093	57907	14
47	35581	64419	38972	61028	42144	57856	13
48	35640	64360	39027	60973	42195	57805	12
49	35698	64302	39082	60918	42246	57754	11
50	9.35757	10.64243	9.39136	10.60864	9.42297	10.57703	10
51	35815	64185	39190	60810	42348	57652	9
52	35873	64127	39245	60755	42399	57601	8
53	35931	64069	39299	60701	42450	57550	7
54	35989	64011	39353	60647	42501	57499	6
55	36047	63953	39407	60593	42552	57448	5
56	36105	63895	39461	60539	42603	57397	4
57	36163	63837	39515	60485	42653	57347	3
58	36221	63779	39569	60431	42704	57296	2
59	36279	63721	39623	60377	42755	57245	1
60	36336	63664	39677	60323	42805	57195	0
/	Cotan	Tan	Cotan	Tan	Cotan	Tan	/
	77°		76°		75°		

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	15°		16°		17°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.42805	10.57195	9.45750	10.54250	9.48534	10.51466	60
1	42856	57144	45797	54203	48579	51421	59
2	42906	57094	45845	54155	48624	51376	58
3	42957	57043	45892	54108	48669	51331	57
4	43007	56993	45940	54060	48714	51286	56
5	43057	56943	45987	54013	48759	51241	55
6	43108	56892	46035	53965	48804	51196	54
7	43158	56842	46082	53918	48849	51151	53
8	43208	56792	46130	53870	48894	51106	52
9	43258	56742	46177	53823	48939	51061	51
10	9.43308	10.56692	9.46224	10.53776	9.48984	10.51016	50
11	43358	56642	46271	53729	49029	50971	49
12	43408	56592	46319	53681	49073	50927	48
13	43458	56542	46366	53634	49118	50882	47
14	43508	56492	46413	53587	49163	50837	46
15	43558	56442	46460	53540	49207	50793	45
16	43607	56393	46507	53493	49252	50748	44
17	43657	56343	46554	53446	49296	50704	43
18	43707	56293	46601	53399	49341	50659	42
19	43756	56244	46648	53352	49385	50615	41
20	9.43806	10.56194	9.46694	10.53306	9.49430	10.50570	40
21	43855	56145	46741	53259	49474	50526	39
22	43905	56095	46788	53212	49519	50481	38
23	43954	56046	46835	53165	49563	50437	37
24	44004	55996	46881	53119	49607	50393	36
25	44053	55947	46928	53072	49652	50348	35
26	44102	55898	46975	53025	49696	50304	34
27	44151	55849	47021	52979	49740	50260	33
28	44201	55799	47068	52932	49784	50216	32
29	44250	55750	47114	52886	49828	50172	31
30	9.44299	10.55701	9.47160	10.52840	9.49872	10.50128	30
31	44348	55652	47207	52793	49916	50084	29
32	44397	55603	47253	52747	49960	50040	28
33	44446	55554	47299	52701	50004	49996	27
34	44495	55505	47346	52654	50048	49952	26
35	44544	55456	47392	52608	50092	49908	25
36	44592	55408	47438	52562	50136	49864	24
37	44641	55359	47484	52516	50180	49820	23
38	44690	55310	47530	52470	50223	49777	22
39	44738	55262	47576	52424	50267	49733	21
40	9.44787	10.55213	9.47622	10.52378	9.50311	10.49689	20
41	44836	55164	47668	52332	50355	49645	19
42	44884	55116	47714	52286	50398	49602	18
43	44933	55067	47760	52240	50442	49558	17
44	44981	55019	47806	52194	50485	49515	16
45	45029	54971	47852	52148	50529	49471	15
46	45078	54922	47897	52103	50572	49428	14
47	45126	54874	47943	52057	50616	49384	13
48	45174	54826	47989	52011	50659	49341	12
49	45222	54778	48035	51965	50703	49297	11
50	9.45271	10.54729	9.48080	10.51920	9.50746	10.49254	10
51	45319	54681	48126	51874	50789	49211	9
52	45367	54633	48171	51829	50833	49167	8
53	45415	54585	48217	51783	50876	49124	7
54	45463	54537	48262	51738	50919	49081	6
55	45511	54489	48307	51693	50962	49038	5
56	45559	54441	48353	51647	51005	48995	4
57	45606	54394	48398	51602	51048	48952	3
58	45654	54346	48443	51557	51092	48908	2
59	45702	54298	48489	51511	51135	48865	1
60	45750	54250	48534	51466	51178	48822	0
/	74°		73°		72°		/
	Cotan	Tan	Cotan	Tan	Cotan	Tan	

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	18°		19°		20°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.51178	10.48822	9.53697	10.46303	9.56107	10.43893	60
1	51221	48779	53738	46262	56146	43864	59
2	51264	48736	53779	46221	56185	43815	58
3	51306	48694	53820	46180	56224	43776	57
4	51349	48651	53861	46139	56264	43736	56
5	51392	48608	53902	46098	56303	43697	55
6	51435	48565	53943	46057	56342	43658	54
7	51478	48522	53984	46016	56381	43619	53
8	51520	48480	54025	45975	56420	43580	52
9	51563	48437	54065	45935	56459	43541	51
10	9.51606	10.48394	9.54106	10.45894	9.56498	10.43502	50
11	51648	48352	54147	45853	56537	43463	49
12	51691	48309	54187	45813	56576	43424	48
13	51734	48266	54228	45772	56615	43385	47
14	51776	48224	54269	45731	56654	43346	46
15	51819	48181	54309	45691	56693	43307	45
16	51861	48139	54350	45650	56732	43268	44
17	51903	48097	54390	45610	56771	43229	43
18	51946	48054	54431	45569	56810	43190	42
19	51988	48012	54471	45529	56849	43151	41
20	9.52031	10.47969	9.54512	10.45488	9.56887	10.43113	40
21	52073	47927	54552	45448	56926	43074	39
22	52115	47885	54593	45407	56965	43035	38
23	52157	47843	54633	45367	57004	42996	37
24	52200	47800	54673	45327	57042	42958	36
25	52242	47758	54714	45286	57081	42919	35
26	52284	47716	54754	45246	57120	42880	34
27	52326	47674	54794	45206	57158	42842	33
28	52368	47632	54835	45165	57197	42803	32
29	52410	47590	54875	45125	57235	42765	31
30	9.52452	10.47548	9.54915	10.45085	9.57274	10.42726	30
31	52494	47506	54955	45045	57312	42688	29
32	52536	47464	54995	45005	57351	42649	28
33	52578	47422	55035	44965	57389	42611	27
34	52620	47380	55075	44925	57428	42572	26
35	52661	47339	55115	44885	57466	42534	25
36	52703	47297	55155	44845	57504	42496	24
37	52745	47255	55195	44805	57543	42457	23
38	52787	47213	55235	44765	57581	42419	22
39	52829	47171	55275	44725	57619	42381	21
40	9.52870	10.47130	9.55315	10.44685	9.57658	10.42342	20
41	52912	47088	55355	44645	57696	42304	19
42	52953	47047	55395	44605	57734	42266	18
43	52995	47005	55434	44566	57772	42228	17
44	53037	46963	55474	44526	57810	42190	16
45	53078	46922	55514	44486	57849	42151	15
46	53120	46880	55554	44446	57887	42113	14
47	53161	46839	55593	44407	57925	42075	13
48	53202	46798	55633	44367	57963	42037	12
49	53244	46756	55673	44327	58001	41999	11
50	9.53285	10.46715	9.55712	10.44288	9.58039	10.41961	10
51	53327	46673	55752	44248	58077	41923	9
52	53368	46632	55791	44209	58115	41885	8
53	53409	46591	55831	44169	58153	41847	7
54	53450	46550	55870	44130	58191	41809	6
55	53492	46508	55910	44090	58229	41771	5
56	53533	46467	55949	44051	58267	41733	4
57	53574	46426	55989	44011	58304	41696	3
58	53615	46385	56028	43972	58342	41658	2
59	53656	46344	56067	43933	58380	41620	1
60	53697	46303	56107	43893	58418	41582	0
/	71°		70°		69°		/
	Cotan	Tan	Cotan	Tan	Cotan	Tan	

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	21°		22°		23°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.58418	10.41582	9.60641	10.39359	9.62785	10.37215	60
1	58455	41545	60677	39323	62820	37180	59
2	58493	41507	60714	39286	62855	37145	58
3	58531	41469	60750	39250	62890	37110	57
4	58569	41431	60786	39214	62926	37074	56
5	58606	41394	60823	39177	62961	37039	55
6	58644	41356	60859	39141	62996	37004	54
7	58681	41319	60895	39105	63031	36969	53
8	58719	41281	60931	39069	63066	36934	52
9	58757	41243	60967	39033	63101	36899	51
10	9.58794	10.41206	9.61004	10.38996	9.63135	10.36865	50
11	58832	41168	61040	38960	63170	36830	49
12	58869	41131	61076	38924	63205	36795	48
13	58907	41093	61112	38888	63240	36760	47
14	58944	41056	61148	38852	63275	36725	46
15	58981	41019	61184	38816	63310	36690	45
16	59019	40981	61220	38780	63345	36655	44
17	59056	40944	61256	38744	63379	36621	43
18	59094	40906	61292	38708	63414	36586	42
19	59131	40869	61328	38672	63449	36551	41
20	9.59168	10.40832	9.61364	10.38636	9.63484	10.36516	40
21	59205	40795	61400	38600	63519	36481	39
22	59243	40757	61436	38564	63553	36447	38
23	59280	40720	61472	38528	63588	36412	37
24	59317	40683	61508	38492	63623	36377	36
25	59354	40646	61544	38456	63657	36343	35
26	59391	40609	61579	38421	63692	36308	34
27	59429	40571	61615	38385	63726	36274	33
28	59466	40534	61651	38349	63761	36239	32
29	59503	40497	61687	38313	63796	36204	31
30	9.59540	10.40460	9.61722	10.38278	9.63830	10.36170	30
31	59577	40423	61758	38242	63865	36135	29
32	59614	40386	61794	38206	63899	36101	28
33	59651	40349	61830	38170	63934	36066	27
34	59688	40312	61865	38135	63968	36032	26
35	59725	40275	61901	38099	64003	35997	25
36	59762	40238	61936	38064	64037	35963	24
37	59799	40201	61972	38028	64072	35928	23
38	59835	40165	62008	37992	64106	35894	22
39	59872	40128	62043	37957	64140	35860	21
40	9.59909	10.40091	9.62079	10.37921	9.64175	10.35825	20
41	59946	40054	62114	37886	64209	35791	19
42	59983	40017	62150	37850	64243	35757	18
43	60019	39981	62185	37815	64278	35722	17
44	60056	39944	62221	37779	64312	35688	16
45	60093	39907	62256	37744	64346	35654	15
46	60130	39870	62292	37708	64381	35619	14
47	60166	39834	62327	37673	64415	35585	13
48	60203	39797	62362	37638	64449	35551	12
49	60240	39760	62398	37602	64483	35517	11
50	9.60276	10.39724	9.62433	10.37567	9.64517	10.35483	10
51	60313	39687	62468	37532	64552	35448	9
52	60349	39651	62504	37496	64586	35414	8
53	60386	39614	62539	37461	64620	35380	7
54	60422	39578	62574	37426	64654	35346	6
55	60459	39541	62609	37391	64688	35312	5
56	60495	39505	62645	37355	64722	35278	4
57	60532	39468	62680	37320	64756	35244	3
58	60568	39432	62715	37285	64790	35210	2
59	60605	39395	62750	37250	64824	35176	1
60	60641	39359	62785	37215	64858	35142	0
/	68°		67°		66°		/
	Cotan	Tan	Cotan	Tan	Cotan	Tan	

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	24°		25°		26°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.64858	10.35142	9.66867	10.33133	9.68818	10.31182	60
1	64892	35108	66900	33100	68850	31150	59
2	64926	35074	66933	33067	68882	31118	58
3	64960	35040	66966	33034	68914	31086	57
4	64994	35006	66999	33001	68946	31054	56
5	65028	34972	67032	32968	68978	31022	55
6	65062	34938	67065	32935	69010	30990	54
7	65096	34904	67098	32902	69042	30958	53
8	65130	34870	67131	32869	69074	30926	52
9	65164	34836	67163	32837	69106	30894	51
10	9.65197	10.34803	9.67196	10.32804	9.69138	10.30862	50
11	65231	34769	67229	32771	69170	30830	49
12	65265	34735	67262	32738	69202	30798	48
13	65299	34701	67295	32705	69234	30766	47
14	65333	34667	67327	32673	69266	30734	46
15	65366	34634	67360	32640	69298	30702	45
16	65400	34600	67393	32607	69329	30671	44
17	65434	34566	67426	32574	69361	30639	43
18	65467	34533	67458	32542	69393	30607	42
19	65501	34499	67491	32509	69425	30575	41
20	9.65535	10.34465	9.67524	10.32476	9.69457	10.30543	40
21	65568	34432	67556	32444	69488	30512	39
22	65602	34398	67589	32411	69520	30480	38
23	65636	34364	67622	32378	69552	30448	37
24	65669	34331	67654	32346	69584	30416	36
25	65703	34297	67687	32313	69615	30385	35
26	65736	34264	67719	32281	69647	30353	34
27	65770	34230	67752	32248	69679	30321	33
28	65803	34197	67785	32215	69710	30290	32
29	65837	34163	67817	32183	69742	30258	31
30	9.65870	10.34130	9.67850	10.32150	9.69774	10.30226	30
31	65904	34096	67882	32118	69805	30195	29
32	65937	34063	67915	32085	69837	30163	28
33	65971	34029	67947	32053	69868	30132	27
34	66004	33996	67980	32020	69900	30100	26
35	66038	33962	68012	31988	69932	30068	25
36	66071	33929	68044	31956	69963	30037	24
37	66104	33896	68077	31923	69995	30005	23
38	66138	33862	68109	31891	70026	29974	22
39	66171	33829	68142	31858	70058	29942	21
40	9.66204	10.33796	9.68174	10.31826	9.70089	10.29911	20
41	66238	33762	68206	31794	70121	29879	19
42	66271	33729	68239	31761	70152	29848	18
43	66304	33696	68271	31729	70184	29816	17
44	66337	33663	68303	31697	70215	29785	16
45	66371	33629	68336	31664	70247	29753	15
46	66404	33596	68368	31632	70278	29722	14
47	66437	33563	68400	31600	70309	29691	13
48	66470	33530	68432	31568	70341	29659	12
49	66503	33497	68465	31535	70372	29628	11
50	9.66537	10.33463	9.68497	10.31503	9.70404	10.29596	10
51	66570	33430	68529	31471	70435	29565	9
52	66603	33397	68561	31439	70466	29534	8
53	66636	33364	68593	31407	70498	29502	7
54	66669	33331	68626	31374	70529	29471	6
55	66702	33298	68658	31342	70560	29440	5
56	66735	33265	68690	31310	70592	29408	4
57	66768	33232	68722	31278	70623	29377	3
58	66801	33199	68754	31246	70654	29346	2
59	66834	33166	68786	31214	70685	29315	1
60	66867	33133	68818	31182	70717	29283	0
/	65°		64°		63°		/
	Cotan	Tan	Cotan	Tan	Cotan	Tan	

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	27°		28°		29°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.70717	10.29283	9.72567	10.27433	9.74375	10.25625	60
1	70748	29252	72598	27402	74405	25595	59
2	70779	29221	72628	27372	74435	25565	58
3	70810	29190	72659	27341	74465	25535	57
4	70841	29159	72689	27311	74494	25506	56
5	70873	29127	72720	27280	74524	25476	55
6	70904	29096	72750	27250	74554	25446	54
7	70935	29065	72780	27220	74583	25417	53
8	70966	29034	72811	27189	74613	25387	52
9	70997	29003	72841	27159	74643	25357	51
10	9.71028	10.28972	9.72872	10.27128	9.74673	10.25327	50
11	71059	28941	72902	27098	74702	25298	49
12	71090	28910	72932	27068	74732	25268	48
13	71121	28879	72963	27037	74762	25238	47
14	71155	28847	72993	27007	74791	25209	46
15	71184	28816	73023	26977	74821	25179	45
16	71215	28785	73054	26946	74851	25149	44
17	71246	28754	73084	26916	74880	25120	43
18	71277	28723	73114	26886	74910	25090	42
19	71308	28692	73144	26856	74939	25061	41
20	9.71339	10.28661	9.73175	10.26825	9.74969	10.25031	40
21	71370	28630	73205	26795	74998	25002	39
22	71401	28599	73235	26765	75028	24972	38
23	71431	28569	73265	26735	75058	24942	37
24	71462	28538	73295	26705	75087	24913	36
25	71493	28507	73326	26674	75117	24883	35
26	71524	28476	73356	26644	75146	24854	34
27	71555	28445	73386	26614	75176	24824	33
28	71586	28414	73416	26584	75205	24795	32
29	71617	28383	73446	26554	75235	24765	31
30	9.71648	10.28352	9.73476	10.26524	9.75264	10.24736	30
31	71679	28321	73507	26493	75294	24706	29
32	71709	28291	73537	26463	75323	24677	28
33	71740	28260	73567	26433	75353	24647	27
34	71771	28229	73597	26403	75382	24618	26
35	71802	28198	73627	26373	75411	24589	25
36	71833	28167	73657	26343	75441	24559	24
37	71863	28137	73687	26313	75470	24530	23
38	71894	28106	73717	26283	75500	24500	22
39	71925	28075	73747	26253	75529	24471	21
40	9.71955	10.28045	9.73777	10.26223	9.75558	10.24442	20
41	71986	28014	73807	26193	75588	24412	19
42	72017	27983	73837	26163	75617	24383	18
43	72048	27952	73867	26133	75647	24353	17
44	72078	27922	73897	26103	75676	24324	16
45	72109	27891	73927	26073	75705	24295	15
46	72140	27860	73957	26043	75735	24265	14
47	72170	27830	73987	26013	75764	24236	13
48	72201	27799	74017	25983	75793	24207	12
49	72231	27769	74047	25953	75822	24178	11
50	9.72262	10.27738	9.74077	10.25923	9.75852	10.24148	10
51	72293	27707	74107	25893	75881	24119	9
52	72325	27677	74137	25863	75910	24090	8
53	72354	27646	74166	25834	75939	24061	7
54	72384	27616	74196	25804	75969	24031	6
55	72415	27585	74226	25774	75998	24002	5
56	72445	27555	74256	25744	76027	23973	4
57	72476	27524	74286	25714	76056	23944	3
58	72506	27494	74316	25684	76086	23914	2
59	72537	27463	74345	25655	76115	23885	1
60	72567	27433	74375	25625	76144	23856	0
/	62°		61°		60°		/
	Cotan	Tan	Cotan	Tan	Cotan	Tan	

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	30°		31°		32°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.76144	10.23856	9.77877	10.22123	9.79579	10.20421	60
1	76173	23827	77906	22094	79607	20393	59
2	76202	23798	77935	22065	79635	20365	58
3	76231	23769	77963	22037	79663	20337	57
4	76261	23739	77992	22008	79691	20309	56
5	76290	23710	78020	21980	79719	20281	55
6	76319	23681	78049	21951	79747	20253	54
7	76348	23652	78077	21923	79776	20224	53
8	76377	23623	78106	21894	79804	20196	52
9	76406	23594	78135	21865	79832	20168	51
10	9.76435	10.23565	9.78163	10.21837	9.79860	10.20140	50
11	76464	23536	78192	21808	79888	20112	49
12	76493	23507	78220	21780	79916	20084	48
13	76522	23478	78249	21751	79944	20056	47
14	76551	23449	78277	21723	79972	20028	46
15	76580	23420	78306	21694	80000	20000	45
16	76609	23391	78334	21666	80028	19972	44
17	76639	23361	78363	21637	80056	19944	43
18	76668	23332	78391	21609	80084	19916	42
19	76697	23303	78419	21581	80112	19888	41
20	9.76725	10.23275	9.78448	10.21552	9.80140	10.19860	40
21	76754	23246	78476	21524	80168	19832	39
22	76783	23217	78505	21495	80195	19805	38
23	76812	23188	78533	21467	80223	19777	37
24	76841	23159	78562	21438	80251	19749	36
25	76870	23130	78590	21410	80279	19721	35
26	76899	23101	78618	21382	80307	19693	34
27	76928	23072	78647	21353	80335	19665	33
28	76957	23043	78675	21325	80363	19637	32
29	76986	23014	78704	21296	80391	19609	31
30	9.77015	10.22985	9.78732	10.21268	9.80419	10.19581	30
31	77044	22956	78760	21240	80447	19553	29
32	77073	22927	78789	21211	80474	19526	28
33	77101	22899	78817	21183	80502	19498	27
34	77130	22870	78845	21155	80530	19470	26
35	77159	22841	78874	21126	80558	19442	25
36	77188	22812	78902	21098	80586	19414	24
37	77217	22783	78930	21070	80614	19386	23
38	77246	22754	78959	21041	80642	19358	22
39	77274	22726	78987	21013	80669	19331	21
40	9.77303	10.22697	9.79015	10.20985	9.80697	10.19303	20
41	77332	22668	79043	20957	80725	19275	19
42	77361	22639	79072	20928	80753	19247	18
43	77390	22610	79100	20900	80781	19219	17
44	77418	22582	79128	20872	80808	19192	16
45	77447	22553	79156	20844	80836	19164	15
46	77476	22524	79185	20815	80864	19136	14
47	77505	22495	79213	20787	80892	19108	13
48	77533	22467	79241	20759	80919	19081	12
49	77562	22438	79269	20731	80947	19053	11
50	9.77591	10.22409	9.79297	10.20703	9.80975	10.19025	10
51	77619	22381	79326	20674	81003	18997	9
52	77648	22352	79354	20646	81030	18970	8
53	77677	22323	79382	20618	81058	18942	7
54	77706	22294	79410	20590	81086	18914	6
55	77734	22266	79438	20562	81113	18887	5
56	77763	22237	79466	20534	81141	18859	4
57	77791	22209	79495	20505	81169	18831	3
58	77820	22180	79523	20477	81196	18804	2
59	77849	22151	79551	20449	81224	18776	1
60	77877	22123	79579	20421	81252	18748	0
/	59°		58°		57°		/
	Cotan	Tan	Cotan	Tan	Cotan	Tan	

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	33°		34°		35°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.81252	10.18748	9.82899	10.17101	9.84523	10.15477	60
1	81279	18721	82926	17074	84550	15450	59
2	81307	18693	82953	17047	84576	15424	58
3	81335	18665	82980	17020	84603	15397	57
4	81362	18638	83008	16992	84630	15370	56
5	81390	18610	83035	16965	84657	15343	55
6	81418	18582	83062	16938	84684	15316	54
7	81445	18555	83089	16911	84711	15289	53
8	81473	18527	83117	16883	84738	15262	52
9	81500	18500	83144	16856	84764	15236	51
10	9.81528	10.18472	9.83171	10.16829	9.84791	10.15209	50
11	81556	18444	83198	16802	84818	15182	49
12	81583	18417	83225	16775	84845	15155	48
13	81611	18389	83252	16748	84872	15128	47
14	81638	18362	83280	16720	84899	15101	46
15	81666	18334	83307	16693	84925	15075	45
16	81693	18307	83334	16666	84952	15048	44
17	81721	18279	83361	16639	84979	15021	43
18	81748	18252	83388	16612	85006	14994	42
19	81776	18224	83415	16585	85033	14967	41
20	9.81803	10.18197	9.83442	10.16558	9.85059	10.14941	40
21	81831	18169	83470	16530	85086	14914	39
22	81858	18142	83497	16503	85113	14887	38
23	81886	18114	83524	16476	85140	14860	37
24	81913	18087	83551	16449	85166	14834	36
25	81941	18059	83578	16422	85193	14807	35
26	81968	18032	83605	16395	85220	14780	34
27	81996	18004	83632	16368	85247	14753	33
28	82023	17977	83659	16341	85273	14727	32
29	82051	17949	83686	16314	85300	14700	31
30	9.82078	10.17922	9.83713	10.16287	9.85327	10.14673	30
31	82106	17894	83740	16260	85354	14646	29
32	82133	17867	83768	16232	85380	14620	28
33	82161	17839	83795	16205	85407	14593	27
34	82188	17812	83822	16178	85434	14566	26
35	82215	17785	83849	16151	85460	14540	25
36	82243	17757	83876	16124	85487	14513	24
37	82270	17730	83903	16097	85514	14486	23
38	82298	17702	83930	16070	85540	14460	22
39	82325	17675	83957	16043	85567	14433	21
40	9.82352	10.17648	9.83984	10.16016	9.85594	10.14406	20
41	82380	17620	84011	15989	85620	14380	19
42	82407	17593	84038	15962	85647	14353	18
43	82435	17565	84065	15935	85674	14326	17
44	82462	17538	84092	15908	85700	14300	16
45	82489	17511	84119	15881	85727	14273	15
46	82517	17483	84146	15854	85754	14246	14
47	82544	17456	84173	15827	85780	14220	13
48	82571	17429	84200	15800	85807	14193	12
49	82599	17401	84227	15773	85834	14166	11
50	9.82626	10.17374	9.84254	10.15746	9.85860	10.14140	10
51	82653	17347	84280	15720	85887	14113	9
52	82681	17319	84307	15693	85913	14087	8
53	82708	17292	84334	15666	85940	14060	7
54	82735	17265	84361	15639	85967	14033	6
55	82762	17238	84388	15612	85993	14007	5
56	82790	17210	84415	15585	86020	13980	4
57	82817	17183	84442	15558	86046	13954	3
58	82844	17156	84469	15531	86073	13927	2
59	82871	17129	84496	15504	86100	13900	1
60	82899	17101	84523	15477	86126	13874	0
/	56°		55°		54°		/
	Cotan	Tan	Cotan	Tan	Cotan	Tan	

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	36°		37°		38°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.86126	10.13874	9.87711	10.12289	9.89281	10.10719	60
1	86153	13847	87738	12262	89307	10693	59
2	86179	13821	87764	12236	89333	10667	58
3	86206	13794	87790	12210	89359	10641	57
4	86232	13768	87817	12183	89385	10615	56
5	86259	13741	87843	12157	89411	10589	55
6	86285	13715	87869	12131	89437	10563	54
7	86312	13688	87895	12105	89463	10537	53
8	86338	13662	87922	12078	89489	10511	52
9	86365	13635	87948	12052	89515	10485	51
10	9.86392	10.13608	9.87974	10.12026	9.89541	10.10459	50
11	86418	13582	88000	12000	89567	10433	49
12	86445	13555	88027	11973	89593	10407	48
13	86471	13529	88053	11947	89619	10381	47
14	86498	13502	88079	11921	89645	10355	46
15	86524	13476	88105	11895	89671	10329	45
16	86551	13449	88131	11869	89697	10303	44
17	86577	13423	88158	11842	89723	10277	43
18	86603	13397	88184	11816	89749	10251	42
19	86630	13370	88210	11790	89775	10225	41
20	9.86656	10.13344	9.88236	10.11764	9.89801	10.10199	40
21	86683	13317	88262	11738	89827	10173	39
22	86709	13291	88289	11711	89853	10147	38
23	86736	13264	88315	11685	89879	10121	37
24	86762	13238	88341	11659	89905	10095	36
25	86789	13211	88367	11633	89931	10069	35
26	86815	13185	88393	11607	89957	10043	34
27	86842	13158	88420	11580	89983	10017	33
28	86868	13132	88446	11554	90009	9991	32
29	86894	13106	88472	11528	90035	9965	31
30	9.86921	10.13079	9.88498	10.11502	9.90061	10.09939	30
31	86947	13053	88524	11476	90086	99914	29
32	86974	13026	88550	11450	90112	99888	28
33	87000	13000	88577	11423	90138	99862	27
34	87027	12973	88603	11397	90164	99836	26
35	87053	12947	88629	11371	90190	99810	25
36	87079	12921	88655	11345	90216	99784	24
37	87106	12894	88681	11319	90242	99758	23
38	87132	12868	88707	11293	90268	99732	22
39	87158	12842	88733	11267	90294	99706	21
40	9.87185	10.12815	9.88759	10.11241	9.90320	10.09680	20
41	87211	12789	88786	11214	90346	99654	19
42	87238	12762	88812	11188	90371	99629	18
43	87264	12736	88838	11162	90397	99603	17
44	87290	12710	88864	11136	90423	99577	16
45	87317	12683	88890	11110	90449	99551	15
46	87343	12657	88916	11084	90475	99525	14
47	87369	12631	88942	11058	90501	99499	13
48	87396	12604	88968	11032	90527	99473	12
49	87422	12578	88994	11006	90553	99447	11
50	9.87448	10.12552	9.89020	10.10980	9.90578	10.09422	10
51	87475	12525	89046	10954	90604	99396	9
52	87501	12499	89073	10927	90630	99370	8
53	87527	12473	89099	10901	90656	99344	7
54	87554	12446	89125	10875	90682	99318	6
55	87580	12420	89151	10849	90708	99292	5
56	87606	12394	89177	10823	90734	99266	4
57	87633	12367	89203	10797	90759	99241	3
58	87659	12341	89229	10771	90785	99215	2
59	87685	12315	89255	10745	90811	99189	1
60	87711	12289	89281	10719	90837	99163	0
/	53°		52°		51°		/
	Cotan	Tan	Cotan	Tan	Cotan	Tan	

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	39°		40°		41°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.90857	10.09163	9.92381	10.07619	9.93916	10.06084	60
1	90863	09137	92407	07593	93942	06058	59
2	90889	09111	92433	07567	93967	06033	58
3	90914	09086	92458	07542	93993	06007	57
4	90940	09060	92484	07516	94018	05982	56
5	90966	09034	92510	07490	94044	05956	55
6	90992	09008	92535	07465	94069	05931	54
7	91018	08982	92561	07439	94095	05905	53
8	91043	08957	92587	07413	94120	05880	52
9	91069	08931	92612	07388	94146	05854	51
10	9.91095	10.08905	9.92638	10.07362	9.94171	10.05829	50
11	91121	08879	92663	07337	94197	05803	49
12	91147	08853	92689	07311	94222	05778	48
13	91172	08828	92715	07285	94248	05752	47
14	91198	08802	92740	07260	94273	05727	46
15	91224	08776	92766	07234	94299	05701	45
16	91250	08750	92792	07208	94324	05676	44
17	91276	08724	92817	07183	94350	05650	43
18	91301	08699	92843	07157	94375	05625	42
19	91327	08673	92868	07132	94401	05599	41
20	9.91353	10.08647	9.92894	10.07106	9.94426	10.05574	40
21	91379	08621	92920	07080	94452	05548	39
22	91404	08596	92945	07055	94477	05523	38
23	91430	08570	92971	07029	94503	05497	37
24	91456	08544	92996	07004	94528	05472	36
25	91482	08518	93022	06978	94554	05446	35
26	91507	08493	93048	06952	94579	05421	34
27	91533	08467	93073	06927	94604	05396	33
28	91559	08441	93099	06901	94630	05370	32
29	91585	08415	93124	06876	94655	05345	31
30	9.91610	10.08390	9.93150	10.06850	9.94681	10.05319	30
31	91636	08364	93175	06825	94706	05294	29
32	91662	08338	93201	06799	94732	05268	28
33	91688	08312	93227	06773	94757	05243	27
34	91713	08287	93252	06748	94783	05217	26
35	91739	08261	93278	06722	94808	05192	25
36	91765	08235	93303	06697	94834	05166	24
37	91791	08209	93329	06671	94859	05141	23
38	91816	08184	93354	06646	94884	05116	22
39	91842	08158	93380	06620	94910	05090	21
40	9.91868	10.08132	9.93406	10.06594	9.94935	10.05065	20
41	91893	08107	93431	06569	94961	05039	19
42	91919	08081	93457	06543	94986	05014	18
43	91945	08055	93482	06518	95012	04988	17
44	91971	08029	93508	06492	95037	04963	16
45	91996	08004	93533	06467	95062	04938	15
46	92022	07978	93559	06441	95088	04912	14
47	92048	07952	93584	06416	95113	04887	13
48	92073	07927	93610	06390	95139	04861	12
49	92099	07901	93636	06364	95164	04836	11
50	9.92125	10.07873	9.93661	10.06339	9.95190	10.04810	10
51	92150	07850	93687	06313	95215	04785	9
52	92176	07824	93712	06288	95240	04760	8
53	92202	07798	93738	06262	95266	04734	7
54	92227	07773	93763	06237	95291	04709	6
55	92253	07747	93789	06211	95317	04683	5
56	92279	07721	93814	06186	95342	04658	4
57	92304	07696	93840	06160	95368	04632	3
58	92330	07670	93865	06135	95393	04607	2
59	92356	07644	93891	06109	95418	04582	1
60	92381	07619	93916	06084	95444	04556	0
/	50°		49°		48°		/
	Cotan	Tan	Cotan	Tan	Cotan	Tan	

TABLE III.—LOG. TANGENTS AND COTANGENTS.

/	42°		43°		44°		/
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.95444	10.04556	9.96966	10.03034	9.98484	10.01516	60
1	95469	04531	96991	03009	98509	01491	59
2	95495	04505	97016	02984	98534	01466	58
3	95520	04480	97042	02958	98560	01440	57
4	95545	04455	97067	02933	98585	01415	56
5	95571	04429	97092	02908	98610	01390	55
6	95596	04404	97118	02882	98635	01365	54
7	95622	04378	97143	02857	98661	01339	53
8	95647	04353	97168	02832	98686	01314	52
9	95672	04328	97193	02807	98711	01289	51
10	9.95698	10.04302	9.97219	10.02781	9.98737	10.01263	50
11	95723	04277	97244	02756	98762	01238	49
12	95748	04252	97269	02731	98787	01213	48
13	95774	04226	97295	02705	98812	01188	47
14	95799	04201	97320	02680	98838	01162	46
15	95825	04175	97345	02655	98863	01137	45
16	95850	04150	97371	02629	98888	01112	44
17	95875	04125	97396	02604	98913	01087	43
18	95901	04099	97421	02579	98939	01061	42
19	95926	04074	97447	02553	98964	01036	41
20	9.95952	10.04048	9.97472	10.02528	9.98989	10.01011	40
21	95977	04023	97497	02503	99015	00985	39
22	96002	03998	97523	02477	99040	00960	38
23	96028	03972	97548	02452	99065	00935	37
24	96053	03947	97573	02427	99090	00910	36
25	96078	03922	97598	02402	99116	00884	35
26	96104	03896	97624	02376	99141	00859	34
27	96129	03871	97649	02351	99166	00834	33
28	96155	03845	97674	02326	99191	00809	32
29	96180	03820	97700	02300	99217	00783	31
30	9.96205	10.03795	9.97725	10.02275	9.99242	10.00758	30
31	96231	03769	97750	02250	99267	00733	29
32	96256	03744	97776	02224	99293	00707	28
33	96281	03719	97801	02199	99318	00682	27
34	96307	03693	97826	02174	99343	00657	26
35	96332	03668	97851	02149	99368	00632	25
36	96357	03643	97877	02123	99394	00606	24
37	96383	03617	97902	02098	99419	00581	23
38	96408	03592	97927	02073	99444	00556	22
39	96433	03567	97953	02047	99469	00531	21
40	9.96459	10.03541	9.97978	10.02022	9.99495	10.00505	20
41	96484	03516	98003	01997	99520	00480	19
42	96510	03490	98029	01971	99545	00455	18
43	96535	03465	98054	01946	99570	00430	17
44	96560	03440	98079	01921	99596	00404	16
45	96586	03414	98104	01896	99621	00379	15
46	96611	03389	98130	01870	99646	00354	14
47	96636	03364	98155	01845	99672	00328	13
48	96662	03338	98180	01820	99697	00303	12
49	96687	03313	98206	01794	99722	00278	11
50	9.96712	10.03288	9.98231	10.01769	9.99747	10.00253	10
51	96738	03262	98256	01744	99773	00227	9
52	96763	03237	98281	01719	99798	00202	8
53	96788	03212	98307	01693	99823	00177	7
54	96814	03186	98332	01668	99848	00152	6
55	96839	03161	98357	01643	99874	00126	5
56	96864	03136	98383	01617	99899	00101	4
57	96890	03110	98408	01592	99924	00076	3
58	96915	03085	98433	01567	99949	00051	2
59	96940	03060	98458	01542	99975	00025	1
60	96966	03034	98484	01516	10.00000	00000	0
/	47°		46°		45°		/
	Cotan	Tan	Cotan	Tan	Cotan	Tan	

TABLE IV.—NATURAL SINES AND COSINES.

/	0°		1°		2°		3°		4°		/
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.00000	Onc.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	Onc.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	Onc.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	Onc.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	Onc.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	Onc.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	Onc.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	Onc.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	Onc.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	Onc.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	Onc.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99706	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99682	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99994	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
/	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	/
	89°		88°		87°		86°		85°		

TABLE IV.—NATURAL SINES AND COSINES.

/	10°		11°		12°		13°		14°		/
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98173	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
/	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	/
	79°		78°		77°		76°		75°		

TABLE IV.—NATURAL SINES AND COSINES.

/	20°		21°		22°		23°		24°		/
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38242	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
/	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	/
	69°		68°		67°		66°		65°		

/	35°		36°		37°		38°		39°		/
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62343	.78188	.63697	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62366	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80265	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62525	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62548	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62593	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76643	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76625	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
/	54°	53°	52°	51°	50°	/					

TABLE IV.—NATURAL SINES AND COSINES.

/	40°		41°		42°		43°		44°		/
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71919	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71875	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71855	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71835	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71815	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75242	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68686	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66305	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66435	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66456	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66478	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
/	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	/
	49°		48°		47°		46°		45°		

TABLE V.—NATURAL TANGENTS AND COTANGENTS.

°	0°		1°		2°		3°		°
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1654	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5069	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
°	89°		88°		87°		86°		°
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	

TABLE V.—NATURAL TANGENTS AND COTANGENTS.

/	4°		5°		6°		7°		/
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9925	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
/	85°		84°		83°		82°		/
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	

TABLE V.—NATURAL TANGENTS AND COTANGENTS.

/	8°		9°		10°		11°		/
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	7.01174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
/	81°		80°		79°		78°		/
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	

TABLE V.—NATURAL TANGENTS AND COTANGENTS.

°	12°		13°		14°		15°		°	
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang		
0	.21256	4.70465	.25087	4.00118	.29555	3.401078	.26795	3.75205	60	
1	.21286	4.69791	.25117	4.02575	.29664	3.400582	.26826	3.72771	59	
2	.21316	4.69121	.25148	4.05001	.29775	3.400085	.26857	3.72538	58	
3	.21347	4.68452	.25179	4.07433	.29886	3.399592	.26888	3.71997	57	
4	.21377	4.67786	.25209	4.09880	.29998	3.399099	.26920	3.71476	56	
5	.21408	4.67121	.25240	4.12329	.30087	3.398607	.26951	3.70956	55	
6	.21438	4.66458	.25271	4.14784	.30181	3.398117	.26982	3.70616	54	
7	.21469	4.65797	.25301	4.17259	.30279	3.397627	.27015	3.70188	53	
8	.21499	4.65138	.25332	4.19735	.30380	3.397139	.27044	3.69761	52	
9	.21529	4.64480	.25363	4.22211	.30481	3.396651	.27076	3.69335	51	
10	.21560	4.63825	.25395	4.24687	.30582	3.396165	.27107	3.68989	50	
11	.21590	4.63171	.25424	4.26911	.30675	3.395680	.27138	3.68485	49	
12	.21621	4.62518	.25455	4.29135	.30771	3.395196	.27169	3.68061	48	
13	.21651	4.61868	.25485	4.31359	.30867	3.394715	.27201	3.67638	47	
14	.21682	4.61219	.25516	4.33583	.30965	3.394232	.27232	3.67217	46	
15	.21712	4.60572	.25547	4.35807	.31063	3.393751	.27263	3.66796	45	
16	.21743	4.59927	.25578	4.38031	.31162	3.393271	.27294	3.66376	44	
17	.21773	4.59283	.25608	4.40255	.31262	3.392791	.27326	3.65957	43	
18	.21804	4.58641	.25639	4.42479	.31362	3.392311	.27357	3.65538	42	
19	.21834	4.58001	.25670	4.44703	.31462	3.391831	.27388	3.65121	41	
20	.21864	4.57365	.25700	4.46927	.31562	3.391354	.27419	3.64705	40	
21	.21895	4.56726	.25731	4.49151	.31662	3.390890	.27451	3.64289	39	
22	.21925	4.56091	.25762	4.51375	.31762	3.390417	.27482	3.63874	38	
23	.21956	4.55458	.25793	4.53600	.31862	3.389945	.27513	3.63461	37	
24	.21986	4.54826	.25823	4.55824	.31962	3.389474	.27545	3.63049	36	
25	.22017	4.54196	.25854	4.58048	.32062	3.389004	.27576	3.62636	35	
26	.22047	4.53568	.25885	4.60272	.32162	3.388536	.27607	3.62224	34	
27	.22078	4.52941	.25916	4.62496	.32262	3.388068	.27638	3.61814	33	
28	.22108	4.52316	.25946	4.64720	.32362	3.387601	.27670	3.61405	32	
29	.22139	4.51695	.25977	4.66944	.32462	3.387136	.27701	3.60996	31	
30	.22169	4.51071	.26008	4.69168	.32562	3.386671	.27732	3.60588	30	
31	.22200	4.50451	.26039	4.71392	.32662	3.386208	.27763	3.60181	29	
32	.22231	4.49832	.26069	4.73616	.32762	3.385745	.27795	3.59775	28	
33	.22261	4.49215	.26100	4.75840	.32862	3.385284	.27826	3.59370	27	
34	.22292	4.48599	.26131	4.78064	.32962	3.384824	.27858	3.58966	26	
35	.22322	4.47986	.26162	4.80288	.33062	3.384364	.27889	3.58562	25	
36	.22353	4.47374	.26193	4.82512	.33162	3.383906	.27921	3.58160	24	
37	.22383	4.46764	.26225	4.84736	.33262	3.383449	.27952	3.57758	23	
38	.22414	4.46155	.26255	4.86960	.33362	3.382992	.27985	3.57357	22	
39	.22444	4.45548	.26285	4.89184	.33462	3.382537	.28015	3.56957	21	
40	.22475	4.44942	.26316	4.91408	.33562	3.382085	.28046	3.56557	20	
41	.22505	4.44338	.26347	4.93632	.33662	3.381630	.28077	3.56159	19	
42	.22536	4.43735	.26377	4.95856	.33762	3.381177	.28109	3.55761	18	
43	.22567	4.43131	.26408	4.98080	.33862	3.380726	.28140	3.55364	17	
44	.22597	4.42528	.26439	5.00304	.33962	3.380276	.28172	3.54968	16	
45	.22628	4.41926	.26470	5.02528	.34062	3.379827	.28203	3.54575	15	
46	.22658	4.41325	.26501	5.04752	.34162	3.379378	.28234	3.54179	14	
47	.22689	4.40725	.26532	5.06976	.34262	3.378931	.28266	3.53785	13	
48	.22719	4.40125	.26562	5.09200	.34362	3.378485	.28297	3.53395	12	
49	.22750	4.39526	.26593	5.11424	.34462	3.378040	.28329	3.53001	11	
50	.22781	4.38926	.26624	5.13648	.34562	3.377595	.28360	3.52609	10	
51	.22811	4.38328	.26655	5.15872	.34662	3.377152	.28391	3.52219	9	
52	.22842	4.37730	.26686	5.18096	.34762	3.376709	.28425	3.51829	8	
53	.22872	4.37132	.26717	5.20320	.34862	3.376268	.28454	3.51441	7	
54	.22903	4.36535	.26747	5.22544	.34962	3.375828	.28486	3.51055	6	
55	.22934	4.35938	.26778	5.24768	.35062	3.375388	.28517	3.50666	5	
56	.22964	4.35342	.26809	5.26992	.35162	3.374950	.28549	3.50279	4	
57	.22995	4.34746	.26840	5.29216	.35262	3.374512	.28580	3.49894	3	
58	.23026	4.34150	.26871	5.31440	.35362	3.374075	.28612	3.49509	2	
59	.23056	4.33555	.26902	5.33664	.35462	3.373640	.28643	3.49125	1	
60	.23087	4.32960	.26933	5.35888	.35562	3.373205	.28675	3.48741	0	
Cotang		Tang	Cotang		Tang	Cotang		Tang	Cotang	
77°			76°			75°			74°	

TABLE V.—NATURAL TANGENTS AND COTANGENTS.

/	16°		17°		18°		19°		/
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
/	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	/
	73°		72°		71°		70°		

TABLE V.—NATURAL TANGENTS AND COTANGENTS.

/	20°		21°		22°		23°		/
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
/	69°		68°		67°		66°		/
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	

TABLE V.—NATURAL TANGENTS AND COTANGENTS.

/	24°		25°		26°		27°		/
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
/	65°		64°		63°		62°		/
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	

TABLE V.—NATURAL TANGENTS AND COTANGENTS.

/	28°		29°		30°		31°		/
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
/	61°		60°		59°		58°		/
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	

TABLE V.—NATURAL TANGENTS AND COTANGENTS.

/	32°		33°		34°		35°		/
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65024	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40713	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45591	.71285	1.40281	31
30	.63707	1.56968	.66189	1.51084	.68728	1.45502	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	/
/	57°		56°		55°		54°		/

TABLE V.—NATURAL TANGENTS AND COTANGENTS.

/	36°		37°		38°		39°		/
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29386	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
/	53°		52°		51°		50°		/
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	

TABLE V.—NATURAL TANGENTS AND COTANGENTS.

/	40°		41°		42°		43°		/
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
/	49°		48°		47°		46°		/
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	

TABLE V.—NATURAL TANGENTS AND COTANGENTS.

44°				44°				44°			
/	Tang		/	/	Tang		/	/	Tang		/
	Cotang				Cotang				Cotang		
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	20
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01112	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02653	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	4
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	3
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	2
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
/	Tang		/	/	Tang		/	/	Tang		/
	Cotang				Cotang				Cotang		
45°				45°				45°			

TABLE VI.—NATURAL VERSED SINES.

			3°	4°	5°	6°	7°	8°	9°
0° 00'	.00000	00'	.00137	.00244	.00381	.00548	.00745	.00973	.01231
05	.00000	01	139	246	383	551	749	977	236
10	.00000	02	140	248	386	554	752	981	240
15	.001	03	142	250	388	557	756	985	245
20	.002	04	143	252	391	560	760	989	249
25	.003	05	.00145	.00254	.00393	.00563	.00763	.00994	.01254
30	.00004	06	146	256	396	566	767	998	259
35	.005	07	148	258	398	569	770	.01002	263
40	.007	08	149	260	401	572	774	006	268
45	.009	09	151	262	404	576	778	010	272
50	.011	10	.00153	.00264	.00406	.00579	.00781	.01014	.01277
55	.013	11	154	266	409	582	785	018	282
1° 00'	.00013	12	156	269	412	585	789	022	286
05	.018	13	158	271	414	588	792	027	291
10	.021	14	159	273	417	591	796	031	296
15	.024	15	.00161	.00275	.00420	.00594	.00800	.01035	.01300
20	.027	16	162	277	422	598	803	039	305
25	.031	17	164	279	425	601	807	043	310
30	.00034	18	166	281	428	604	811	047	314
35	.038	19	167	284	430	607	814	052	319
40	.042	20	.00169	.00286	.00433	.00610	.00818	.01056	.01324
45	.047	21	171	288	436	614	822	060	329
50	.051	22	173	290	438	617	825	064	333
55	.056	23	174	292	441	620	829	069	338
2° 00'	.00061	24	176	295	444	623	833	073	343
05	.056	25	.00178	.00297	.00447	.00626	.00837	.01077	.01348
10	.060	26	179	299	449	630	840	081	352
15	.064	27	181	301	452	633	844	086	357
20	.068	28	183	304	455	636	848	090	362
25	.072	29	185	306	458	640	852	094	367
30	.076	30	.00187	.00308	.00460	.00643	.00856	.01098	.01371
35	.080	31	188	311	463	646	859	103	376
40	.084	32	190	313	466	649	863	107	381
45	.088	33	192	315	469	653	867	111	386
50	.092	34	194	317	472	656	871	116	391
55	.096	35	.00190	.00320	.00474	.00659	.00875	.01120	.01396
1° 05'	.00071	36	197	322	477	663	878	124	400
10	.074	37	199	324	480	666	882	129	405
15	.078	38	201	327	483	669	886	133	410
20	.081	39	203	329	486	673	890	137	415
25	.085	40	.00205	.00332	.00489	.00676	.00894	.01142	.01420
30	.089	41	207	334	492	680	898	146	425
35	.093	42	208	336	494	683	902	151	430
40	.097	43	210	339	497	686	906	155	435
45	.101	44	212	341	500	690	909	159	439
50	.105	45	.00214	.00343	.00503	.00693	.00913	.01164	.01444
55	.109	46	216	346	506	697	917	168	449
1° 10'	.113	47	218	348	509	700	921	173	454
15	.117	48	220	351	512	703	925	177	459
20	.121	49	222	353	515	707	929	182	464
25	.125	50	.00224	.00356	.00518	.00710	.00933	.01186	.01469
30	.129	51	226	358	521	714	937	191	474
35	.133	52	228	361	524	717	941	195	479
40	.137	53	230	363	527	721	945	200	484
45	.141	54	232	365	530	724	949	204	489
50	.145	55	.00234	.00368	.00533	.00728	.00953	.01209	.01494
55	.149	56	236	370	536	731	957	213	499
1° 15'	.153	57	238	373	539	735	961	218	504
20	.157	58	240	375	542	738	965	222	509
25	.161	59	242	378	545	742	969	227	514
30	.165	60	.00244	.00381	.00548	.00745	.00973	.01231	.01519

TABLE VI.—NATURAL VERSED SINES.

	10°	11°	12°	13°	14°	15°	16°	17°
00'	.01510	.01837	.02185	.02563	.02970	.03407	.03874	.04370
01	524	843	191	570	977	475	882	378
02	529	848	197	576	985	422	890	387
03	534	854	203	583	992	430	898	395
04	539	860	209	589	999	438	906	404
05	.01545	.01865	.02216	.02596	.03006	.03445	.03914	.04412
06	550	871	222	602	013	453	922	421
07	555	876	228	609	020	460	930	429
08	560	882	234	616	027	468	938	438
09	565	888	240	622	034	476	946	446
10	.01570	.01893	.02246	.02629	.03041	.03483	.03954	.04455
11	575	899	252	635	048	491	963	464
12	580	904	258	642	055	498	971	472
13	586	910	265	649	063	506	979	481
14	591	916	271	655	070	514	987	489
15	.01596	.01921	.02277	.02662	.03077	.03521	.03995	.04498
16	601	927	283	669	084	529	.04003	507
17	606	933	289	675	091	537	011	515
18	611	939	295	682	098	544	019	524
19	617	944	302	689	106	552	028	533
20	.01622	.01950	.02308	.02696	.03113	.03560	.04036	.04541
21	627	956	314	702	120	567	044	550
22	632	961	320	709	127	575	052	559
23	638	967	327	716	134	583	060	567
24	643	973	333	722	142	590	069	576
25	.01648	.01979	.02339	.02729	.03149	.03598	.04077	.04585
26	653	984	345	736	156	606	085	593
27	659	990	352	743	163	614	093	602
28	664	996	358	749	171	621	102	611
29	669	.02002	364	756	178	629	110	620
30	.01675	.02008	.02370	.02763	.03185	.03637	.04118	.04628
31	680	013	377	770	193	645	126	637
32	685	019	383	777	200	653	135	646
33	690	025	389	783	207	660	143	655
34	696	031	396	790	214	668	151	663
35	.01701	.02037	.02402	.02797	.03222	.03676	.04159	.04672
36	706	042	408	804	229	684	168	681
37	712	048	415	811	236	692	176	690
38	717	054	421	818	244	699	184	699
39	723	060	427	824	251	707	193	707
40	.01728	.02066	.02434	.02831	.03258	.03715	.04201	.04716
41	733	072	440	838	266	723	209	725
42	739	078	447	845	273	731	218	734
43	744	084	453	852	281	739	226	743
44	750	090	459	859	288	747	234	752
45	.01755	.02095	.02466	.02866	.03295	.03754	.04243	.04760
46	760	101	472	873	303	762	251	769
47	766	107	479	880	310	770	260	778
48	771	113	485	887	318	778	268	787
49	777	119	492	894	325	786	276	796
50	.01782	.02125	.02498	.02900	.03333	.03794	.04285	.04805
51	788	131	504	907	340	802	293	814
52	793	137	511	914	347	810	302	823
53	799	143	517	921	355	818	310	832
54	804	149	524	928	362	826	319	841
55	.01810	.02155	.02530	.02935	.03370	.03834	.04327	.04850
56	815	161	537	942	377	834	336	858
57	821	167	543	949	385	850	344	867
58	826	173	550	956	392	858	353	876
59	832	179	556	963	400	866	361	885
60	.01837	.02185	.02563	.02970	.03407	.03874	.04370	.04894

TABLE VII. — LENGTHS OF CIRCULAR ARCS: RADIUS

Sec.	Length.	Min.	Length.	Deg.	Length.	Deg.	Length.
1	.0000048	1	.0002909	1	.0174533	61	1.0646508
2	.0000097	2	.0005818	2	.0349066	62	1.0821041
3	.0000145	3	.0008727	3	.0523599	63	1.0995574
4	.0000194	4	.0011636	4	.0698132	64	1.1170107
5	.0000242	5	.0014544	5	.0872665	65	1.1344640
6	.0000291	6	.0017453	6	.1047198	66	1.1519173
7	.0000339	7	.0020362	7	.1221730	67	1.1693706
8	.0000388	8	.0023271	8	.1396263	68	1.1868239
9	.0000436	9	.0026180	9	.1570796	69	1.2042772
10	.0000485	10	.0029089	10	.1745329	70	1.2217305
11	.0000533	11	.0031998	11	.1919862	71	1.2391838
12	.0000582	12	.0034907	12	.2094395	72	1.2566371
13	.0000630	13	.0037815	13	.2268928	73	1.2740904
14	.0000679	14	.0040724	14	.2443461	74	1.2915436
15	.0000727	15	.0043633	15	.2617994	75	1.3089969
16	.0000776	16	.0046542	16	.2792527	76	1.3264502
17	.0000824	17	.0049451	17	.2967060	77	1.3439035
18	.0000873	18	.0052360	18	.3141593	78	1.3613568
19	.0000921	19	.0055269	19	.3316126	79	1.3788101
20	.0000970	20	.0058178	20	.3490659	80	1.3962634
21	.0001018	21	.0061087	21	.3665191	81	1.4137167
22	.0001067	22	.0063995	22	.3839724	82	1.4311700
23	.0001115	23	.0066904	23	.4014257	83	1.4486233
24	.0001164	24	.0069813	24	.4188790	84	1.4660766
25	.0001212	25	.0072722	25	.4363323	85	1.4835299
26	.0001261	26	.0075631	26	.4537856	86	1.5009832
27	.0001309	27	.0078540	27	.4712389	87	1.5184364
28	.0001357	28	.0081449	28	.4886922	88	1.5358897
29	.0001406	29	.0084358	29	.5061455	89	1.5533430
30	.0001454	30	.0087266	30	.5235988	90	1.5707963
31	.0001503	31	.0090175	31	.5410521	91	1.5882496
32	.0001551	32	.0093084	32	.5585054	92	1.6057029
33	.0001600	33	.0095993	33	.5759587	93	1.6231562
34	.0001648	34	.0098902	34	.5934119	94	1.6406095
35	.0001697	35	.0101811	35	.6108652	95	1.6580628
36	.0001745	36	.0104720	36	.6283185	96	1.6755161
37	.0001794	37	.0107629	37	.6457718	97	1.6929694
38	.0001842	38	.0110538	38	.6632251	98	1.7104227
39	.0001891	39	.0113446	39	.6806784	99	1.7278760
40	.0001939	40	.0116355	40	.6981317	100	1.7453293
41	.0001988	41	.0119264	41	.7155850	101	1.7627825
42	.0002036	42	.0122173	42	.7330383	102	1.7802358
43	.0002085	43	.0125082	43	.7504916	103	1.7976891
44	.0002133	44	.0127991	44	.7679449	104	1.8151424
45	.0002182	45	.0130900	45	.7853982	105	1.8325957
46	.0002230	46	.0133809	46	.8028515	106	1.8500490
47	.0002279	47	.0136717	47	.8203047	107	1.8675023
48	.0002327	48	.0139626	48	.8377580	108	1.8849556
49	.0002376	49	.0142535	49	.8552113	109	1.9024089
50	.0002424	50	.0145444	50	.8726646	110	1.9198622
51	.0002473	51	.0148353	51	.8901179	111	1.9373155
52	.0002521	52	.0151262	52	.9075712	112	1.9547688
53	.0002570	53	.0154171	53	.9250245	113	1.9722221
54	.0002618	54	.0157080	54	.9424778	114	1.9896753
55	.0002666	55	.0159989	55	.9599311	115	2.0071286
56	.0002715	56	.0162897	56	.9773844	116	2.0245819
57	.0002763	57	.0165806	57	.9948377	117	2.0420352
58	.0002812	58	.0168715	58	1.0122910	118	2.0594885
59	.0002860	59	.0171624	59	1.0297443	119	2.0769418
60	.0002909	60	.0174533	60	1.0471976	120	2.0943951

VERTICAL HEIGHTS

Min- utes	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
0...	0.00	1.74	3.49	5.23	6.96	8.68	10.40	12.10	13.78	15.45
2...	0.06	1.80	3.55	5.28	7.02	8.74	10.45	12.15	13.84	15.51
4...	0.12	1.86	3.60	5.34	7.07	8.80	10.51	12.21	13.89	15.56
6...	0.17	1.92	3.66	5.40	7.13	8.85	10.57	12.26	13.95	15.62
8...	0.23	1.98	3.72	5.46	7.19	8.91	10.62	12.32	14.01	15.67
10...	0.29	2.04	3.78	5.52	7.25	8.97	10.68	12.38	14.06	15.73
12...	0.35	2.09	3.84	5.57	7.30	9.03	10.74	12.43	14.12	15.78
14...	0.41	2.15	3.90	5.63	7.36	9.08	10.79	12.49	14.17	15.84
16...	0.47	2.21	3.95	5.69	7.42	9.14	10.85	12.55	14.23	15.89
18...	0.52	2.27	4.01	5.75	7.48	9.20	10.91	12.60	14.28	15.95
20...	0.58	2.33	4.07	5.80	7.53	9.25	10.96	12.66	14.34	16.00
22...	0.64	2.38	4.13	5.86	7.59	9.31	11.02	12.72	14.40	16.06
24...	0.70	2.44	4.18	5.92	7.65	9.37	11.08	12.77	14.45	16.11
26...	0.76	2.50	4.24	5.98	7.71	9.43	11.13	12.83	14.51	16.17
28...	0.81	2.56	4.30	6.04	7.76	9.48	11.19	12.88	14.56	16.22
30...	0.87	2.62	4.36	6.09	7.82	9.54	11.25	12.94	14.62	16.28
32...	0.93	2.67	4.42	6.15	7.88	9.60	11.30	13.00	14.67	16.33
34...	0.99	2.73	4.48	6.21	7.94	9.65	11.36	13.05	14.73	16.39
36...	1.05	2.79	4.53	6.27	7.99	9.71	11.42	13.11	14.79	16.44
38...	1.11	2.85	4.59	6.33	8.05	9.77	11.47	13.17	14.84	16.50
40...	1.16	2.91	4.65	6.38	8.11	9.83	11.53	13.22	14.90	16.55
42...	1.22	2.97	4.71	6.44	8.17	9.88	11.59	13.28	14.95	16.61
44...	1.28	3.02	4.76	6.50	8.22	9.94	11.64	13.33	15.01	16.66
46...	1.34	3.08	4.82	6.56	8.28	10.00	11.70	13.39	15.06	16.72
48...	1.40	3.14	4.88	6.61	8.34	10.05	11.76	13.45	15.12	16.77
50...	1.45	3.20	4.94	6.67	8.40	10.11	11.81	13.50	15.17	16.83
52...	1.51	3.26	4.99	6.73	8.45	10.17	11.87	13.56	15.23	16.88
54...	1.57	3.31	5.05	6.79	8.51	10.22	11.93	13.61	15.28	16.94
56...	1.63	3.37	5.11	6.84	8.57	10.28	11.98	13.67	15.34	16.99
58...	1.69	3.43	5.17	6.90	8.63	10.34	12.04	13.73	15.40	17.05
60...	1.74	3.49	5.23	6.96	8.68	10.40	12.10	13.78	15.45	17.10

HORIZONTAL CORRECTIONS

Dist.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
100..	0.0	0.0	0.1	0.3	0.5	0.8	1.1	1.5	1.9	2.5
200..	0.0	0.1	0.2	0.5	1.0	1.5	2.2	3.0	3.9	4.9
300..	0.0	0.1	0.4	0.8	1.5	2.3	3.3	4.5	5.8	7.4
400..	0.0	0.1	0.5	1.1	2.0	3.0	4.4	6.0	7.8	9.8
500..	0.0	0.2	0.6	1.4	2.5	3.8	5.5	7.5	9.7	12.3
600..	0.0	0.2	0.7	1.6	2.9	4.6	6.5	8.9	11.6	14.7
700..	0.0	0.2	0.8	1.9	3.4	5.3	7.6	10.4	13.6	17.2
800..	0.0	0.2	1.0	2.2	3.9	6.1	8.7	11.9	15.5	19.6
900..	0.0	0.3	1.1	2.4	4.4	6.8	9.8	13.4	17.5	22.1
1000..	0.0	0.3	1.2	2.7	4.9	7.6	10.9	14.9	19.4	24.5

TABLE VIII.—STADIA REDUCTIONS

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VERTICAL HEIGHTS

Min- utes	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°
0...	17.10	18.73	20.34	21.92	23.47	25.00	26.50	27.96	29.39	30.78
2...	17.16	18.78	20.39	21.97	23.52	25.05	26.55	28.01	29.44	30.83
4...	17.21	18.84	20.44	22.02	23.58	25.10	26.59	28.06	29.48	30.87
6...	17.26	18.89	20.50	22.08	23.63	25.15	26.64	28.10	29.53	30.92
8...	17.32	18.95	20.55	22.13	23.68	25.20	26.69	28.15	29.58	30.97
10...	17.37	19.00	20.60	22.18	23.73	25.25	26.74	28.20	29.62	31.01
12...	17.43	19.05	20.66	22.23	23.78	25.30	26.79	28.25	29.67	31.06
14...	17.48	19.11	20.71	22.28	23.83	25.35	26.84	28.30	29.72	31.10
16...	17.54	19.16	20.76	22.34	23.88	25.40	26.89	28.34	29.76	31.15
18...	17.59	19.21	20.81	22.39	23.93	25.45	26.94	28.39	29.81	31.19
20...	17.65	19.27	20.87	22.44	23.99	25.50	26.99	28.44	29.86	31.24
22...	17.70	19.32	20.92	22.49	24.04	25.55	27.04	28.49	29.90	31.28
24...	17.76	19.38	20.97	22.54	24.09	25.60	27.09	28.54	29.95	31.33
26...	17.81	19.43	21.03	22.60	24.14	25.65	27.13	28.58	30.00	31.38
28...	17.86	19.48	21.08	22.65	24.19	25.70	27.18	28.63	30.04	31.42
30...	17.92	19.54	21.13	22.70	24.24	25.75	27.23	28.68	30.09	31.47
32...	17.97	19.59	21.18	22.75	24.29	25.80	27.28	28.73	30.14	31.51
34...	18.03	19.64	21.24	22.80	24.34	25.85	27.33	28.77	30.19	31.56
36...	18.08	19.70	21.29	22.85	24.39	25.90	27.38	28.82	30.23	31.60
38...	18.14	19.75	21.34	22.91	24.44	25.95	27.43	28.87	30.28	31.65
40...	18.19	19.80	21.39	22.96	24.49	26.00	27.48	28.92	30.32	31.69
42...	18.24	19.86	21.45	23.01	24.55	26.05	27.52	28.96	30.37	31.74
44...	18.30	19.91	21.50	23.06	24.60	26.10	27.57	29.01	30.41	31.78
46...	18.35	19.96	21.55	23.11	24.65	26.15	27.62	29.06	30.46	31.83
48...	18.41	20.02	21.60	23.16	24.70	26.20	27.67	29.11	30.51	31.87
50...	18.46	20.07	21.66	23.22	24.75	26.25	27.72	29.15	30.55	31.92
52...	18.51	20.12	21.71	23.27	24.80	26.30	27.77	29.20	30.60	31.96
54...	18.57	20.18	21.76	23.32	24.85	26.35	27.81	29.25	30.65	32.01
56...	18.62	20.23	21.81	23.37	24.90	26.40	27.86	29.30	30.69	32.05
58...	18.68	20.28	21.87	23.42	24.95	26.45	27.91	29.34	30.74	32.09
60...	18.73	20.34	21.92	23.47	25.00	26.50	27.96	29.39	30.78	32.14

HORIZONTAL CORRECTIONS

Dist.	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°
100..	3.0	3.6	4.3	5.1	5.9	6.7	7.6	8.5	9.5	10.6
200..	6.0	7.3	8.6	10.1	11.7	13.4	15.2	17.1	19.1	21.2
300..	9.1	10.9	13.0	15.2	17.6	20.1	22.8	25.6	28.6	31.8
400..	12.1	14.6	17.3	20.2	23.4	26.8	30.4	34.2	38.2	42.4
500..	15.1	18.2	21.6	25.3	29.3	33.5	38.0	42.7	47.7	53.0
600..	18.1	21.8	25.9	30.4	35.1	40.2	45.6	51.3	57.3	63.6
700..	21.1	25.5	30.2	35.4	41.0	46.9	53.2	59.8	66.8	74.2
800..	24.2	29.1	34.6	40.5	46.8	53.6	60.8	68.4	76.4	84.8
900..	27.2	32.8	38.9	45.5	52.7	60.3	68.4	76.9	85.9	95.4
1000..	30.2	36.4	43.2	50.6	58.5	67.0	76.0	85.5	95.3	106.0

TABLE VIII.—STADIA REDUCTIONS

VERTICAL HEIGHTS

Min- utes	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°
0...	32.14	33.46	34.73	35.97	37.16	38.30	39.40	40.45	41.45	42.40
2...	32.18	33.50	34.77	35.01	37.20	38.34	39.44	40.49	41.48	42.43
4...	32.23	33.54	34.82	36.05	37.23	38.38	39.47	40.52	41.52	42.46
6...	32.27	33.59	34.86	36.09	37.27	38.41	39.51	40.55	41.55	42.49
8...	32.32	33.63	34.90	36.13	37.31	38.45	39.54	40.59	41.58	42.53
10...	32.36	33.67	34.94	36.17	37.35	38.49	39.58	40.62	41.61	42.56
12...	32.41	33.72	34.98	36.21	37.39	38.53	39.61	40.66	41.65	42.59
14...	32.45	33.76	35.02	36.25	37.43	38.56	39.65	40.69	41.68	42.62
16...	32.49	33.80	35.07	36.29	37.47	38.60	39.69	40.72	41.71	42.65
18...	32.54	33.84	35.11	36.33	37.51	38.64	39.72	40.76	41.74	42.68
20...	32.58	33.89	35.15	36.37	37.54	38.67	39.76	40.79	41.77	42.71
22...	32.63	33.93	35.19	36.41	37.58	38.71	39.79	40.82	41.81	42.74
24...	32.67	33.97	35.23	36.45	37.62	38.75	39.83	40.86	41.84	42.77
26...	32.72	34.01	35.27	36.49	37.66	38.78	39.86	40.89	41.87	42.80
28...	32.76	34.06	35.31	36.53	37.70	38.82	39.90	40.92	41.90	42.83
30...	32.80	34.10	35.36	36.57	37.74	38.86	39.93	40.96	41.93	42.86
32...	32.85	34.14	35.40	36.61	37.77	38.89	39.97	40.99	41.97	42.89
34...	32.89	34.18	35.44	36.65	37.81	38.93	40.00	41.02	42.00	42.92
36...	32.93	34.23	35.48	36.69	37.85	38.97	40.04	41.06	42.03	42.95
38...	32.98	34.27	35.52	36.73	37.89	39.00	40.07	41.09	42.06	42.98
40...	33.02	34.31	35.56	36.77	37.93	39.04	40.11	41.12	42.09	43.01
42...	33.07	34.35	35.60	36.80	37.96	39.08	40.14	41.16	42.12	43.04
44...	33.11	34.40	35.64	36.84	38.00	39.11	40.18	41.19	42.15	43.07
46...	33.15	34.44	35.68	36.88	38.04	39.15	40.21	41.22	42.19	43.10
48...	33.20	34.48	35.72	36.92	38.08	39.18	40.24	41.26	42.22	43.13
50...	33.24	34.52	35.76	36.96	38.11	39.22	40.28	41.29	42.25	43.16
52...	33.28	34.57	35.80	37.00	38.15	39.26	40.31	41.32	42.28	43.18
54...	33.33	34.61	35.85	37.04	38.19	39.29	40.35	41.35	42.31	43.21
56...	33.37	34.65	35.89	37.08	38.23	39.33	40.38	41.39	42.34	43.24
58...	33.41	34.69	35.93	37.12	38.26	39.36	40.42	41.42	42.37	43.27
60...	33.46	34.73	35.97	37.16	38.30	39.40	40.45	41.45	42.40	43.30

HORIZONTAL CORRECTIONS

Dist.	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°
100..	11.7	12.8	14.0	15.3	16.5	17.9	19.2	20.6	22.0	23.5
200..	23.4	25.7	28.1	30.5	33.1	35.7	38.4	41.2	44.1	47.0
300..	35.1	38.5	42.1	45.8	49.6	53.6	57.7	61.8	66.1	70.5
400..	46.8	51.4	56.1	61.1	66.2	71.4	76.9	82.4	88.2	94.0
500..	58.5	64.2	70.2	76.4	82.7	89.3	96.1	103.1	110.2	117.5
600..	70.2	77.0	84.2	91.6	99.2	107.2	115.3	123.7	132.2	141.0
700..	81.9	89.9	98.2	106.9	115.8	125.0	134.5	144.3	154.3	164.5
800..	93.6	102.7	112.2	122.2	132.3	142.9	153.8	164.9	176.3	188.0
900..	105.3	115.6	126.3	137.4	148.9	160.7	173.0	185.5	198.4	211.5
1000..	117.0	128.4	140.3	152.7	165.4	178.6	192.2	206.1	220.4	235.0

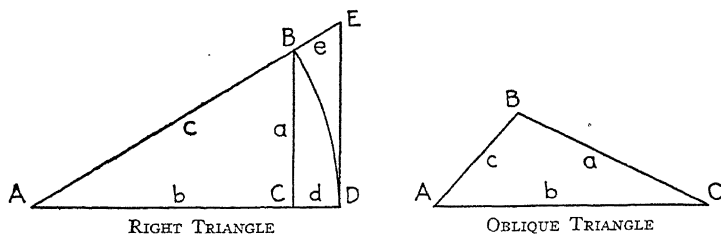
VERTICAL HEIGHTS.

Minutes	30°	31°	32°	33°	34°	35°	36°	37°	38°	39°
0....	43.30	44.15	44.94	45.68	46.36	46.98	47.55	48.06	48.52	48.91
2....	43.33	44.17	44.97	45.70	46.38	47.00	47.57	48.08	48.53	48.92
4....	43.36	44.20	44.99	45.72	46.40	47.02	47.59	48.10	48.54	48.93
6....	43.39	44.23	45.02	45.75	46.42	47.04	47.61	48.11	48.56	48.94
8....	43.42	44.26	45.04	45.77	46.45	47.06	47.62	48.13	48.57	48.96
10....	43.45	44.28	45.07	45.80	46.47	47.08	47.64	48.14	48.58	48.97
12....	43.47	44.31	45.09	45.82	46.49	47.10	47.66	48.16	48.60	48.98
14....	43.50	44.34	45.12	45.84	46.51	47.12	47.68	48.17	48.61	48.99
16....	43.52	44.36	45.14	45.86	46.53	47.14	47.69	48.19	48.63	49.00
18....	43.56	44.39	45.17	45.89	46.55	47.16	47.71	48.21	48.64	49.01
20....	43.59	44.42	45.19	45.91	46.57	47.18	47.73	48.22	48.65	49.03
22....	43.62	44.44	45.22	45.93	46.60	47.20	47.75	48.24	48.67	49.04
24....	43.65	44.47	45.24	45.96	46.62	47.22	47.76	48.25	48.68	49.05
26....	43.67	44.50	45.27	45.98	46.64	47.24	47.78	48.27	48.69	49.06
28....	43.70	44.52	45.29	46.00	46.66	47.26	47.80	48.28	48.71	49.07
30....	43.73	44.55	45.32	46.03	46.68	47.28	47.82	48.30	48.72	49.08
32....	43.76	44.58	45.34	46.05	46.70	47.30	47.83	48.31	48.73	49.09
34....	43.79	44.60	45.36	46.07	46.72	47.31	47.85	48.33	48.74	49.10
36....	43.82	44.63	45.39	46.09	46.74	47.33	47.87	48.34	48.76	49.11
38....	43.84	44.66	45.41	46.12	46.76	47.35	47.88	48.36	48.77	49.13
40....	43.87	44.68	45.44	46.14	46.78	47.37	47.90	48.37	48.78	49.14
42....	43.90	44.71	45.46	46.16	46.80	47.39	47.92	48.39	48.80	49.15
44....	43.93	44.74	45.49	46.18	46.82	47.41	47.93	48.40	48.81	49.16
46....	43.95	44.76	45.51	46.21	46.84	47.43	47.95	48.41	48.82	49.17
48....	43.98	44.79	45.53	46.23	46.86	47.44	47.97	48.43	48.83	49.18
50....	44.01	44.81	45.56	46.25	46.88	47.46	47.98	48.44	48.85	49.19
52....	44.04	44.84	45.58	46.27	46.90	47.48	48.00	48.46	48.86	49.20
54....	44.07	44.86	45.61	46.29	46.92	47.50	48.01	48.47	48.87	49.21
56....	44.09	44.89	45.63	46.32	46.94	47.52	48.03	48.49	48.88	49.22
58....	44.12	44.91	45.65	46.34	46.96	47.54	48.05	48.50	48.90	49.23
60....	44.15	44.94	45.68	46.36	46.98	47.55	48.06	48.52	48.91	49.24

HORIZONTAL CORRECTIONS.

Dist.	30° 00'	30° 30'	31° 00'	31° 30'	32° 00'	32° 30'	33° 00'	33° 30'	34° 00'	34° 30'
100....	25.0	25.8	26.5	27.3	28.1	28.9	29.7	30.5	31.3	32.1
200....	50.0	51.5	53.1	54.6	56.2	57.7	59.3	60.9	62.5	64.2
300....	75.0	77.3	79.6	81.9	84.2	86.6	89.0	91.4	93.8	96.2
400....	100.0	103.0	106.1	109.2	112.3	115.5	118.6	121.8	125.1	128.3
500....	125.0	128.8	132.6	136.5	140.4	144.3	148.3	152.3	156.3	160.4
Dist.	35° 00'	35° 30'	36° 00'	36° 30'	37° 00'	37° 30'	38° 00'	38° 30'	39° 00'	39° 30'
100....	32.9	33.7	34.6	35.4	36.2	37.1	37.9	38.7	39.6	40.5
200....	65.8	67.4	69.1	70.8	72.4	74.1	75.8	77.5	79.2	80.9
300....	98.7	101.2	103.7	106.1	108.7	111.2	113.7	116.2	118.8	121.4
400....	131.6	134.9	138.2	141.5	144.9	148.2	151.6	155.0	158.4	161.8
500....	164.5	168.6	172.8	176.9	181.1	185.3	189.5	193.7	198.0	202.3

TABLE IX. TRIGONOMETRIC FORMULAS.



SOLUTION OF RIGHT TRIANGLES.

$$\begin{aligned} \sin A &= \frac{a}{c} = \cos B & \cos A &= \frac{b}{c} = \sin B & \text{vers } A &= \frac{c-b}{c} = \frac{d}{c} \\ \tan A &= \frac{a}{b} = \cot B & \cot A &= \frac{b}{a} = \tan B \\ \sec A &= \frac{c}{b} = \text{cosec } B & \text{cosec } A &= \frac{c}{a} = \sec B & \text{exsec } A &= \frac{e}{c} \\ a &= c \sin A = b \tan A = c \cos B = b \cot B = \sqrt{(c+b)(c-b)} \\ b &= c \cos A = a \cot A = c \sin B = a \tan B = \sqrt{(c+a)(c-a)} = c - c \text{ vers } A \\ d &= c \text{ vers } A & e &= c \text{ exsec } A \\ c &= \frac{a}{\cos B} = \frac{b}{\sin B} = \frac{a}{\sin A} = \frac{b}{\cos A} = \frac{d}{\text{vers } A} = \frac{e}{\text{exsec } A} = b + b \text{ exsec } A \end{aligned}$$

SOLUTION OF OBLIQUE TRIANGLES.

Given.	Sought.	Formulas.
A, B, a	b, c	$b = \frac{a}{\sin A} \cdot \sin B, \quad c = \frac{a}{\sin A} \sin (A + B)$
A, a, b	B, c	$\sin B = \frac{\sin A}{a} \cdot b, \quad c = \frac{a}{\sin A} \cdot \sin C.$
C, a, b	$A - B$	$\tan \frac{1}{2} (A - B) = \frac{a-b}{a+b} \tan \frac{1}{2} (A + B)$
a, b, c	A	$\cos A = (b^2 + c^2 - a^2) \div 2bc$ $\text{If } s = \frac{1}{2} (a + b + c), \sin \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{bc}}$ $\cos \frac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}}; \tan \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ $\sin A = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{bc};$ $\text{vers } A = \frac{2(s-b)(s-c)}{bc}$
	area	$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$
A, B, C, a	area	$\text{area} = \frac{a^2 \sin B \cdot \sin C}{2 \sin A}$
C, a, b	area	$\text{area} = \frac{1}{2} a b \sin C.$

TABLE IX. TRIGONOMETRIC FORMULAS.

GENERAL TRIGONOMETRIC FORMULAS.

$$\sin A = 2 \sin \frac{1}{2} A \cos \frac{1}{2} A = \sqrt{1 - \cos^2 A} = \tan A \cos A = \sqrt{\frac{1}{2}(1 - \cos 2A)}$$

$$\cos A = 2 \cos^2 \frac{1}{2} A - 1 = 1 - 2 \sin^2 \frac{1}{2} A = \cos^2 \frac{1}{2} A - \sin^2 \frac{1}{2} A = 1 - \text{vers } A$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\sqrt{1 - \cos^2 A}}{\cos A} = \frac{\sin 2A}{1 + \cos 2A}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\sin 2A}{1 - \cos 2A} = \frac{\sin 2A}{\text{vers } 2A}$$

$$\text{vers } A = 1 - \cos A = \sin A \tan \frac{1}{2} A = 2 \sin^2 \frac{1}{2} A$$

$$\text{exsec } A = \sec A - 1 = \tan A \tan \frac{1}{2} A = \frac{\text{vers } A}{\cos A}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2 \cos^2 A - 1 = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A$$

$$\sin^2 A + \cos^2 A = 1$$

TABLE X. GEOMETRIC FORMULAS.

Required.	Given.	Formulas.
Area of		
Circle	Radius = r	πr^2
Sector of Circle	Radius = r , Arc = L_c	$\frac{L_c}{2}$
Segment of Circle	chord = C , Middle Ordinate = M	$\frac{2}{3} CM$ (Approximate)
Ellipse	Semi-axes = a and b	πab
Surface of		
Cone	Radius of Base = r ; Slant Height = s	πrs
Cylinder	Radius = r , Height = h	$2\pi rh$
Sphere	Radius = r	$4\pi r^2$
Zone	Radius of Sphere = r , Height of Zone = h	$2\pi rh$
Volume of		
Prism or Cylinder	Area of Base = b ; Height = h	bh
Pyramid or Cone	Area of Base = b ; Height = h	$\frac{bh}{3}$
Frustum of Pyramid or Cone	Area of bases = b and b' ; Height = h	$\frac{h}{3}(b + b' + \sqrt{bb'})$
Sphere	Radius = r	$\frac{4}{3}\pi r^3$

TABLE XI. LINEAR MEASURE.

1 foot	= 12 inches
1 yard	= 3 feet
1 rod	= $5\frac{1}{2}$ yards = $16\frac{1}{2}$ feet
1 mile	= 320 rods = 1760 yards : : 5280 feet
1 furlong	= 660 feet = 220 yards
1 fathom	= 6 feet = 2 yards

TABLE XII. SQUARE MEASURE.

1 sq. foot	= 144 sq. inches
1 sq. yard	= 9 sq. feet = 1296 sq. inches
1 sq. rod	= $30\frac{1}{4}$ sq. yards = $272\frac{1}{4}$ sq. feet
1 acre	= 160 sq. rods = 4840 sq. yards = 43,560 sq. feet
1 sq. mile	= 640 acres = 102,400 sq. rods = 27,878,400 sq. feet

TABLE XIII. LINEAR MEASURE — METRIC SYSTEM.

1 myriameter	= : 10 kilometers
1 kilometer	= : 10 hectometers
1 hectometer	= : 10 decameters
1 decameter	= : 10 meters
1 meter	= : 10 decimeters
1 decimeter	= : 10 centimeters
1 centimeter	= : 10 millimeters

TABLE XIV. SQUARE MEASURE — METRIC SYSTEM.

1 centare	= 1 sq. meter
1 are	= 100 sq. meters
1 hectare	= 100 ares = 10,000 sq. meters

TABLE XV. CONSTANTS.

	Number.	Logarithm.
Ratio of circumference to diameter (π)	3.14159	0.49715
Base of hyperbolic logarithms	2.71828	0.43429
Modulus of common system of logs	0.43429	9.63778-10
Acceleration due to gravity at N. Y.	32.15949	1.50731
Cubic inches in 1 U. S. gallon	231	2.36361
Cubic feet in 1 U. S. gallon	0.1337	9.12613-10
Cubic centimeters in 1 U. S. gallon	3785.	3.57807
U. S. gallons in 1 cubic foot	7.4805	0.87393
U. S. gallons in 1 Imperial gallon	1.201	0.07954
Pounds of water in 1 cubic foot	62.5	1.79588
Pounds of water in 1 U. S. gallon	8.355	0.92195
Pounds per square inch due to 1 atmosphere	14.7	1.16732
Pounds per square inch due to 1 foot head of water	0.434	9.63749-10
Feet of head for pressure of 1 pound per square inch	2.304	0.36248
Inches in 1 centimeter	0.3937	9.59517-10
Centimeters in 1 inch	2.5400	0.40483
Feet in 1 meter	3.2808	0.51598
Meters in 1 foot	0.3048	9.48402-10
Miles in 1 kilometer	0.62137	9.79335-10
Kilometers in 1 mile	1.60935	0.20665
Statute miles in 1 nautical mile (knot)	1.15156	0.06129
Square inches in 1 square centimeter	0.1550	9.19033-10
Square centimeters in 1 square inch	6.4520	0.80969
Square feet in 1 square meter	10.764	1.03197
Square meters in 1 square foot	0.09290	8.96802-10
Cubic feet in 1 cubic meter	35.3156	1.54797
Pounds (av.) in 1 kilogram	2.2046	0.34333
Kilograms in 1 pound (av.)	0.4536	9.65667-10
Ft.-lbs. in 1 kilogram-meter	7.23308	0.85932

APPROXIMATE VALUES OF SINES.

$$\text{Natural sine of } 1^\circ = \frac{1.75 \text{ ft.}}{100 \text{ ft.}} = \frac{1}{60} \text{ (roughly)}$$

$$\text{Natural sine of } 0^\circ 1' = \frac{0.03 \text{ ft.}}{100 \text{ ft.}}$$

$$\text{Natural sine of } 0^\circ 00' 01'' = \frac{0.3 \text{ inch}}{1 \text{ mile}}$$

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